Higher Nationals -
Internal verification of assessment decisions – BTEC (RQF)

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<tr><td>Programme title</td><td colspan="4">BTEC Higher National Diploma in Computing</td></tr>
<tr><td>Assessor</td><td>Mrs.Shyamali</td><td>Internal Verifier</td><td colspan="2"></td></tr>
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<tr><td rowspan="2">List which assessment criteria the Assessor has awarded.</td><td>Pass</td><td>Merit</td><td colspan="2">Distinction</td></tr>
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<tr><td>Do the assessment criteria awarded match those shown in the assignment brief?</td><td>Y/N</td><td colspan="3"></td></tr>
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<tr><td>Is the feedback to the student:
Give details:
• Constructive?
• Linked to relevant assessment criteria?
• Identifying opportunities for improved performance?
• Agreeing actions?</td><td>Y/N
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Y/N</td><td colspan="3"></td></tr>
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Confirm action completed			
Remedial action taken Give details:			
Assessor signature		**Date**	
Internal Verifier		**Date**	
Programme Leader signature (if		**Date**	

Higher Nationals – Summative Assignment Feedback Form

Student Name/ID	Chandratheebam Suresh Gobi/ KAN00078776		
Unit Title	Unit 11 : Maths for Computing		
Assignment Number	1	**Assessor**	Mrs.Shyamali
Submission Date	28/06/2022	**Date Received 1st submission**	
Re-submission Date		**Date Received 2nd submission**	

Assessor Feedback:

LO1 Use applied number theory in practical computing scenarios.

Pass, Merit & Distinction Descripts	P1 ☐	P2 ☐	M1 ☐	D1 ☐

LO2 Analyse events using probability theory and probability distributions.

Pass, Merit & Distinction Descripts	P3 ☐	P4 ☐	M2 ☐	D2 ☐

LO3 Determine solutions of graphical examples using geometry and vector methods.

Pass, Merit & Distinction Descripts	P5 ☐	P6 ☐	M3 ☐	D3 ☐

LO4 Evaluate problems concerning differential and integral calculus.

Pass, Merit & Distinction Descripts	P7 ☐	P8 ☐	M4 ☐	D4 ☐

Grade:	**Assessor Signature:**	**Date:**
Resubmission Feedback:		
Grade:	**Assessor Signature:**	**Date:**
Internal Verifier's Comments:		
Signature & Date:		

* Please note that grade decisions are provisional. They are only confirmed once internal and external moderation has taken place and grades decisions have been agreed at the assessment board.

General Guidelines

1. A Cover page or title page – You should always attach a title page to your assignment. Use previous page as your cover sheet and make sure all the details are accurately filled.
2. Attach this brief as the first section of your assignment.
3. All the assignments should be prepared using a word processing software.
4. All the assignments should be printed on A4 sized papers. Use single side printing.
5. Allow 1" for top, bottom, right margins and 1.25" for the left margin of each page.

Word Processing Rules

1. The font size should be **12** point, and should be in the style of **Time New Roman**.
2. **Use 1.5 line spacing**. Left justify all paragraphs.
3. Ensure that all the headings are consistent in terms of the font size and font style.
4. Use **footer function in the word processor to insert Your Name, Subject, Assignment No, and Page Number on each pag**e. This is useful if individual sheets become detached for any reason.
5. Use word processing application spell check and grammar check function to help editing your assignment.

Important Points:

1. It is strictly prohibited to use textboxes to add texts in the assignments, except for the compulsory information. eg: Figures, tables of comparison etc. Adding text boxes in the body except for the before mentioned compulsory information will result in rejection of your work.
2. Avoid using page borders in your assignment body.
3. Carefully check the hand in date and the instructions given in the assignment. Late submissions will not be accepted.
4. Ensure that you give yourself enough time to complete the assignment by the due date.
5. Excuses of any nature will not be accepted for failure to hand in the work on time.
6. You must take responsibility for managing your own time effectively.
7. If you are unable to hand in your assignment on time and have valid reasons such as illness, you may apply (in writing) for an extension.
8. Failure to achieve at least PASS criteria will result in a REFERRAL grade .
9. Non-submission of work without valid reasons will lead to an automatic RE FERRAL. You will then be asked to complete an alternative assignment.
10. If you use other people's work or ideas in your assignment, reference them properly using HARVARD referencing system to avoid plagiarism. You have to provide both in-text citation and a reference list.
11. If you are proven to be guilty of plagiarism or any academic misconduct, your grade could be reduced to A REFERRAL or at worst you could be expelled from the course.

Student Declaration

I hereby, declare that I know what plagiarism entails, namely, to use another's work and to present it as my own without attributing the sources in the correct way. I further understand what it means to copy another's work.

1. I know that plagiarism is a punishable offence because it constitutes theft.
2. I understand the plagiarism and copying policy of the Edexcel UK.
3. I know what the consequences will be if I plagiaries or copy another's work in any of the assignments for this program.
4. I declare therefore that all work presented by me for every aspects of my program, will be my own, and where I have made use of another's work, I will attribute the source in the correct way.
5. I acknowledge that the attachment of this document signed or not, constitutes a binding agreement between myself and Edexcel UK.
6. I understand that my assignment will not be considered as submitted if this document is not attached to the attached.

Student's Signature: **sureshgobi34@gmail.com**

Date: **28/06/2022**

Feedback Form

Formative Feedback : Assessor to Student			
The given answers are correct.			
Action Plan			
Summative feedback			
Feedback: Student to Assessor.			
Assessor's Signature		**Date**	
Student's Signature	**sureshgobi34@gmail.com**	**Date**	28/06/2022

Assignment Brief

Student Name /ID Number	Chandratheebam Suresh Gobi/ KAN00078776
Unit Number and Title	Unit 11 : Maths for Computing
Academic Year	2021/2022
Unit Tutor	Mrs.Shyamali
Assignment Title	Importance of Maths in the Field of Computing
Issue Date	31/01/2022
Submission Date	28/06/2022
IV Name & Date	
Submission Format:	
This assignment should be submitted at the end of your lesson, on the week stated at the front of this brief. The assignment can either be word-processed or completed in legible handwriting. If the tasks are completed over multiple pages, ensure that your name and student number are present on each sheet of paper.	

Unit Learning Outcomes:
LO1 Use applied number theory in practical computing scenarios. **LO2** Analyse events using probability theory and probability distributions. **LO3** Determine solutions of graphical examples using geometry and vector methods. **LO4** Evaluate problems concerning differential and integral calculus.

Assignment Brief and Guidance:

Activity 01

Part 1

1. A tailor wants to make square shaped towels. The required squared pieces of cloth will be cut from a ream of cloth which is 20 meters in length and 16 meters in width.
   a) Find the minimum number of squared pieces that can be cut from the ream of cloth without wasting any cloth.
   b) Briefly explain the technique you used to solve (a).

2. On the first day of the month, 4 customers come to a restaurant. Afterwards, those 4 customers come to the same restaurant once in 2,4,6 and 8 days respectively.
   a) On which day of the month, will all the four customers come back to the restaurant together?
   b) Briefly explain the technique you used to solve (a).

Part 2

3. Logs are stacked in a pile with 24 logs on the bottom row and 10 on the top row. There are 15 rows in all with each row having one more log than the one above it.
   a) How many logs are in the stack?
   b) Briefly explain the technique you used to solve (a).

4. A company is offering a job with a salary of Rs. 50,000.00 for the first year and a 4% raise each year after that. If that 4% raise continues every year,
   a) Find the total amount of money an employee would earn in a 10-years career.
   b) Briefly explain the technique you used to solve (a).

Part 3

5. Define the multiplicative inverse in modular arithmetic and identify the multiplicative inverse of 6 mod 13 while explaining the algorithm used.

6. Prime numbers are important to many fields. In the computing field also prime numbers are applied. Provide examples and in detail explain how prime numbers are important in the field of computing.

Activity 02

Part 1

1. Define 'Conditional Probability' with a suitable example.

2. The manager of a supermarket collected the data of 25 customers on a certain date. Out of them 5 purchased Biscuits, 10 purchased Milk, 8 purchased Fruits, 6 purchased both Milk and Fruits.
   *Let **B represents the randomly selected customer purchased Biscuits, M represents the randomly selected customer purchased Milk** and **F represents the randomly selected customer purchased Fruits.***
   Represent the given information in a Venn diagram. Use that Venn diagram to answer the following questions.
   a) Find the probability that a randomly selected customer either purchased Biscuits or Milk.
   b) Show that the events "The randomly selected customer purchased Milk" and "The randomly selected customer purchased Fruits" are independent.

3. Suppose a voter poll is taken in three states. Of the total population of the three states, 45% live in state A, 20% live in state B, and 35% live in state C. In state A, 40% of voters support the liberal candidate, in state B, 30% of the voters support the liberal candidate, and in state C, 60% of the voters support the liberal candidate.
   *Let **A represents the event that voter is from state A**, **B represents the event that voter is from state B** and **C represents the event that voter is from state C**. Let **L represents the event that a voter supports the liberal candidate**.*
   a) Find the probability that a randomly selected voter does not support the liberal candidate and lives in state A.
   b) Find the probability that a randomly selected voter supports the liberal candidate.
   c) Given that a randomly selected voter supports the liberal candidate, find the probability that the selected voter is from state B.

4. In a box, there are 4 types [Hearts, Clubs, Diamonds, Scorpions] of cards. There are 6 Hearts cards, 7 Clubs cards, 8 Diamonds cards and 5 Scorpions cards in the box. Two cards are selected randomly <u>without replacement</u>.
   a) Find the probability that the both selected cards are Hearts.
   b) Find the probability that one card is Clubs and the other card is Diamonds.
   c) Find the probability that the both selected cards are from the same type.

Part 2

5. Differentiate between 'Discrete Random Variable' and 'Continuous Random Variable".

6. Two fair cubes are rolled. The random variable X represents the difference between the values of the two cubes.

   a) Find the mean of this probability distribution. (i.e. Find E[X])
   b) Find the variance and standard deviation of this probability distribution. (i.e. Find V[X] and SD[X])

   The random variables A and B are defined as follows:
   A = X-10 and B = [(1/2)X]-5
   c) Show that E[A] and E[B].
   d) Find V[A] and V[B].
   e) Arnold and Brian play a game using two fair cubes. The cubes are rolled, and Arnold records his score using the random variable A and Brian uses the random variable B. They repeat this for a large number of times and compare their scores. Comment on any likely differences or similarities of their scores.

7. A discrete random variable Y has the following probability distribution.

Y=y	1	2	3	4	5
P(Y=y)	1/3	1/6	1/4	k	1/6

   where k is a constant.
   a) Find the value of k.
   b) Find $P(Y\leq 3)$.
   c) Find $P(Y>2)$.

Part 3

10. The "Titans" cricket team has a winning rate of 75%. The team is planning to play 10 matches in the next season.

    a) Let X be the number of matches that will be won by the team. What are the possible values of X?
    b) What is the probability that the team will win exactly 6 matches?
    c) What is the probability that the team will lose 2 or less matches?
    d) What is the mean number of matches that the team will win?
    e) What are the variance and the standard deviation of the number of matches that the team will win?

11. In a boys' school, there are 45 students in grade 10. The height of the students was measured. The mean height of the students was 154 cm and the standard deviation was 2 cm. Alex's height was 163 cm. Would his height be considered an outlier, if the height of the students were normally distributed? Explain your answer.

12. The battery life of a certain battery is normally distributed with a mean of 90 days and a standard deviation of 3 days.

    For each of the following questions, construct a normal distribution curve and provide the answer.

    a) About what percent of the products last between 87 and 93 days?
    b) About what percent of the products last 84 or less days?

    For each of the following questions, use the standard normal table and provide the answer.

    c) About what percent of the products last between 89 and 94 days?
    d) About what percent of the products last 95 or more days?

13. In the computing field, there are many applications of Probability theories. Hashing and Load Balancing are also included to those. Provide an example for an application of Probability in Hashing and an example for an application of Probability in Load Balancing. Then, evaluate in detail how Probability is used for each application while assessing the importance of using Probability to those applications.

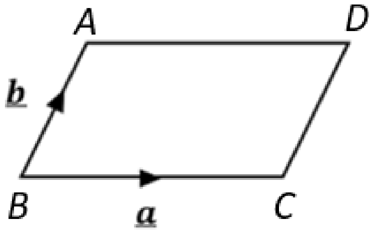
Activity 03

Part 1

1. Find the equation (formula) of a circle with radius r and center C(h,k) and if the Center of a circle is at (3,-1) and a point on the circle is (-2,1) find the formula of the circle.

2. Find the equation (formula) of a sphere with radius r and center C(h, k, l) and show that $x^2 + y^2 + z^2 - 6x + 2y + 8z - 4 = 0$ is an equation of a sphere. Also, find its center and radius.

3. Following figure shows a Parallelogram.



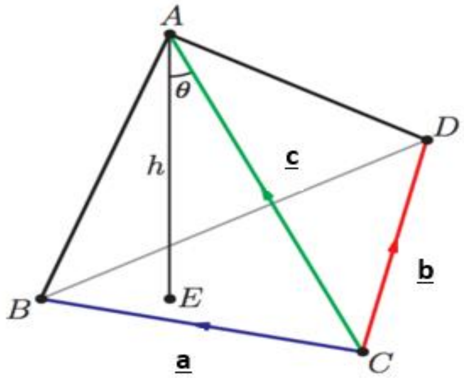
If **a**=(i+3j-k) , **b**=(7i-2j+4k), find the area of the Parallelogram.

Part 2

4. If 2x - 4y =3, 5y = (-3)x + 10 are two functions. Evaluate the x, y values using graphical method.

5. Evaluate the surfaces in $\mathbb{R}^3$ that are represented by the following equations.
   i. y = 4
   ii. z = 5

6. Following figure shows a Tetrahedron.



Construct an equation to find the volume of the given Tetrahedron using vector methods and if the vectors of the Tetrahedron are **a**=(i+4j-2k) , **b**=(3i-5j+k) and **c**=(-4i+3j+6k), find the volume of the Tetrahedron using the above constructed equation..

Activity 04

Part 1

1. Determine the slope of the following functions.
   i. $f(x) = 2x - 3x^4 + 5x + 8$
   ii. $f(x) = \cos(2x) + 4x^2 - 3$

2. Let the displacement function of a moving object is $S(t) = 5t^3 - 3t^2 + 6t$. What is the function for the velocity of the object at time t.

Part 2

3. Find the area between the two curves $f(x) = 2x^2 + 1$ and $g(x) = 8 - 2x$ on the interval $(-2) \leq x \leq 1$.

4. It is estimated that *t* years from now the tree plantation of a certain forest will be increasing at the rate of $3t^2 + 5t + 6$ hundred trees per year. Environmentalists have found that the level of Oxygen in the forest increases at the rate of approximately 4 units per 100 trees. By how much will the Oxygen level in the forest increase during the next 3 years?

Part 3

5. Sketch the graph of $f(x) = x^5 - 6x^3 + 3$ by applying differentiation methods for analyzing where the graph is increasing/decreasing, local maximum/minimum points [Using the second derivative test], concave up/down intervals with inflection points.

6. Identify the maximum and minimum points of the function $f(x)= 2x^3 - 4x^4 + 5x^2$ by further differentiation. [i.e Justify your answer using both first derivative test and second derivative test.]

Grading Rubric

Grading Criteria	Achievement (Yes/No)	Feedback
LO1 : Use applied number theory in practical computing scenarios.		
P1 : Calculate the greatest common divisor and least common multiple of a given pair of numbers.		
P2 : Use relevant theory to sum arithmetic and geometric progressions.		
M1 : Identify multiplicative inverses in modular arithmetic.		
D1 : Produce a detailed written explanation of the importance of prime numbers within the field of computing.		
LO2 : Analyse events using probability theory and probability distributions.		
P3 : Deduce the conditional probability of different events occurring within independent trials.		
P4 : Identify the expectation of an event occurring from a discrete, random variable.		
M2 : Calculate probabilities within both binomially distributed and normally distributed random variables.		
D2 : Evaluate probability theory to an example involving hashing and load balancing.		

LO3 : Determine solutions of graphical examples using geometry and vector methods.		
P5 : Identify simple shapes using co-ordinate geometry.		
P6 : Determine shape parameters using appropriate vector methods.		
M3 : Evaluate the coordinate system used in programming a simple output device.		
D3 : Construct the scaling of simple shapes that are described by vector coordinates.		
LO4 : Evaluate problems concerning differential and integral calculus.		
P7 : Determine the rate of change within an algebraic function.		
P8 : Use integral calculus to solve practical problems involving area.		
M4 : Analyse maxima and minima of increasing and decreasing functions using higher order derivatives.		
D4 : Justify, by further differentiation, that a value is a minimum.		

Acknowledgment

In preparation for my assignment, I had to take the help and guidance of some respected persons, who deserve my deepest gratitude. As the completion of this assignment gave me much pleasure, I would like to show my gratitude to Mrs.Shyamali, Lecturer, on Esoft Metro Campus for giving me good guidelines for assignments throughout numerous consultations. I would also like to expand my gratitude to all those who have directly and indirectly guided me in writing this assignment.

Sincerely,

Chandratheebam Suresh Gobi

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Activity 1

Part 1

1.

a) **Find the minimum number of squared pieces**

Consider **20** and **16**

2	**20**
2	10
5	5
	1

2	**16**
2	8
2	4
2	2
	1

The factors of 20 is $2 \times 2 \times 5$

The factors of 16 is $2 \times 2 \times 2 \times 2$

The highest common factor of 20 and 16 is 2×2 = **4 squared pieces**

Area of the Towel = $20 \times 16 = 320\ m^2$

Are of a single piece of Towel = $4 \times 4 = 16\ m^2$

Total pieces of Towel can cut from the ream of cloth = $\frac{320}{16} = \mathbf{20\ Towels}$

b) **Explanation**

The **highest common factor** of two numbers is the largest whole number which is a factor of both. Take, for example, common factors of 10 and 20.

10 = $\mathbf{2} \times \mathbf{5}$

20 = $2 \times \mathbf{2} \times \mathbf{5}$

The common factors of 10 and 20 are thus 2 and 5. As a result, the highest common factor is 10. As an example, the question asks to find the minimum number of square pieces that can be cut from a length of 20 meters and a width of 16 meters. First and foremost, the common factors of 20 and 16 should be found.

20 = $\mathbf{2} \times \mathbf{2} \times 5$

$16 = 2 \times 2 \times \mathbf{2} \times \mathbf{2}$

Hence, the common factors are 2×2 and highest common factor if **4**. So 4 pieces of squared pieces can be cut from the given ream of cloth.

2.

a) Finding the date on which consumers will together

Consider **2, 4, 6** and **8**

2	**2, 4, 6, 8**
2	1, 2, 3, 4
2	1, 1, 3, 2
3	1, 1, 3, 1
	1, 1, 1, 1

2, 4, 6, 8 L.C.M = $2 \times 2 \times 2 \times 3 = \mathbf{24^{th}}$

1st customer visit dates = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, **25**, 27, 29

2nd customer visit dates = 1, 5, 9, 13, 17, 21, **25**, 29

3rd customer visit dates = 1, 7, 13, 19, **25**

4th customer visit dates = 1, 9, 17, **25**

Common date of all four customers is 25^{th}. So, all the four customers come back to the restaurant together on 25^{th} of the month.

b) Explanation

The "**smallest non-zero common number**" that is a multiple of both numbers is the **least common multiple of two numbers**. Consider the numbers 6 and 8 as an example.

2	**6, 8**
2	3,4
2	3,2

3	3,1
	1,1

So, the common multiple numbers are $2 \times 2 \times 2 \times 3$ and the least common multiple is **24**. Similarly, the task is to find when the four customers will meet again. Finding the least common multiple is the most important step in this process.

The least common multiple of 2, 4, 6, and 8 is 24. The least common multiple of visited dates, according to customer visits to the restaurant, is the 24^{th} of that month. As a result, on the 25^{th} of the month, the four customers come together.

Part 2

3.

a) Finding the summation of logs are in the stack

$\mathbf{n} = 15$, $\mathbf{a_1} = 24$, $\mathbf{a_n} = 10$

$$\mathbf{S_n = \frac{n}{2}\{a_1 + a_n\}}$$

$$S_{15} = \frac{15}{2}\{24 + 10\}$$

$$S_{15} = \frac{15}{2}\{34\}$$

$$S_{15} = 15 \times 17 = \mathbf{255\ logs}$$

255 logs are in the row.

b) Explanation

The **Arithmetic sequence** is the set of terms in which the common difference between any two succeeding terms remains constant. The arithmetic sequence formula is as follows:

a_n = n^{th} term in the sequence

a_1 = 1^{st} term in the sequence

n = Number of terms

d = Common difference

s_n= Sum of n terms

- Common difference **d = a_n - a_{n-1}**
- n^{th} term **a_n = a + (n-1) d**
- Sum of n term (There have two equations)

$$Sn = \frac{n}{2}\{a_1 + a_n\}$$

Or

$$Sn = (\frac{n}{2})[2a + (n - 1)d]$$

Above question say that find total number of stack in the logs. For that, identifying the **n (number of rows), a_1 (logs on the bottom row), a_n (logs on the top row)** that 15, 24, 10 in manner. To find the logs in the stack, first identify the correct equation that is **$S_n = \frac{n}{2}\{a_1 + a_n\}$** and add the correct values to the equation to find the summation for the first 15 rows. Hence, the final answer is **255 logs where in the stack**.

4.

a) Finding the total amount of money earned by an employee

The first-year salary a_1 = 50,000 × 12 = 600,000

Second-year salary a_2 = 600,000 × 100% + 600,000 × 4%

= 600,000 (104%)

The growth factor **r** = $\frac{104}{100} = \mathbf{1.04}$

$$s_n = a_1 \frac{(1 - r^n)}{1 - r}$$

$$s_{10} = 600{,}000 \frac{(1 - 1.04^{10})}{1 - 1.04}$$

$$s_{10} = \frac{600{,}000\ (1 - 1.48)}{-0.04}$$

$$s_{10} = \frac{600{,}000\ (-0.48)}{-0.04}$$

$$\mathbf{s_{10} = 600{,}000 \times 12 = 7{,}200{,}000\ /=}$$

In 10 years, the employee will earn a total amount of 7,200,000 / =

b) Explanation

The ratio between any two successive terms is the same throughout a series of terms called **Geometric progression**. For instance, 1, 3, 9, 27,.. The ratio is a multiple of 3 in this instance. And the following equation is used to get the summation of the first **n terms**:

$$s_n = a_1 \frac{(1 - r^n)}{1 - r}$$

a_1 = 1st term in the sequence

n = Number of terms

r = Common ratio

s_n= Sum of n terms

This equation is used in question (a) to solve. Finding the first year (a_1) and second year salary (a_2). Also the common growth factor of salary is 1.04. As per the geometric progression finding the summation of 10 years of salary using above equation. The employee get the total amount of 10 years' salary is 7,200,000/=.

Part 3

5. Multiplicative inverse in modular arithmetic

The inverse in the groups is created by adding up the whole numbers that remain after one number has been divided by another. The solution to xy=1(mod n), or alternatively, XY mod n = 1, is the modular multiplicative inverse of x modulo n. Y= x-1 is the modular inverse. The task in 5. To calculate the multiplicative inverse of 6 mod 13. For that,

6×0 mod 13= 0

6×1 mod 13= 6

6×2 mod 13= 12

6×3 mod 13= 5

6×4 mod 13= 11

6×5 mod 13= 4

6×6 mod 13= 10

6×7 mod 13= 3

6×8 mod 13= 9

6×9 mod 13= 2

6×10 mod 13= 8

6×11 mod 13= 1 ← **inverse**

6×12 mod 13= 7

So, The Multiplicative modular inverse of 6 mod 13 is 11 since 6 * 11 = 1 mod 13.

6. Prime Numbers

Prime numbers are the numbers that have only two factors, that are, 1 and the number itself. For example, the number 5 has two positive divisors, 5 and 1, so it is a prime number. The number 8 has three positive divisors, 8, 3, and 1, so it is not a prime number. 2, 3, 5, 7, 11, 13, 17, 19 … are list of prime numbers under 20.

Prime Numbers in Computer field

Prime numbers are essential for communications and most computer cryptography works through them. One of the major applications and technologies (cryptography and hash functions) for prime numbers is in cyber security, which makes the sharing of information over the internet safer. Software engineers use prime numbers to create algorithms that encrypt data such as credit card numbers, medical records, and even some messaging platforms like WhatsApp.

- **Cryptography**
- **Hashing**

Cryptography

Hackers nowadays are constantly attempting to steal or damage the information of people or businesses. Modern technology uses cryptography, which Julius Caesar already used, to safeguard data from being stolen (100 B.C to 44 B.C). Cryptography, which increases data communication security, is an important factor in secure communications.

The study of secure information communication methods, such as encryption and decryption, formed from mathematics concepts and a set of algorithms is known as cryptography. When a user sends Cleartext (also known as plaintext) to another user's device. Initially, the plaintext transmits to an encryption server, where encryption algorithms are implemented with the secret key and give encrypted outputs as Ciphertext. This Ciphertext is the encrypted output that goes from the public domain to the description server, where it is decrypted using the secret key and sent back to the destination as Plaintext (Richards, 2021).

There are two types of Cryptography

Symmetric Cryptography - It uses a common key as the basis for both the encryption and decryption of plaintext and Ciphertext.

Asymmetric Cryptography (public key cryptography) - The encryption and decryption processes in this type of cryptography use two different keys. When sending plaintext to a recipient, the sender must have access to the recipient's public key. It is accessible from a dependable source. Using an encryption server, the transmitted plaintext is encrypted using an asymmetric algorithm and a public key, producing ciphertext as the result. This ciphertext will be forwarded to the decryption server, which will perform asymmetric algorithms on the ciphertext along with a private key to decrypt that again and provide the plaintext to the receiver.

How prime numbers are used in Cryptography

Prime numbers are important because many encryption methods' security depends on the fact that multiplying two large prime numbers rapidly gives the desired result, whereas doing the reverse takes a lot of computing power. It is quite challenging to find the product of two prime numbers. Finding a fast algorithm for this problem, known as prime factorization, is one of the unsolved problems in computer science.

For example, multiplying large integers numerous times in encryption to obtain the largest number and multiplying this huge number by N. Therefore, the encrypted information that will be sent to the end-point is a reminder. A secret prime number serves as the basis for the calculation in the decryption as well. After receiving the encrypted value, the endpoint multiplies it by itself using a secret prime number. The result will be divided by N and finds the remainder. So, the result is what actual value that the user sends. The RSA system employs this technique to encrypt and decrypt both public and private keys. (Smith, 2022).

Hashing

A hash is a mathematical operation that transforms a value input into a fixed-length encrypted output. Its unique hash will therefore always be the same size, regardless of the actual data or file size involved. The fact that hash functions are "**one-way**" means that it is impossible to "**reverse-engineer**" the input from the hashed result. The message-passing abilities of hash functions are combined with security features in a cryptographic hash function. For purposes like verifying the accuracy of communications and authenticating data, computer systems frequently use hash functions as data structures (Frankenfield, 2022).

How prime numbers are used in Hashing

Key-value pairs are the most common type of data to be stored in hash tables. Hash tables are a great choice for data access, insertion, and removal since they make it simple to rapidly discover information by analyzing its corresponding key. Hashing tables use prime integers to reduce clustering inside the hashed table.

In the case that the default length is set to 15, any integers that are multiples of 3 or 5, but not multiples of 15, will be hashed into the corresponding indexes of 3 and 5, respectively. For example, keys that generate integers of 0, 15, 30, etc. will be given index 0, keys that generate integers of 3, 18, 33, etc. will be given index 3, and keys that generate integers of 5, 20, 35, and etc. will be given index 5. For non-random data, the most common distribution of integers to indices will be produced by a hash table with a prime number length. Choosing a length with the fewest possible elements is advantageous. A prime number sequence they are renowned for being only divisible by one and themselves. Therefore, selecting a high prime number as the length of the hash table will significantly lower the probability of collisions.

Activity 2

Part 1

1. **Conditional Probability**

   The probability that an event will occur given its relationship to one or more other events is known as conditional probability. It is determined by multiplying the probability of the preceding event by the renewed probability of the conditional or event.

   Definition: Conditional probability is the probability that any event A will occur in the presence of another event B that is related to A. It is depicted by $\mathbf{P(A|B)}$.

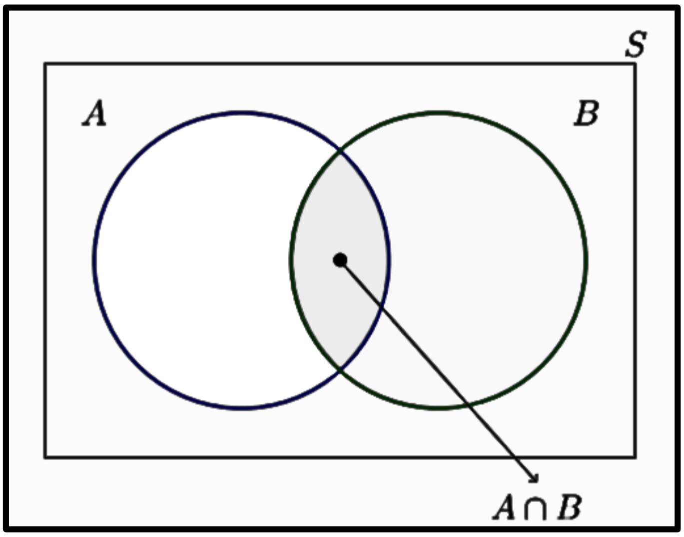

Figure 1 : Conditional Probability

In depth, S represents the sample space, and events A and B are present. Once the event B has already happened, sample space S is automatically reduced to B because an event's probability of happening now resides inside B. The probability that A will occur after B has already happened can only be represented by the portion that both A and B express. The common portion of the events is depicted by the intersection of both the events A and B, i.e. A ∩ B. Then the conditional probability of $\mathbf{P(A|B)}$ is defined as,

$$\mathbf{P(A|B)} = \frac{\boldsymbol{P(A \cap B)}}{\boldsymbol{P(B)}}\ \boldsymbol{, when\ P(B) > 0}$$

Similarly, P(B|A) can be defined as,

$$\mathbf{P(B|A) = \frac{\mathit{P(A \cap B)}}{\mathit{P(A)}}}\ \mathbf{,\mathit{when}\ \mathit{P(A)} > 0}$$

2. Probability of items that customers randomly selected

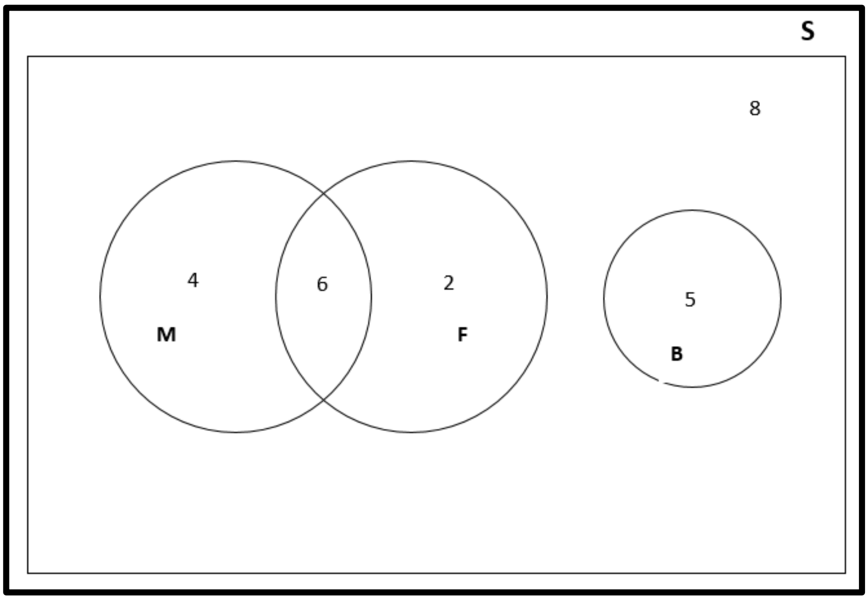

Figure 2 : Probability of items that customers randomly selected

B represent customer purchased **"Biscuits"**

M represent customer purchased **"Milk"**

F represent customer purchased **"Fruits"**

$n(B) = 5\ ,n(M) = 10\ ,n(F) = 8$

$n(M \cap F) = 6$

$n(S) = 25$

a) The probability that randomly selected customers either purchased Biscuits or Milk.

$$P(B \cup M) = \frac{5 + 10}{25} = \frac{15}{25} = \mathbf{\frac{3}{5}}$$

b) Randomly selected Milk and Fruit

$$P(M) = \frac{10}{25} = \frac{2}{5}$$

$$P(F) = \frac{8}{25}$$

$$P(M).P(F) = \frac{2}{5} \times \frac{8}{25} = \frac{16}{125}$$

$$P(M \cap F) = \frac{6}{25} \neq \frac{16}{25} = P(M).P(F)$$

So, the events "The randomly selected customer purchased Milk" and "The randomly selected customer purchased Fruits" are not independent.

3.

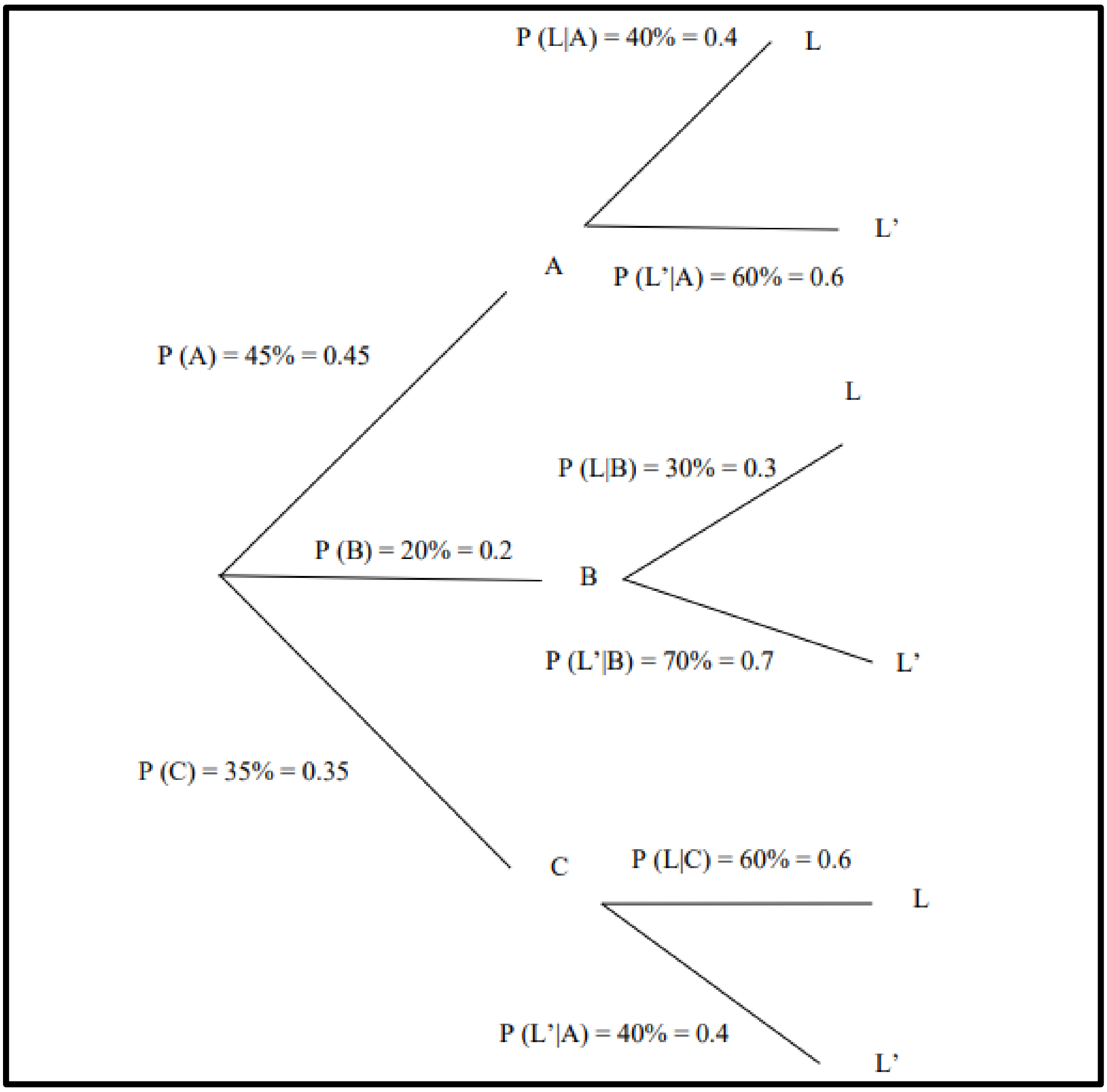

Figure 3 : Tree Diagram

a) **Probability that a randomly selected voter does not support the liberal candidate and lives in state A.**

$$P\ (A \cap L') = P(A).P(L'|A)$$
$$= 0.45 \times 0.6$$
$$= 0.27$$

b) **Probability that a randomly selected voter supports the liberal candidate.**

$$P(L) = P\ (A \cap L) + P\ (B \cap L) + P\ (C \cap L)$$
$$= P\ (A).P\ (L|A) + P\ (B).P\ (L|B) + P(C).P\ (L|C)$$
$$= 0.45 \times 0.4 + 0.2 \times 0.3 + 0.35 \times 0.6$$
$$= 0.18 + 0.06 + 0.21$$
$$= 0.45$$

c) **Probability that the selected voter is from state B**

$$P\ (B|L) = \frac{P\ (B \cap L)}{P\ (L)}$$
$$= \frac{P\ (B).P\ (L|B)}{P\ (L)}$$
$$= \frac{0.2 \times 0.3}{0.45}$$
$$= \frac{6}{45} = \mathbf{\frac{2}{15}}$$

4.

6 Hearts cards

7 Clubs cards

8 Diamonds cards

5 Scorpions cards

a) **Probability of selected cards are Heart**

$$P = \frac{C_6^2}{C_{26}^2} = \frac{\frac{6!}{2! \times (6-2)!}}{\frac{26!}{2! \times (26-2)!}}$$

$$P = \frac{\frac{6 \times 5 \times 4!}{2! \times 4!}}{\frac{26 \times 25 \times 24!}{2! \times 24!}}$$

$$P = \frac{\frac{6 \times 5}{2!}}{\frac{26 \times 25}{2!}} = \frac{\frac{30}{2}}{\frac{650}{2}}$$

$$p = \frac{15}{325} = \frac{3}{65} = \mathbf{0.0461538}$$

b) **Probability of selected card one is Clubs and other cards is Diamond**

$$P = \frac{7 \times 8}{C_{26}^{2}} = \frac{56}{\frac{26!}{2! \times (26-2)}}$$

$$P = \frac{56}{\frac{26 \times 25 \times 24!}{2! \times 24!}} = \frac{56}{\frac{26 \times 25}{2!}}$$

$$P = \frac{56}{\frac{650}{2}} = \frac{56}{325} = 0.17230769$$

c) **Probability of both selected cards are same type**

$$P = \frac{C_{6}^{2} + C_{7}^{2} + C_{8}^{2} + C_{5}^{2}}{C_{26}^{2}}$$

$$P = \frac{\frac{30}{2} + \frac{42}{2} + \frac{56}{2} + \frac{20}{2}}{\frac{650}{2}}$$

$$P = \frac{15 + 21 + 28 + 10}{325} = \frac{74}{325} = \mathbf{0.22769230}$$

Part 2

5. **Random Variable**

A real-valued function that is defined across the sample space of a random experiment is a random variable. To look at it another way, the values of the random variable match the outcomes of the random experiment. There are possibilities for discrete and continuous random variables. Two types of random variables include:

- **Discrete Random Variable**
- **Continuous Random Variable**

Discrete Random Variable

The value of a discrete random variable is a sort of variable whose value depends on the numerical results of particular random phenomena. A stochastic variable is another term for this. All discrete random variables are whole integers that may be counted. A discrete random variable's probability distribution is described by the probability mass function.

Continuous Random Variable

The definition of a continuous random variable is a random variable with an infinite number of potential values. Because of this, there is no chance for a continuous random variable to have an exact value. A continuous random variable's features are described using the cumulative distribution function and the probability density function.

Difference between Discrete Random Variable and Continuous Random Variable

Discrete Random Variable	**Continuous Random Variable**
An exact value can be assigned to a discrete random variable.	A continuous random variable's value will fall between specific intervals.
A discrete random variable is described by a probability mass function.	A continuous random variable is described by a probability density function.
The binomial, geometric, Bernoulli, and poison random variables are a few examples of distributions with discrete random variables.	Normal and exponential random variables are two types of continuous random variable distributions.

6.

X = x	-5	-4	-3	-2	-1	0	1	2	3	4	5
P(X=X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

a) **Finding E (X)**

The random variable X represents the difference between the values of the two cubes.

$$E(X) = \sum[x.P(X=x)] = (-5)\times\left(\frac{1}{36}\right) + (-4)\times\left(\frac{2}{36}\right) + (-3)\times\left(\frac{3}{36}\right) + (-2)\times\left(\frac{4}{36}\right) + (-1)\times\left(\frac{5}{36}\right) + 0\times\left(\frac{6}{36}\right) + 1\times\left(\frac{5}{36}\right) + 2\times\left(\frac{4}{36}\right) + 3\times\left(\frac{3}{36}\right) + 4\times\left(\frac{2}{36}\right) + 5\times\left(\frac{1}{36}\right) = 0$$

b) **Variance and Standard Deviation**

$$\mathbf{V[X]} = \boldsymbol{E(X^2) - [E(X)]^2} = \sum[x^2.P(X=x)] - \{\sum[x.P(X=x)]\}^2 =$$

$$=\left\{(-5)^2\times\left(\frac{1}{36}\right) + (-4)^2\times\left(\frac{2}{36}\right) + (-3)^2\times\left(\frac{3}{36}\right) + (-2)^2\times\left(\frac{4}{36}\right) + (-1)^2\times\left(\frac{5}{36}\right) + (0)^2\times\left(\frac{6}{36}\right) + 1^2\times\left(\frac{5}{36}\right) + (2)^2\times\left(\frac{4}{36}\right) + (3)^2\times\left(\frac{3}{36}\right) + (4)^2\times\left(\frac{2}{36}\right) + 5^2\times\left(\frac{1}{36}\right)\right\} - 0^2$$

$$=\{2x[25x(\frac{1}{36})] + 2x[16x(\frac{2}{36})] + 2x[9x(\frac{3}{36})] + 2x[4x(\frac{4}{36})] + 2x[1x(\frac{5}{36})] + 0\} - 0$$

$$=2x\,[\frac{25+32+27+16+5}{36}]$$

$$= 2x\,[\frac{105}{36}] = \frac{105}{18} = \mathbf{5.83}$$

Standard deviation SD[X],

$$SD(X) = \sqrt{V(X)} = \sqrt{5.83} = \mathbf{2.14}$$

c) **E[A] and E[B]**

E (A)

E (A) = E(X-10)

=E(X + (-10))

=E(X) + E (-10)

=E(X) + (-10) = 0 + (-10) = **(-10)**

E (B)

E (B) = E ((1/2) X – 5)

=E ((1/2) X + (-5))

=E ((1/2) X) + E (-5)

= (1/2) E(X) + (-5)

$=(1/2) \times 0 + (-5) = 0 + (-5) = (\mathbf{-5})$

Therefore, E (A) and E (B) are -10 and -5 respectively.

d) V[A] and V[B]

A = X-10 and B = [(1/2) X] – 5

V (A)

V (A) = V(X-10)

=V(X + (-10))

= V(X) + V (-10)

= V(X) + 0 = 5.83 + 0 = **5.83**

V (B)

V (B) = V ((1/2) X – 5)

=V ((1/2) X + (-5))

$=V((1/2)X) + V(-5) = 1/2^2\, V(X) + 0 = (1/4) \times 5.83 + 0 = (1/4) \times 5.83 = \mathbf{1.46}$

e) Differences or similarities between the random variables A and B

Arnold → A = X-10

Brian → B = (1/2) X - 5

Use E (A), E (B) and V (A), V (B)

If Arnold and Brian do this game for large number of times, then Expected Value of Arnold's score is twice as the Expected Value of Brian's score. E (A) = (-10) and E (B) = (-5)

If Arnold and Brian do this game for a large number of times, the variance of the scores of Arnold is greater than the variance of score of Brian. V (A) = 5.83 and V (B) = 1.46 → V (A) > V (B)

7.

a) Value of k

Sum of all probability is 1,

So,

$$P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) + P = 1$$

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{4} + k + \frac{1}{6} = 1$$

$$\frac{11}{12} + k = 1$$

$$\boldsymbol{k = \frac{1}{12}}$$

$$\boldsymbol{P(Y = 4) = \frac{1}{12}}$$

b) P (Y<=3)

$$P(Y \leq 3) = P(Y = 1) + P(Y = 2) + P(Y = 3)$$

$$P(Y \leq 3) = \frac{1}{3} + \frac{1}{6} + \frac{1}{4}$$

$$\boldsymbol{P(Y \leq 3) = \frac{3}{4}}$$

c) P (Y > 2)

$$P(Y > 2) = P(Y = 3) + P(Y = 4) + P(Y = 5)$$

$$P(Y > 2) = \frac{1}{4} + \frac{1}{12} + \frac{1}{6}$$

$$\boldsymbol{P(Y > 2) = \frac{1}{2}}$$

Part 3

10.

a) Possible values for X

X = {0,1,2,3,4,5,6,7,8,9,10}

b) Probability of wining 6 matches

W represent the win

L represent the lose

Winning percentage 0.75

$X \sim Bin(n, p)$

$n = 10, p = 0.75$

$q = 1 - p = 1 - 0.75 = 0.25$

$P(X = 6) = \binom{10}{6} (0.75^6)(0.25)^{10-6}$

$\boldsymbol{P(X = 6) = 0.1459980011}$

c) **Probability of lose 2 or less matches**

$P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10)$

$P(X \geq 8) = \binom{10}{8} (0.75)^8(0.25)^{10-8} + \binom{10}{9} (0.75)^9(0.25)^{10-9}$
$+ \binom{10}{10} (0.75)^{10}(0.25)^{10-10}$

$\boldsymbol{P(X \geq 8) = 0.52559280396}$

d) **Number of matches will win**

$E(X) = mean = np = 10(0.75) = \mathbf{7.5 = 8}$

e) **Variance and standard deviation**

$\mathrm{Var(X)} = \sigma^2 = \mathrm{npq} = 10(0.75)(0.25) = \mathbf{1.875 = 2}$

$\sigma = \sqrt{\sigma^2} = \sqrt{1.875} = \mathbf{1.3693 = 1}$

9.

Mean height(μ) = 154cm

Alex height (X) = 163cm

Standard deviation (o) = 2

Normal Curve for Random Variable X

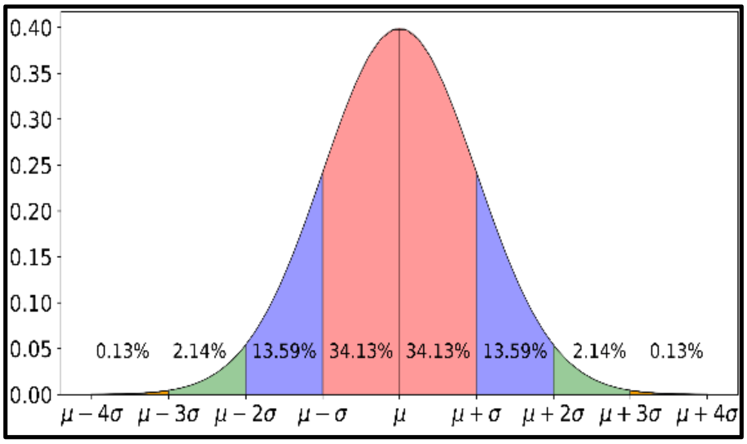

Figure 4 : Normal Curve for Random Variable X

What is an Outlier?

A single data point that significantly deviates significantly from the average value of a set of statistics is referred to as an outlier. Because they can significantly affect the overall results, outliers are a crucial consideration in statistics. A single outlier can have a significant impact on averages and affect the study's conclusions is particularly small sample sizes.

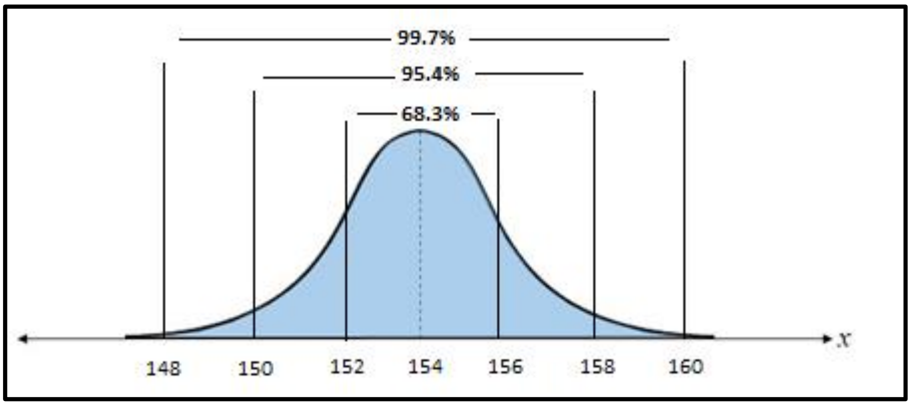

Figure 5 : Normal Curve for Answer

A standard cut-off value for finding outliers is less than 148 or more than 160. Since 163 > 160, the Alex height is an outlier.

10.

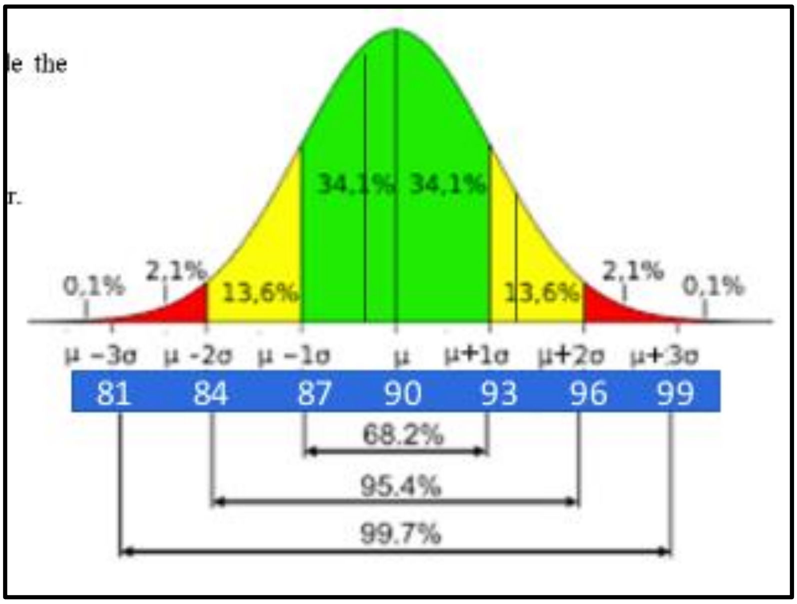

Figure 6 : Normal Curve for Battery Life

a) **Percent of the products last between 87 and 93 days**

$P(87 \leq x \leq 93)$

$= p\left(\frac{87-90}{3} \leq z \leq 1\right)$

$= p(-1 \leq z \leq 1)$

$= 0.841 - 0.159 = 0.682 = \mathbf{68.2}\%$

b) Percent of the products last 84 or less days

2.1% + 0.1% = 2.2% = **0.022**

c) **Percentage pf the product last between 89 and 94 days**

$\boldsymbol{P}\ (\mathbf{89}\ <\ \boldsymbol{X}\ <\ \mathbf{94})$

$z = (x-\mu)/\sigma = (89-90)/3 = -1/3 = -0.33$

$z = (x-\mu)/\sigma = (94-90)/3 = 4/3 = 1.33$

$P(z < 1.33) - P(z < -0.33)$

$= 0.9082 - 0.3707$

$P(89 < x < 94) = 0.5375$

$= \ \mathbf{53.75}\%$

d) Percent of the product last 95 or more days

P (X > 95)

$$z = \frac{x-\mu}{\sigma} = \frac{95-90}{3} = \frac{5}{3} = 1.67$$

$P(z \geq 1.67)$

$1 - P(z < 1.67)$

$= 1 - 0.9525$

$Range = 0.0475 = \mathbf{4.75}\%$

11. Hash Function

Using a cryptographic method called hashing, any type of data may be converted into a unique text string. A hashing method produces the same results for both the sender and the receiver when a user provides plaintext. A text that can be read is transformed into a secure text that cannot be read by humans. The process of hashing is effective, but it is very hard to reverse. Users that don't know the data's content can get data authorization by hashing. Using safe hashing techniques to save passwords is another option. An obstacle to potential attackers is created by cryptographic hashing. If an attacker tries to access the database and see the hashes, they are unable to convert the hash value back to the original value.

Probability in Hashing

The hash function produces an integer hash value within a specified range for a given object of a specific type. Strings, files, compiled shade programs, and event directories are all acceptable input types. A hash collision occurred when the same input resulted in the same hash value every time. The idea of probability is used by a hash function. Therefore, creating hash values for a set of inputs is quite similar to creating a set of random integers.

Given a space of N possible hash values, suppose user already picked a single value. After that, there N – 1 reaming values that are unique from the first. In view of this, the likelihood that two distinct integers will be generated at random is $\frac{N-1}{N}$. N – 2 is unique reaming values, which means that the probability of randomly generating three integers that are all unique is $\frac{N-1}{N} \times \frac{N-2}{N}$.

In general, the probability of randomly generating k integers that are all unique is (preshing, 2011) :

$$\frac{\mathbf{N-1}}{\boldsymbol{N}} \times \frac{\mathbf{N-2}}{\boldsymbol{N}} \times \cdots \times \frac{\mathbf{N-(k-2)}}{\boldsymbol{N}} \times \frac{\mathbf{N-(k-1)}}{\boldsymbol{N}}$$

Load Balancing

The performance and dependability of websites, apps, databases, and other services are often improved by load balancing, a crucial part of the highly-available infrastructure that distributes the workload over numerous servers. The efficient and speedier delivery of services is made possible by the fair distribution of processing tasks across two or more computers, CPUs, network connections, and storage devices. The user will no longer be able to access the website if the web server signal disappears while they are using it. Computer clusters are frequently used in load balancing, which can be carried out via hardware, software, or both.

Probability in Load balancing

In a distributed operating system, the probability can be used to determine the likelihood of load balancing success. This likelihood provides information on how the system is used and helps determine how effective the system is. Assume there are n balls and n bins, then distribute the balls among the bins at random. In this instance, knowing how many balls can enter a certain bin. The simplest random load balancing approach is to put each ball in a bin that is selected at random. The network traffic will load and break the connection if a user tries to access a website server at the same time. The load balancer is now activated to lighten the server's workload. This burden is distributed among servers using probability theories. The website server in this scenario is the bin, and the balls are the users. Through the use of load balancing, the users are connected to the server as random.

Probability theory used in other techniques in computing field

Other than hashing and load balancing, the probability may be applied in a variety of scenarios, such as analyzing the probability of different events occurring, understanding algorithms, or modeling systems that operate in an asynchronous environment governed by uncertainty. The modeling of online data, speech recognition, robotics, network traffic, and dependability, as well as probabilistic analysis of algorithms and graphs, machine learning, data mining, cryptography, and other fields, all make use of probability.

Probability theory is rapidly being used in computing applications as well as being a key tool for studying complicated systems. One method of exploiting smartcards, remote authentication services, or encryption running on a cloud virtual machine next to infer the secret key from cache duration. Probability theory is applied outside of reputation-based systems in various fields. For sensor networks, one may choose a peer-to-peer sharing system or a reputation-based routing algorithm.

Activity 3

Part 1

1. Circle

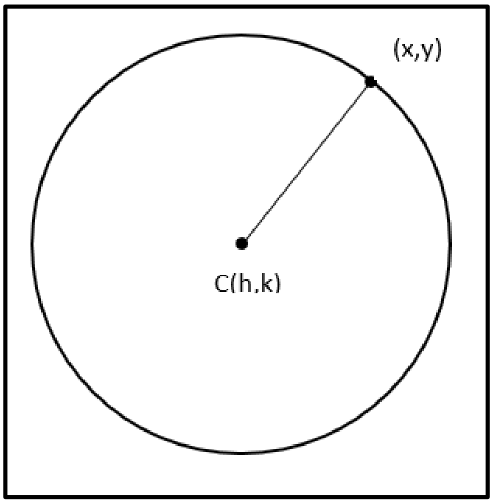

Figure 7 : Circle Definition

Formula of Circle,

$(x-h)^2+(y-k)^2=r^2$ The center is (h, k)

$$\boldsymbol{r = \sqrt{(x-h)^2+(y-k)^2}}$$

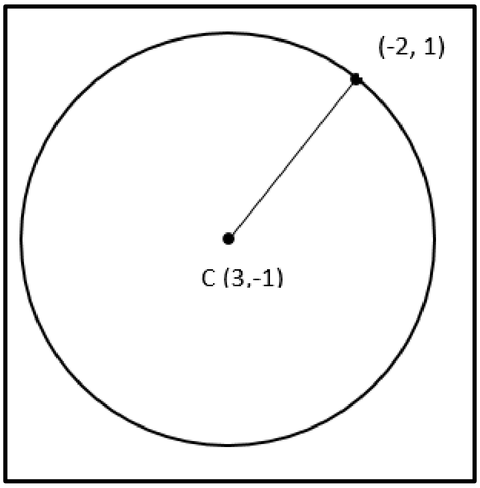

Figure 8 : Circle Answer

$\boldsymbol{r^2 = (x-h)^2 + (y-k)^2}$

$\boldsymbol{r^2 = (-2-3)^2 + (1-(-1))^2}$

$r^2 = (-5)^2 + (2)^2$

$r^2 = 25 + 4$

$\boldsymbol{r^2 = 29}$

$(x-h)^2 + (y-k)^2 = r^2$
$(x-3)^2 + \left(y-(-1)\right)^2 = 29$
$\boldsymbol{(x-3)^2 + (y+1)^2 = 29}$

If the Center of a circle is at (3,-1) and a point on the circle is (-2, 1) , the formula of the circle is $\boldsymbol{(x-3)^2 + (y+1)^2 = 29}$

2. Sphere

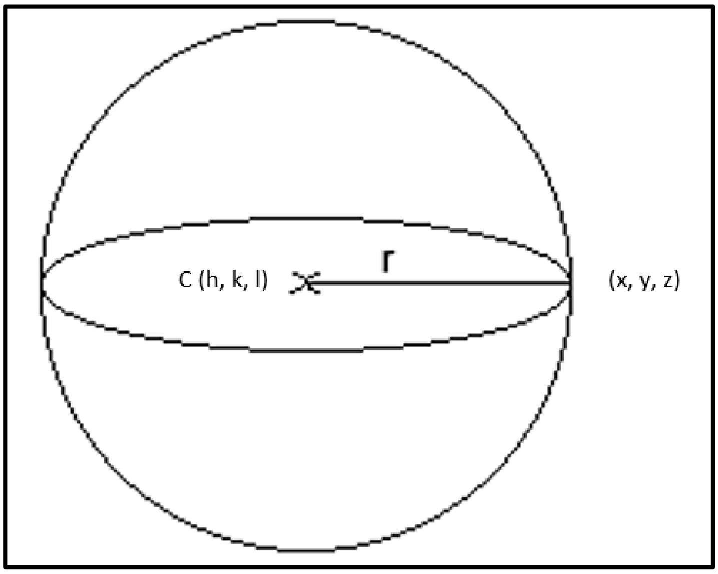

Figure 9 : Sphere Definition

Formula of a Sphere,

$(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2+(\boldsymbol{z}-\boldsymbol{l})^2=\boldsymbol{r}^{\boldsymbol{2}}$

$(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2+(\boldsymbol{z}-\boldsymbol{l})^2=\boldsymbol{r}^{\boldsymbol{2}}$

$x^2 + y^2 + z^2 - 6\mathrm{x} + 2\mathrm{y} + 8\mathrm{z} - 4 = 0$

$x^2-6x+y^2+2\mathrm{y}+z^2+8\mathrm{z}-4=0$

$x^2-6x+9-9+y^2+2y+1-1+z^2+8z+16-16-4=0$

$(\boldsymbol{x}-\boldsymbol{3})^{\boldsymbol{2}}+(\boldsymbol{y}+\boldsymbol{1})^{\boldsymbol{2}}+(\boldsymbol{z}+\boldsymbol{4})^{\boldsymbol{2}}=\boldsymbol{28}$

The equation of a sphere with radius r = $\boldsymbol{2\sqrt{7}}$ and center $\boldsymbol{C(+3,-1,-4)}$ is

$(\boldsymbol{x}-\boldsymbol{3})^{\boldsymbol{2}}+(\boldsymbol{y}+\boldsymbol{1})^{\boldsymbol{2}}+(\boldsymbol{z}+\boldsymbol{4})^{\boldsymbol{2}}=\boldsymbol{28}$

3. Parallelogram

Area = $|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}|$

a $= \mathrm{i}+3\mathrm{j}-\mathrm{k}$, **b** $= 7\mathrm{i}-2\mathrm{j}+4\mathrm{k}$, then

$$\boldsymbol{a}\times\boldsymbol{b}=\begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 7 & -2 & 4 \end{vmatrix}$$

$=(12-2)i-(4+7)j+(-2-21)k$

$=10i-11j-23k$

$|a\times b|=\sqrt{10^2+(-11)^2+(-23)^2}$

$||\boldsymbol{a} \times \boldsymbol{b}|| = \mathbf{27.38613}\ \boldsymbol{square\ units}$

Part 2

4. Evaluation of x,y using graphical method

$\mathbf{2x - 4y = 3}$

When x=0, $2x - 4y - 3 = 0$ $4y = 3$ $y = -\frac{3}{4}$ So, $\left(\mathbf{0}, -\frac{\mathbf{3}}{\mathbf{4}}\right)$	When y=0, $2x - 3 = 0$ $x = \frac{3}{2}$ So,$\left(\frac{\mathbf{3}}{\mathbf{2}}, \mathbf{0}\right)$

$\mathbf{5y = (-3)x + 10}$

$\mathbf{5\boldsymbol{y} + 3\boldsymbol{x} - 10 = 0}$

When x=0, $5y = 10$ $y = 2$ So, $(\mathbf{0}, \mathbf{2})$	When y=0, $3x - 10 = 0$ $x = \frac{10}{3}$ So, $\left(\frac{\mathbf{10}}{\mathbf{3}}, \mathbf{0}\right)$

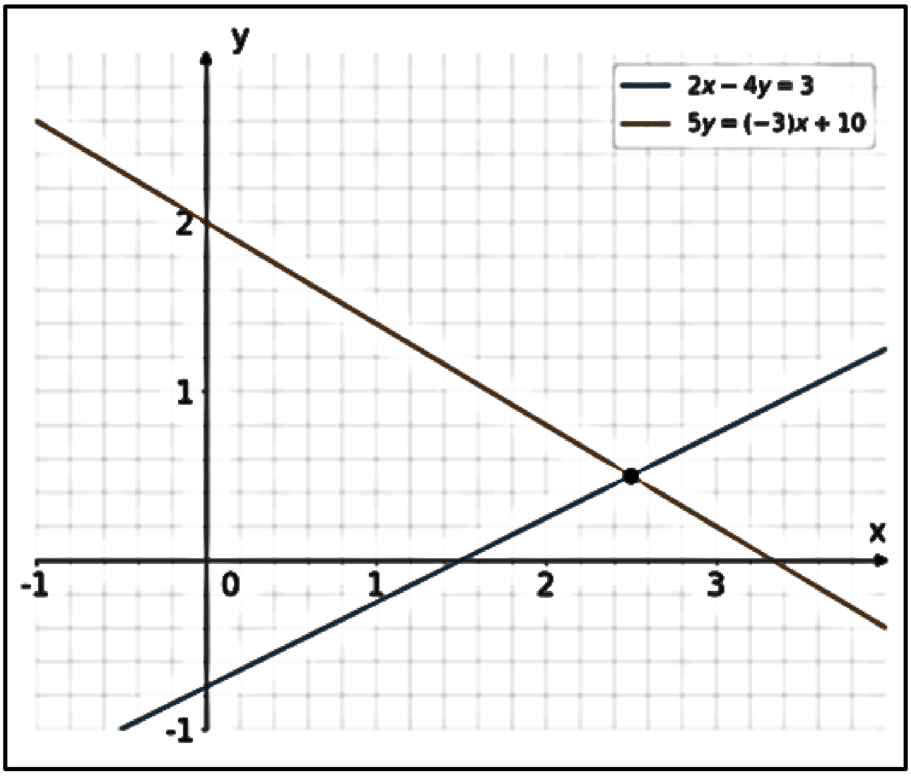

Figure 10 : Graph

Intersection point of $2x - 4y = 3$, $5y = (-3)x + 10$

$5y = 10 - 3x$
$4y = 2x - 3$

$20y = 40 - 12x$
$20y = 10x - 15$

$40 - 12x = 10x - 15$
$22x = 55$
$\boldsymbol{x = \frac{55}{22} = \frac{5}{2}}$

$3x = 10 - 5y$
$2x = 4y + 3$

$6x = 20 - 10y$
$6x = 12y + 9$

$20 - 10y = 12y + 9$
$22y = 11$
$\boldsymbol{y = \frac{11}{22} = \frac{1}{2}}$

Hence, the interaction points of $2x - 4y = 3, 5y = (-3)x + 10$ is $\left(\frac{5}{2}, \frac{1}{2}\right)$

5.

i. y =4

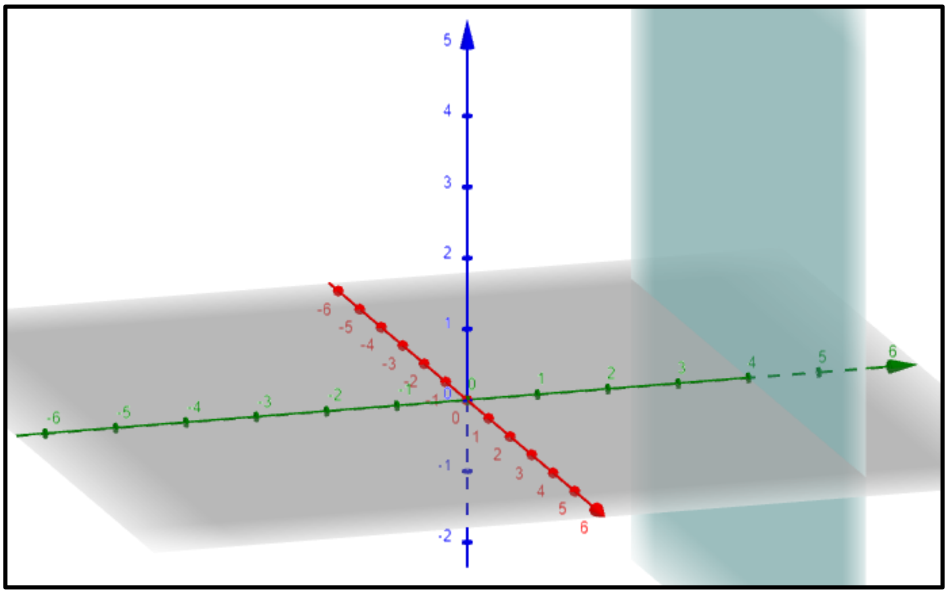

Figure 11 : Surface 01

Y = 4 is a plane parallel to plane XOZ with an intersection with y-axis at y=4

ii. z = 5

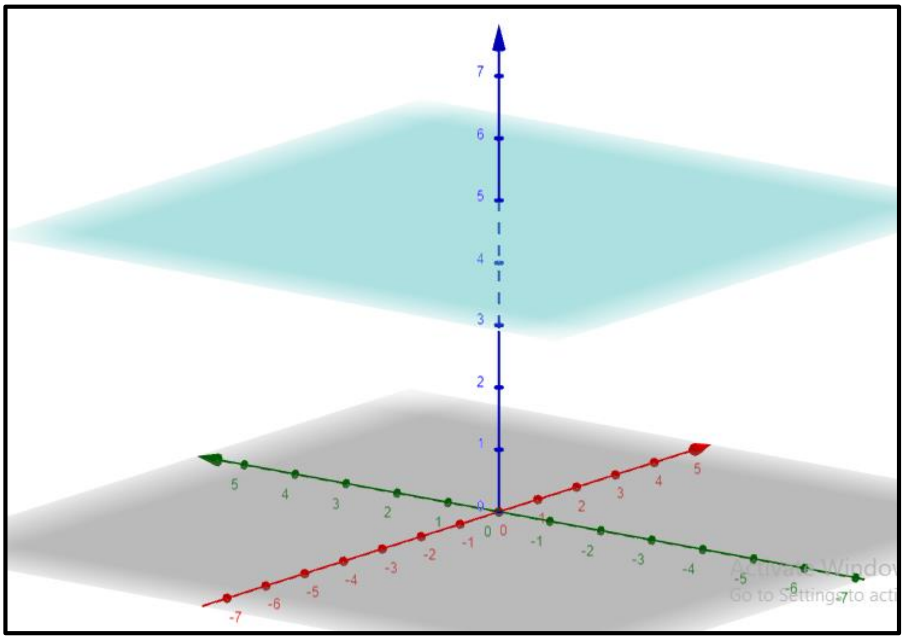

Figure 12 : Surface 02

z=5 is a plane parallel to plane XOY with an intersection with z-axis at z=5

6. Tetrahedron

a = (i+4j-2k)

b = (3i-5j+k)

c = (-4i+3j+6k)

.$[a, b, c] = c.(a \times b)$

.$\begin{bmatrix} 1 & 4 & -2 \\ 3 & -5 & 1 \\ -4 & 3 & 6 \end{bmatrix}$

$=1[(-5)\times 6 - 1\times 3] - 4[3\times 6 - 1\times(-4)] + (-2)[3\times 3 - (-5)\times(-4)]$

=1[-30-3]-4[18-(-4)] + (-2) [9-20]

=1× (-33)-4×22 + (-2) × (-11)

=-33-88+22

= **- 99**

So, $\mathbf{c}.(\mathbf{a}\times\mathbf{b})$ = -99

The formula of the tetrahedron

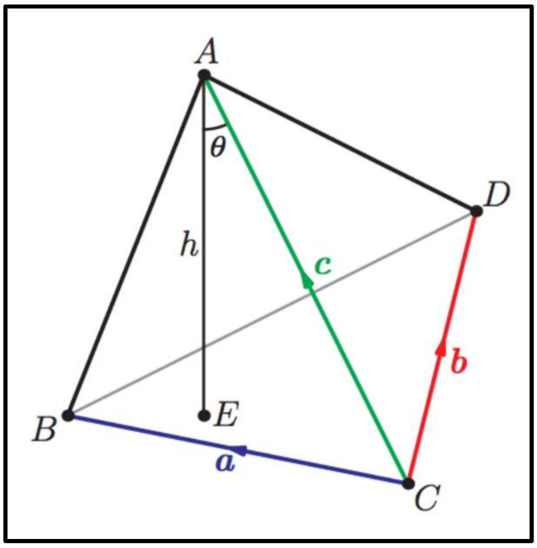

Figure 13 : Formula for the Tetrahedron

Here the tetrahedron is $\frac{1}{6}$ of the volume of the parallelepiped formed by $\vec{a}$, $\vec{b}$, $\vec{c}$. The volume of the parallelepiped is the scalar triple product | (a × b) ·c|| (a × b) ·c|.

Thus, the volume of a tetrahedron is,

$$\mathbf{V} = \frac{\mathbf{1}}{\mathbf{3}}[| \ \frac{\mathbf{1}}{\mathbf{2}}\ (|\mathbf{a} \times \mathbf{b}\ |)\,.\,|\mathbf{c}|\mathbf{cos\theta}|] = \frac{\|\mathbf{c}.(\mathbf{a} \times \mathbf{b})\|}{\mathbf{6}}$$

Applying **c.** (**a** × **b**) = -99 to V = $\frac{\|c.(a \times b)\|}{6}$

V = $\frac{\mathbf{99}}{\mathbf{6}}$ **= 16.5 units**

The volume of Tetrahedron is 16.5 units.

Activity 4

Part 1

1.

i. **f(x) = 2x – 3x4 + 5x + 8**

$$f'(x) = \frac{d}{dx}(\ 2x - 3x^4 + 5x + 8)$$

$$f'(x) = \frac{d}{dx}(7x - 3x^4 + 8)$$

$$f'(x) = \frac{d}{dx}(7x) + \frac{d}{dx}(-3x^4 + \frac{d}{dx}(8))$$

$$f'(x) = 7 - 3 \times 4x^3 + 0$$

$$\boldsymbol{f'(x) = 7 - 12x^3}$$

iii. **f(x) = cos(2x) + 4x2 – 3**

$$f'(x) = \frac{d}{dx}(\cos(2x) + 4x^2 - 3)$$

$$f'(x) = \frac{d}{dx}(\cos(2x)) + \frac{d}{dx}(4x^2) - \frac{d}{dx}(3)$$

$$f'(x) = -2\sin(2x) + 8x$$

$$\boldsymbol{f'(x) = 8x - 2\sin(2x)}$$

2. **Function of velocity**

Displacement is S (t) = 5t^3 – 3t^2 + 6t

Velocity is V (t)

V = $\frac{s}{t}$

V (t) = $\frac{d[st]}{dt}$

$=\frac{d(5t^3)}{dt} - \frac{d(3t^2)}{dt} + \frac{d(6t)}{dt}$

$=5(3t^2) - 3(2t) + 6(1)$

$=\mathbf{15t^2 - 6t + 6}$

Part 2

3. **Area between two curves**

   $\boldsymbol{f(x) = 2x^2 + 1}$

   $\boldsymbol{g(x) = 8 - 2x}$

   The boundaries are x = -2 and x = 1

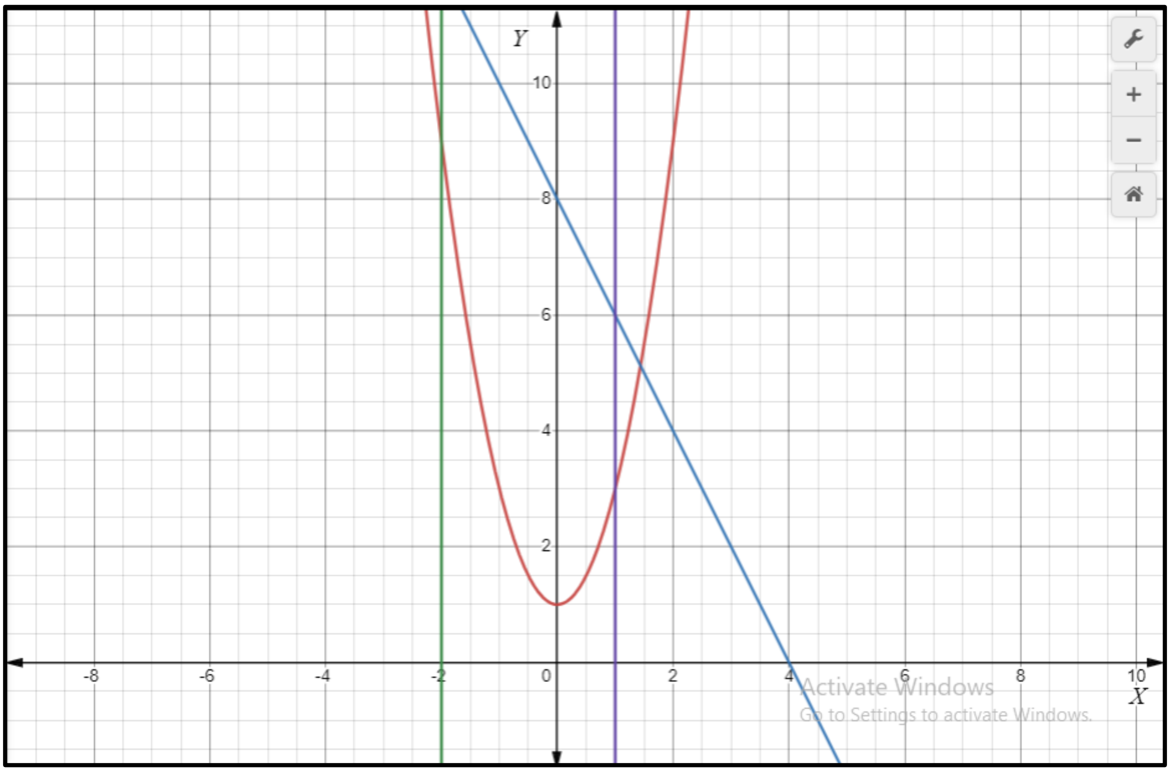

Figure 14 : Graph

Area = $\int_{(-2)}^{1}[(8-2x)-(2x^2+1)]dx$

$=\int_{(-2)}^{1}[7-2x-2x^2]dx$

$=\int_{(-2)}^{1}7dx - \int_{(-2)}^{1}(2x)dx - \int_{(-2)}^{1}(2x^2)dx$

$=\left[7x - 2x\left(\frac{x^2}{2}\right) - 2\times\left(\frac{x^3}{3}\right)\right]_{-2}^{1}$

$=\left[7x - x^2 - \frac{2x^3}{3}\right]\begin{matrix}1\\-2\end{matrix}$

$=\left[7(1) - (1^2) - \frac{2(1)^3}{3}\right] - \left[7(-2) - (-2)^2 - \frac{2(-2)^3}{3}\right]$

=**18 square units**

4. **Oxygen level in 3 years**

Let f (t) is the tree plantation of the forest at time t.

Therefore, the increasing rate is:

$F'(x) = 3t^2 + 5t + 6$

The increase of the tree plantation during the next 3 years = $\int_0^3 f'(x)dt$

$$\int_0^3 (3t^2 + 5t + 6)dt$$

$=\int_0^3 (3t^2)dt + \int_0^3 (3t)dt + \int_0^3 6\,dt$

$=\left[3 \times \left(\frac{t^3}{3}\right) + 5 \times \left(\frac{t^2}{2}\right) + 6t\right]\begin{matrix}3\\0\end{matrix}$

$=\left[t^3 + \frac{5t^2}{6} + 6t\right]\begin{matrix}3\\0\end{matrix}$

$=\left[(3^3) + \frac{5(3)^2}{6} + 6(3)\right] - \left[(0)^3 - \frac{5(0)^2}{2} + 6(0)\right]$

=67.5×100

Therefore, the increase of the Oxygen level during the next 3 years = $\frac{4}{100} \times 6750$

= **270 units**

Part 3

5. **Graph Sketch**

$f\ (x)\ = x^5 - \ 6x^3\ +\ 3$

$f'(x) = \frac{d}{dx}(x^5 - \ 6x^3\ +\ 3)$

$f'(x) = \frac{d}{dx}(x^5) + \frac{d}{dx}(-6x^3) + \frac{d}{dx}(3)$

$f'(x) = 5x^4 - 6 \times 3x^2 + 0$

$\boldsymbol{f'(x) = 5x^4 - 18x^2}$

Intervals of Increase / Decrease

$f'(x) = 0$

$f'(x) = 5x^4 - 18x^2$

$= x^2(5x^2 - 18) = 0$

$x_1 = 0, \quad x_2 = {}^{-}\sqrt{\frac{18}{5}}, \quad x_3 = {}^{+}\sqrt{\frac{18}{5}}$

Critical Numbers:

$x_1 = 0, \quad x_2 = -1.89737\ , x_3 = \ 1.89737$

Interval	Test (x=?)	$\boldsymbol{f'(x) = 5x^4 - 18x^2}$	$\boldsymbol{f(x)\ is\ ...}$
$x < -1.89737$	$x = -2$	f(-2) > 0	Increasing
$-1.89737 < x < 0$	$x = -1$	f(-1) < 0	Decreasing
$0 < x < 1.89737$	$x = 1$	f(1) < 0	Decreasing
$x > 1.89737$	$x = 2$	f(2) > 0	Increasing

Interval of increase: $(-\infty, \boldsymbol{-1.89737})$ and $(\boldsymbol{1.89737}, +\infty)$

Interval of decrease: $(\boldsymbol{-1.89737, 0})$ and $(\boldsymbol{0, 1.89737})$

First Derivative Test

$\mathrm{x = -1.89737: f'(x)}$ goes from $(-)$ to $(+) \rightarrow$ local min at x

$= \ -1.89737$ and the value is $\boldsymbol{f\ (-1.89737) = 19.39}$

$x = 0: f'(x)$ goes from $(+)$ to $(-)$ $\rightarrow$ local min at $x = -1.89737$ and the value is $\mathbf{f\ (0) = 3}$

$x = 1.89737: f'(x)$ goes from $(-)$ to $(+)$ $\rightarrow$ local min at $x = 1.89737$ and the value is $\mathbf{f\ (1.89737) = -13.39}$

2nd Derivative Test

$f'(x) = 5x^4 - 18x^2$

$$f''(x) = \frac{d}{dx}(5x^4) - \frac{d}{dx}(18x^2)$$

$$\boldsymbol{f''(x) = 20x^3 - 36x}$$

$x = -1.89737: f''(-1.89737) < 0$

$x = 0: f''(0) > 0$

$x = 1.89737: f''(1.89737) < 0$

Interval of Concavity

$f''(x) = 0$

$f''(x) = 20x^3 - 36x = 0$

$$20x^3 - 36x = 0$$

$$x = 0, x = \frac{-3\sqrt{5}}{5}, x = \frac{3\sqrt{5}}{5}$$

$$\boldsymbol{x = 0, x = (-1.34), x = (1.34)}$$

Interval	Test (x=?)	$\boldsymbol{f''(x) = 20x^3 - 36x}$	$\boldsymbol{f(x)\ is\ concave\ ...}$
$x < -1.34$	$x = -2$	f(-2) < 0	Down
$-1.34 < x < 0$	$x = -1$	f(-1) > 0	Up
$0 < x < 1.34$	$x = 1$	f(1) < 0	Down
$x > 1.34$	$x = 2$	f(2) > 0	Up

Concave Up: $(\mathbf{1.34, 0})$ **and** $(\mathbf{1.34}, +\infty)$

Concave Down: $(-\infty, \mathbf{-1.34})$ **and** $(\mathbf{0.1.34})$

Inflection Points

When x = (-1.34), y = f (1.34)

When x = 0, y f (0) = 3

When x = 1.34, y = f (1.34) = (-7.12)

Intercept Points of x and y/f(x) axis

$f\ (x)\ = x^5 - 6x^3 + 3$

y/f(x) axis intercept points :

When x = 0, f (0) = 3; **(0, 3)**

X axis intercept points:

When f(x) = 0, x = 0.83, x = -2.49, x = 2.41

Local Maximum Points – (-1.89, 19.39)

Local Minimum Points – (1.89, -13.39)

Inflection Points – (0, 3), (-1.34, 14.64) and (1.34, -7.12)

Y and X axis intercept Points

Y axis – (0, 3)

X axis - (0.83, 0), (-2.49, 0), (2.41, 0)

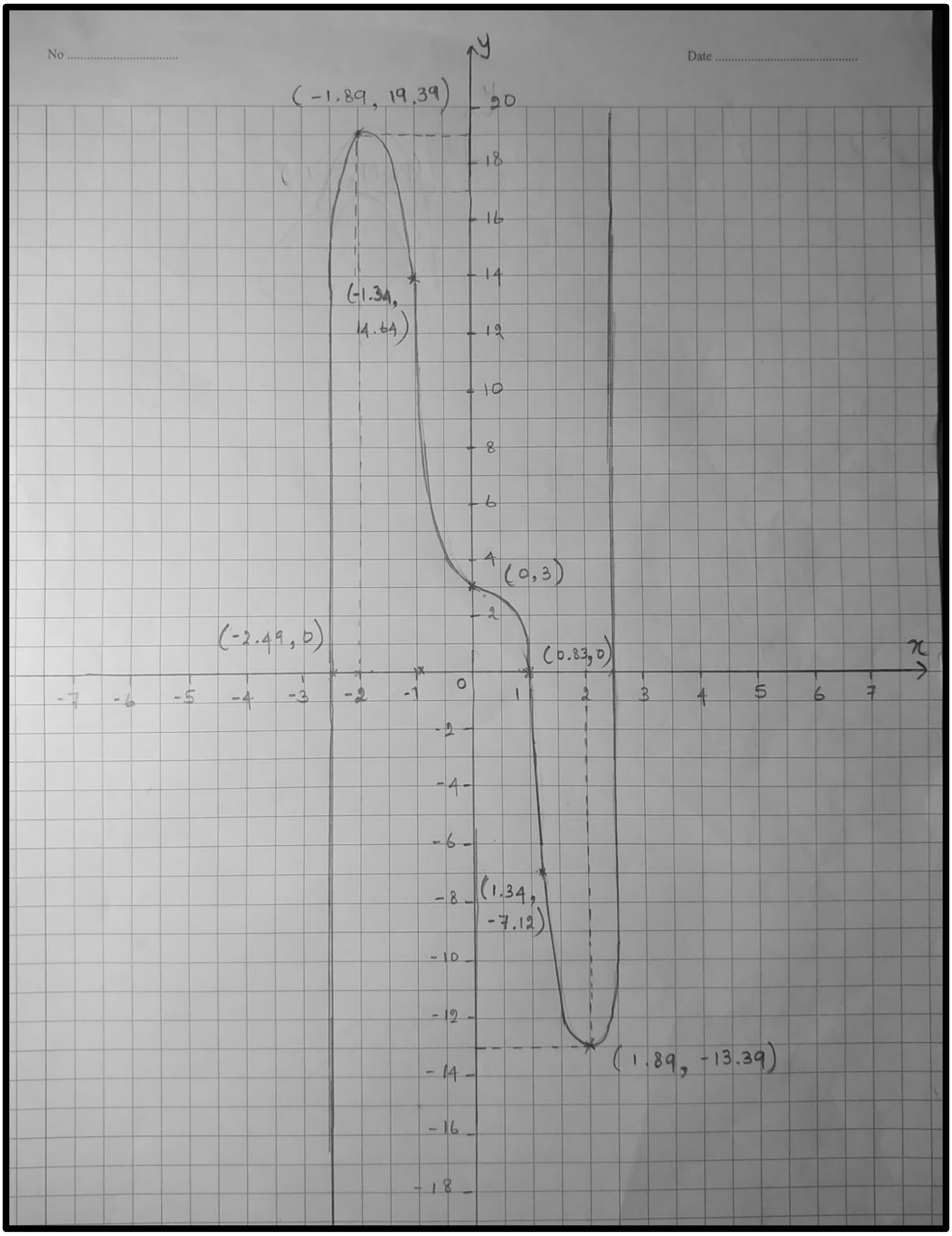

Figure 15 : Graph Answer

6. Maximum and minimum points

f (x)= $2x^3 - 4x^4 + 5x^2$

1st Derivative Test

$$f'(x) = \frac{d}{dx}(2x^3 - 4x^4 + 5x^2)$$

$$f'(x) = \frac{d}{dx}(2x^3) + \frac{d}{dx}(-4x^4) + \frac{d}{dx}(5x^2)$$

$$f'(x) = 2 \times 3x^2 - 4 \times 4x^3 + 5 \times 2x$$

$$\boldsymbol{f'(x) = 6x^2 - 16x^3 + 10x}$$

$$= 2x(3x - 8x^2 + 5) = 0$$

$$\boldsymbol{x_1 = -\frac{5}{8}}, \quad \boldsymbol{x_2 = 0}, \ \boldsymbol{x_3 = 1}$$

When f ($-\frac{5}{8}$) = 0.85

When f (0) = 0

When f (1) = 3

Interval	**Test (x=?)**	$\boldsymbol{f'(x) = 6x^2 - 16x^3 + 10x}$	$\boldsymbol{f(x)\ is}$ ...
$x < -\frac{5}{8}$	$x = -10$	f(-10) > 0	Increasing
$-\frac{5}{8} < x < 0$	$x = -0.5$	f(-0.5) < 0	Decreasing
$0 < x < 1$	$x = 0.5$	f(0.5) > 0	Increasing
$x > 1$	$x = 2$	f(2) < 0	Decreasing

Local Maximum Points – ($-\frac{5}{8}$, 0.85) and (1, 3)

Local Minimum Points – (0, 0)

2nd Derivation Test

$$f''(x) = \frac{d}{dx}(6x^2 - 16x^3 + 10x)$$

$$f''(x) = \frac{d}{dx}(6x^2) + \frac{dy}{dx}(-16x^3) + \frac{dy}{dx}(10x)$$

$$f''(x) = 6 \times 2x - 16 \times 3x^2 + 10$$

$$\boldsymbol{f''(x) = 12x - 48x^2 + 10}$$

$$= 2(6x - 24x^2 + 5) = 0$$

$$\boldsymbol{x_1 = \frac{3 - \sqrt{129}}{24}}, \quad \boldsymbol{x_2 = \frac{3 + \sqrt{129}}{24}}$$

$$\boldsymbol{x_1 \approx -0.348}, \quad \boldsymbol{x_2 \approx 0.598}$$

When f (0.59) = 4.7

When f (0.34) = 3.46

Local Minimum Points – $(\mathbf{0.59}, \mathbf{4.7})$ and (0.34, 3.46)

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