Principle Component

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Determine the first and second principal components Y1 and Y2for the covariance matrix of the random vector $X'=[X1\ X2\ X3]$ sigma =matrix(c(1,-2,0,-2,5,0,0,0,2),nrow=3,ncol=3)

Also calculate the variance of the first and second principal components and proportion of the total population variance explained by the first principal component. Also calculate correlation coefficient between first and second(y1,y2) and random variable x1,x2,x3.

formulation of Var Cov Matrix

```
SIGMA=matrix(c(1,-2,0,-2,5,0,0,0,2),nrow = 3,ncol=3)
SIGMA
```

```
## [,1] [,2] [,3]
## [1,] 1 -2 0
## [2,] -2 5 0
## [3,] 0 0 2
```

finding Eigen Value and Eigen Vector

```
EIGEN_VALUE=eigen(SIGMA)
EIGEN_VALUE
```

for normalize eigen vector

```
VECTORS=EIGEN_VALUE$vectors
VECTORS
```

```
##
               [,1] [,2]
                               [,3]
 ## [1,] -0.3826834
                     0 0.9238795
 ## [2,] 0.9238795
                        0 0.3826834
 ## [3,] 0.0000000
                       1 0.0000000
 E1=VECTORS[,1]
 E1
 ## [1] -0.3826834 0.9238795 0.0000000
 E2=VECTORS[,2]
 E2
 ## [1] 0 0 1
 E3=VECTORS[,3]
 E3
 ## [1] 0.9238795 0.3826834 0.0000000
##Finding varaiance of Principal components
 #Y1 = E1.X = -0.383(X1) - .924(X3)
 #VAR_Y1=var(Y1)
 VAR_Y1=(((-0.3826834)^2)*(1)+((0.9238795 )^2)*(5)+2*(-0.3826834)*(0.9238795 )*(-2))
 VAR_Y1
 ## [1] 5.828427
 #Y3=E3.X=0.924(X1)+0.383(X3)
 VAR_{Y3} = (((0.9238795)^2)^*(1) + ((0.3826834)^2)^*(5) + 2^*(0.9238795)^*(0.3826834)^*(-2))
 VAR_Y3
 ## [1] 0.1715729
 #Y2=E2.X=1(X3)
   VAR_Y2=1*2
   VAR_Y2
 ## [1] 2
```

###correlation=((eii*sqrt(lamda1))/sqrt(var1)

```
var_x1=1
var_x2=5
var_x3=2
cov_x1_x2=-2
cov_x1_x3=0
cov_x2_x3=0

cor_y1_x1=(-0.3826834 *(sqrt(5.8284271))/sqrt(1))
cor_y1_x1
```

```
## [1] -0.9238795
```

```
cor_y1_x2=(0.9238795*(sqrt(5.8284271))/sqrt(5))
cor_y1_x2
```

[1] 0.9974842

```
cor_y1_x3=0*(sqrt(5.8284271))/sqrt(2)
cor_y1_x3
```

[1] 0