SADS-II PROBLEM-3 **Date: 14-02-2024**

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You are given the random vector $X'=[X_1, X_2, .X_3, X_4]$ with mean vector $\mu_x^{'}=[4,3,2,1]$ and variance-covariance matrix

$$\sum \square \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

Partition of X as $X=\begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ \dots. \\ X^{(2)} \end{bmatrix}$

Let $A=[1 \quad 2]$, $B=\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and consider the linear transformation $\mathrm{AX}^{(1)}$ and $\mathrm{BX}^{(2)}$.

Find (a)$E\left(X^{(1)}\right)$, (b)$E\left(\mathrm{AX}^{(1)}\right)$ (c) $\mathrm{Cov}\left(X^{(1)}\right)$(d) $\mathrm{Cov}\left(\mathrm{AX}^{(1)}\right)$ (e) $E\left(X^{(2)}\right)$ (f) $E\left(\mathrm{BX}^{(2)}\right)$ (g) $\mathrm{Cov}\left(X^{(2)}\right)$(h) $\mathrm{Cov}\left(\mathrm{BX}^{(2)}\right)$ (i) $\mathrm{Cov}\left(X^{(1)}, X^{(2)}\right)$(j) $\mathrm{Cov}\left(\mathrm{AX}^{(1)}, \mathrm{BX}^{(2)}\right)$

SOLUTION:-

The mean vector is,

```
> Mean_vec=matrix(c(4,3,2,1), nrow=4,ncol=1)
> Mean_vec
      [,1]
[1,]    4
[2,]    3
[3,]    2
[4,]    1
```

The variance and covariance matrix is,

```
> Co_var_mat=matrix(c(3,0,2,2,0,1,1,0,2,1,9,-2,2,0,-2,4),nrow=4)
> Co_var_mat
      [,1] [,2] [,3] [,4]
[1,]    3    0    2    2
[2,]    0    1    1    0
[3,]    2    1    9   -2
[4,]    2    0   -2    4
```

The matrix A and matrix B is given by,

```
> mat_A=matrix(c(1,2),ncol=2)
> mat_A
      [,1] [,2]
[1,]    1    2
```

```
> mat_B=matrix(c(1,2,-2,-1),nrow=2)
> mat_B
     [,1] [,2]
[1,]    1   -2
[2,]    2   -1
```

(a) $E(X^{(1)})$

```
> expt_x1=matrix(c(Mean_vec[1:2]))

> expt_x1
     [,1]
[1,]    4
[2,]    3
```

(b) $E(AX^{(1)})$

```
> exp_AX1=mat_A %*%expt_x1
> exp_AX1
     [,1]
[1,]   10
```

(c) $COV(X^{(1)})$

```
> cov_X1=matrix(c(3,0,0,1),nrow=2,ncol=2)
> cov_X1
     [,1] [,2]
[1,]    3    0
[2,]    0    1
```

(d) $COV(AX^{(1)})$

```
> cov_AX1=(mat_A) %*% (cov_X1) %*% t(mat_A)
> cov_AX1
     [,1]
[1,]    7
```

(e) $E(X^{(2)})$

```
> expt_x2=matrix(c(Mean_vec[3:4]))
> expt_x2
     [,1]
[1,]    2
[2,]    1
```

(f) $E(BX^{(2)})$

```
> exp_BX2=(mat_B) %*%expt_x2
> exp_BX2
     [,1]
[1,]    0
[2,]    3
```

(g) $COV(X^{(2)})$

```
> cov_X2=matrix(c(9,-2,-2,4),nrow=2,ncol=2)
> cov_X2
     [,1] [,2]
[1,]    9   -2
[2,]   -2    4
```

(h) $COV(BX^{(2)})$

```
> cov_BX2=(mat_B) %*% (cov_X2) %*% t(mat_B)
> cov_BX2
     [,1] [,2]
```

```
[1,]   33   36
[2,]   36   48
```

(i)COV($X^{(1)}$,$X^{(2)}$)

```
> cov_X1X2=Co_var_mat[1:2,3:4]
> cov_X1X2
      [,1] [,2]
[1,]    2    2
[2,]    1    0
```

(j)COV($AX^{(1)}$,$AX^{(2)}$)

```
> cov_AX1.BX1=mat_A %*% (cov_X1X2) %*% (mat_B)
> cov_AX1.BX1
      [,1] [,2]
[1,]    8  -10
```