ELEVATE
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CLASS 9

MOCK PAPER

MATHEMATICS [SA1]

Time : 3 Hrs. *MM : 90*

GENERAL INSTRUCTIONS

I. *All questions are compulsory.*

II. *The question paper consists of 34 questions divided into four sections A, B, C and D.*

III. *Section A contains 8 questions of 1 mark each, which are multiple choice type questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each, Section D contains 10 questions of 4 marks each.*

IV. *There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, three questions of 3 marks and two questions of 4 marks.*

V. *Use of calculator is not permitted.*

SECTION-A

1. Rationalizing factor of $1+\sqrt{2}+\sqrt{3}$

(a) $1+\sqrt{2}-\sqrt{3}$ (b) 2

(c) 4 (d) $1+\sqrt{2}+\sqrt{3}$

2. If $a+b+c=0$, then $a^3+b^3+c^3$ is equal to

(a) 5 abc (b) $2abc$

(c) $abc+(abc)^0$ (d) $3abc$

3. Factors of polynomial $12x^2-7x+1$ are

(a) $(3x-1)(4x-1)$ (b) $(4x+1)(3x-1)$

(c) $12\left(x+\frac{1}{3}\right)\left(x-\frac{1}{4}\right)$ (d) $12\left(x+\frac{1}{4}\right)\left(x-\frac{1}{3}\right)$

4. An angle is 14° more than its complementary angle then angle is

(a) 38° (b) 52°

(c) 50° (d) None of these

5. Expansion of $\left(x+\frac{1}{x}\right)^2$ is

(a) $x^2+2x+\frac{1}{x^2}$ (b) $x^2-2x+\frac{1}{x^2}$

(c) $x^2+2+\frac{1}{x^2}$ (d) $x^2-2+\frac{1}{x^2}$

6. In a triangle ABC, $\angle A+\angle B=144°$ and $\angle A+\angle C=124°$ then $\angle B=$

(a) 56° (b) 60°

(c) 65° (d) 45°

7. The side of an isosceles right triangle of hypotenuse $4\sqrt{2}$ cm is

(a) 8 cm (b) 6 cm

(c) 4 cm (d) $4\sqrt{3}$ cm

8. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be

(a) 24 cm^2 (b) 40 cm^2

(c) 48 cm^2 (d) 80 cm^2

SECTION-B

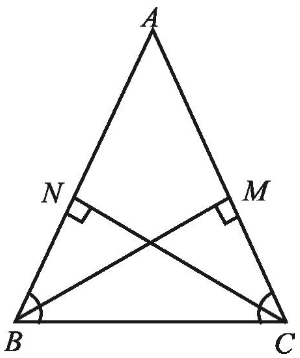
9. If $\dfrac{3+\sqrt{5}}{4-2\sqrt{5}} = p+q\sqrt{5}$, where p and q are rational numbers, find the value of p and q.

10. Factorize : $x^2 - x\left(\dfrac{a^2-1}{a}\right) - 1$.

11. Simplify: $(a+b)^3 + (a-b)^3 + 6a(a^2-b^2)$

12. The side BC of a triangle ABC is produced to D. The bisector of the $\angle$ A meets BC in L. Prove that $\angle ABC + \angle ACD = 2\angle ALC$

13. In the adjoining figure, AB = AC. Prove that BM = CN



Or

Prove that the sides opposite to equal angles of a triangle are equal.

14. Points (6, –6) and (–6, 6) lie in the same quadrant, State true or false and justify your answer.

SECTION-C

15. Find three rational numbers between $\dfrac{1}{5}$ and $\dfrac{7}{10}$.

Or

Express $0.\overline{001}$ in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

16. Simplify the following by rationalising the denominators : $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \dfrac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}}$

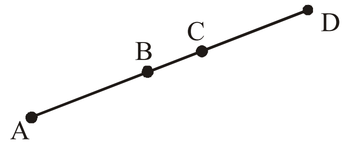
17. Factorize : $p^3 - p^2q + \dfrac{1}{3}pq^2 - \dfrac{1}{27}q^3$

Or

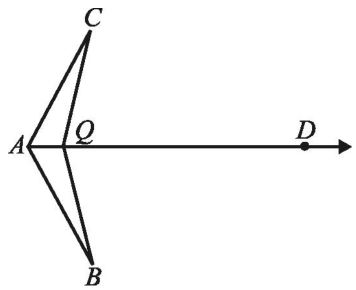
If $\boldsymbol{x} = \dfrac{\mathbf{4}}{\mathbf{3}}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$, then find the value of k.

18. Use the factor theorem to factorize $x^3 + x^2 - 4x - 4$ completely.

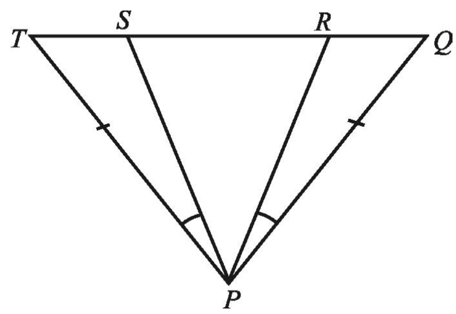
19. If $AC = BD$, then prove that $AB = CD$.



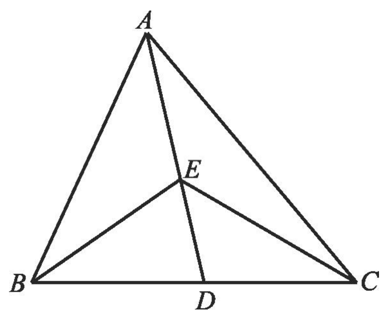
20. In figure, $\angle CQD = \angle BQD$ and AD is the bisector of $\angle BAC$. Prove that $\Delta CAQ \cong \Delta BAQ$ and hence $CQ = BQ$.



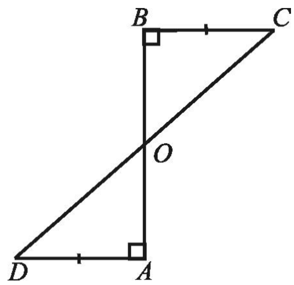
21. If in figure, $PQ = PT$ and $\angle TPS = \angle QPR$, prove that triangle PRS is isosceles.



22. In figure, E is any point on median AD of a ΔABC. Show that ar (ΔABE) = ar (ΔACE).

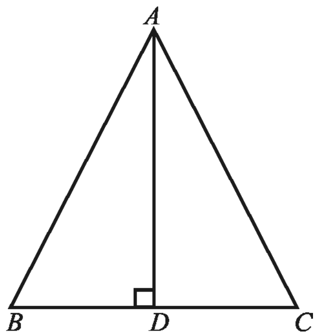


23. *AD* and *BC* are equal perpendiculars to a line segment *AB* (See figure). Show that *CD* bisects *AB*.



Or

In ΔABC, *AD* is the perpendicular bisector of *BC*. Show that ΔABC is an isosceles triangle in which $AB = AC$.



24. The sides of a triangular plot are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area.

SECTION-D

25. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

26. The teacher asked the students to write $0.19\overline{6}$ in the form of $\frac{p}{q}, q \neq 0,$ p and q are integers. Ravi said $\frac{79}{900}$ while Anu said no it is $\frac{59}{300}$. Is Anu correct? Justify your answer. Which values are shown by Anu?

27. Find α and β if x + 1 and x + 2 are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Or

Find the value of p and q, if (x + 3) and (x – 4) are factors of $x^3 - px^2 - qx + 24$.

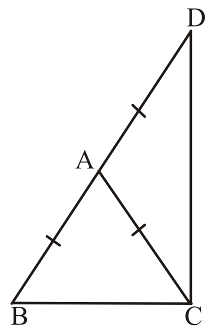
28. Evaluate each of the following using suitable identities :

(*i*) $(999)^3$ (*ii*) 95×96

29. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by $(x + 1)$ and $(x - 1)$ leaves the remainders 19 and 5 respectively. Find the values of *a* and *b*. Hence, find remainder when $p(x)$ is divided by $(x + 2)$.

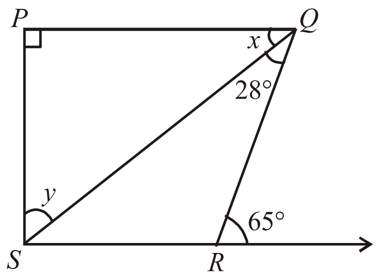
30. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

31. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB.



Show that $\angle$ BCD is a right angle.

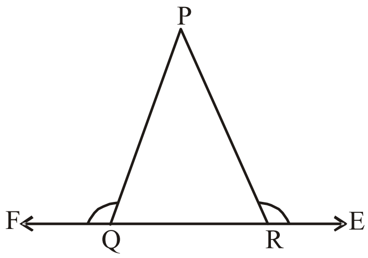
32. In figure, if $PQ \perp PS, PQ \parallel SR, \angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of *x* and *y*.



33. An exterior angle of a triangle is 115° and one of the opposite angles is 35°. Find the other two angles.

Or

Side QR of a ΔPQR is produced in both the directions. Prove that the sum of the two exterior angles so formed is greater than 180°.



34. Plot the points (2, 0), (2, 3), (0, 6), (–2, 3) and (–2, 0) and join them in order. Find the type of figure thus formed.

HINTS & SOLUTIONS

SECTION-A

1. (a) Since rationalising factor of $(a+b) = a - b$

similarly rationalising factor of $(1+\sqrt{2})+\sqrt{3}$

$= (1+\sqrt{2})-\sqrt{3}$ **(1 mark)**

2. (d) $a^3 + b^3 + c^2 - 3abc = (a+b+c)$

$(a^3 + b^3 + c^3 - ab - bc - ca)$ **(½ mark)**

$\Rightarrow a^3 + b^3 + c^3 = 3abc$ **(½ mark)**

3. (a) $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x-1) - (3x-1)$

$= (4x-1)(3x-1)$ **(1 mark)**

4. (a) $x + x + 14 = 90°$

$2x = 76 \Rightarrow x = 38°$ **(1 mark)**

5. (c) $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2\cdot x\cdot\frac{1}{x} = x^2 + \frac{1}{x^2} + 2$

(1 mark)

6. (a) $\angle A + \angle C = 124°, \angle A + \angle B = 144°$

$\Rightarrow 144° - \angle B + \angle B + \angle B - 20 = 180$

(½ mark)

$\Rightarrow \angle B = 56°$ **(½ mark)**

7. (c) Let side be 'x'

$x^2 + x^2 = (4\sqrt{2})^2$ **(½ mark)**

$\Rightarrow x^2 = 16 \Rightarrow x = 4\text{cm}$ **(½ mark)**

8. (a) Third side $= \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$

$= \sqrt{100-64} = 6\text{ cm}$

$s = \frac{10+8+6}{2}$

$s = 12\text{ cm}$ **(½ mark)**

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-10)(12-8)(12-6)}$

$= \sqrt{12\times2\times4\times6}\text{ cm}^2 = 24\text{ cm}^2$ **(½ mark)**

SECTION-B

9. We have $\frac{3+\sqrt{5}}{4-2\sqrt{5}} = p + q\sqrt{5}$

LHS $\frac{3+\sqrt{5}}{4-2\sqrt{5}} \times \frac{4+2\sqrt{5}}{4+2\sqrt{5}}$ **(½ mark)**

$= \frac{12+10\sqrt{5}+10}{16-20}$

$= \frac{22+10\sqrt{5}}{-4}$ **(½ mark)**

Now, $-\left(\frac{22}{4}+\frac{10\sqrt{5}}{4}\right) = p + q\sqrt{5}$

$\Rightarrow \frac{-11}{2} - \frac{5\sqrt{5}}{2} = p + q\sqrt{5}$ **(½ mark)**

$\Rightarrow p = -\frac{11}{2}$ and $q = \frac{-5}{2}$ **(½ mark)**

10. $x^2 - x\left(\frac{a^2-1}{a}\right) - 1 = x^2 - x\left(a - \frac{1}{a}\right) - 1$

(½ mark)

$= x^2 - ax + \frac{x}{a} - 1$ **(½ mark)**

$= x(x-a) + \frac{1}{a}(x-a)$ **(½ mark)**

$= (x-a)\left(x+\frac{1}{a}\right)$ **(½ mark)**

11. Consider $(a+b)^3 + (a-b)^3 + 6a(a^2-b^2)$

$= (a+b)^3 + (a-b)^3 + 3(2a)(a+b)(a-b).$

(½ mark)

(Using Identity $(a^2 - b^2) = (a-b)(a+b)$)

$= (a+b)^3 + (a-b)^3 + 3(a+b)(a-b)\{(a+b)+(a-b)\}$

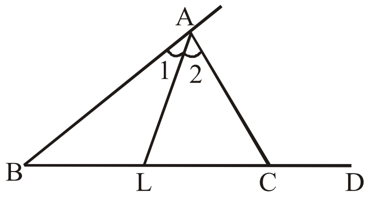
(½ mark)

$= \{(a+b)+(a-b)\}^3$ **(½ mark)**

$= (2a)^3 = 8a^3$ **(½ mark)**

12. $\angle ALC = \angle 1 + \angle B$

$\Rightarrow \quad 2\angle ALC = 2\angle 1 + 2\angle B$ **(½ mark)**



$\Rightarrow \quad 2\angle ALC = \angle A + 2\angle B$ **(½ mark)**

$\Rightarrow \quad 2\angle ALC = (\angle ACD - \angle B) + 2\angle B$ **(½ mark)**

$(\because \angle ACD = \angle A + \angle B)$

$\Rightarrow \quad 2\angle ALC = \angle ACD + \angle B = \angle ACD + \angle ABC$ **(½ mark)**

13. In ΔABC, $AB = AC$ (given)

$\angle ABC = \angle ACB$ **(½ mark)**

(angles opposite to equal sides are equal)

In Δs BCM and CBN,

$\angle N = \angle M$ (each $= 90°$)

$\angle ABC = \angle ACB$ (from above)

$BC = BC$ (common)

$\therefore \quad \Delta BCM \cong \Delta CBN$ **(1 mark)**

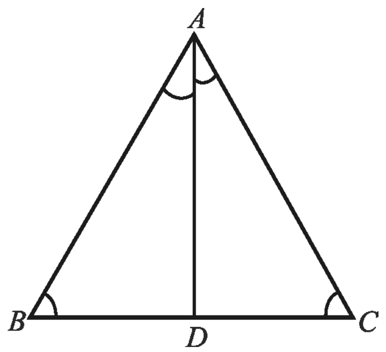
(A.A.S. rule of congruency)

$\Rightarrow \quad BM = CN$ (CPCT) **(½ mark)**

Or

Given : ΔABC, in which $\angle B = \angle C$

To prove : $AB = AC$



(½ mark)

Construction : Draw AD, the bisector of angle $\angle BAC$ which meets BC at D.

(½ mark)

Proof : In ΔABD and ΔACD

$\angle B = \angle C$ (Given)

$AD = AD$ (Common side)

$\angle BAD = \angle CAD$ (By construction)

Therefore $\Delta ABD \cong \Delta ACD$ (By ASA)

(½ mark)

Hence corresponding sides, $AB = AC$

(½ mark)

14. False, because (6, –6) lies in IV quadrant and (–6, 6) lie in II quadrant. **(2 marks)**

SECTION-C

15. One rational number between $\frac{1}{5}$ and $\frac{7}{10}$

$$= \frac{1}{2}\left(\frac{1}{5} + \frac{7}{10}\right) = \frac{1}{2}\left[\frac{2+7}{10}\right] = \frac{9}{20}$$ **(1 mark)**

Second rational number between $\frac{1}{5}$ and $\frac{7}{10}$

$$= \frac{1}{2}\left(\frac{1}{5} + \frac{7}{10}\right)$$

$$= \frac{1}{2}\left(\frac{1}{5} + \frac{9}{20}\right) = \frac{1}{2}\left(\frac{4+9}{20}\right) = \frac{13}{40}$$ **(1 mark)**

Third rational number between $\frac{1}{5}$ and

$$\frac{7}{10} = \frac{1}{2}\left(\frac{13}{40} + \frac{1}{5}\right) = \frac{1}{2}\left(\frac{13+8}{40}\right) = \frac{21}{80}$$

The required three rational and numbers bewteen

$\frac{1}{5}$ and $\frac{7}{10}$ are $\frac{9}{20}, \frac{13}{40}$ and $\frac{21}{80}$ **(1 mark)**

Or

Let $x = 0.\overline{001} = 0.001001001........$

Multiplying both sides by 1000 (since three digits are repeating), we get

$1000x = 1.001001....$ **(½ mark)**

$\Rightarrow \quad 1000x = 1 + 0.001001001....$ **(½ mark)**

$\Rightarrow \quad 1000x = 1 + x$ **(½ mark)**

$\Rightarrow \quad 999x = 1$ **(½ mark)**

$\Rightarrow \quad x = \frac{1}{999}$ **(½ mark)**

Thus, $0.\overline{001} = \frac{1}{999}$ which is of the form $\frac{p}{q}$,

Where, $p = 1$ **(½ mark)**

$q = 999 \ (\neq 0)$.

16. Consider, $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\dfrac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}}$

$$=\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}+\frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}$$ **(1 mark)**

$$=\frac{2\sqrt{12}-2\sqrt{18}}{2-3}+\frac{6\sqrt{12}-6\sqrt{6}}{6-3}$$ **(½ mark)**

$=2\sqrt{18}-2\sqrt{12}+2\sqrt{12}-2\sqrt{6}$ **(½ mark)**

$=2\sqrt{18}-2\sqrt{6}$ **(½ mark)**

$=2\sqrt{6}(\sqrt{3}-1)$ **(½ mark)**

17. Consider $p^3-p^2q+\dfrac{1}{3}pq^2-\dfrac{1}{27}q^3$

$$=p^3-\frac{1}{27}q^3-p^2q+\frac{1}{3}pq^2$$ **(½ mark)**

$$=(p)^3-\left(\frac{1}{3}q\right)^3-3p\left(\frac{1}{3}q\right)\left(p-\frac{1}{3}q\right)$$ **(1 mark)**

$$=\left(p-\frac{1}{3}q\right)^3$$ **(1 mark)**

[Using identity: $a^3-b^3-3ab\,(a-b)$]

$$=\left(p-\frac{1}{3}q\right)\left(p-\frac{1}{3}q\right)\left(p-\frac{1}{3}q\right)$$ **(½ mark)**

Or

$f(x)=6x^3-11x^2+kx-20$

$$f\left(\frac{4}{3}\right)=6\left(\frac{4}{3}\right)^3-11\left(\frac{4}{3}\right)^2+k\left(\frac{4}{3}\right)-20=0$$ **(½ mark)**

$$\Rightarrow 6\left(\frac{64}{27}\right)-11\left(\frac{16}{9}\right)+\frac{4k}{3}-20=0$$ **(½ mark)**

$\Rightarrow 128-176+12k-180=0$ **(½ mark)**

$\Rightarrow 12k+128-356=0$ **(½ mark)**

$\Rightarrow 12k=228$ **(½ mark)**

$\Rightarrow \quad k=19$ **(½ mark)**

18. Let $f(x)=x^3+x^2-4x-4$

The constant term in $f(x)$ is -4

Its factors are $1, -1, 2, -2, 4$ and -4 **(½ mark)**

Now, $f(2)=2^3+2^2-4\times2-4=0$

$\therefore$ $(x-2)$ is a factor of $f(x)=x^3+x^2-4x-4$ **(½ mark)**

On dividing $f(x)$ by $(x-2)$,

$$\begin{array}{r|l|l}
x-2 & x^3+x^2-4x-4 & x^2+3x+2 \\
 & x^3-2x^2 & \\
 & -\quad+ & \\
\hline
 & 3x^2-4x-4 & \\
 & 3x^2-6x & \\
 & -\quad+ & \\
\hline
 & 2x-4 & \\
 & 2x-4 & \\
 & -\quad+ & \\
\hline
 & 0 &
\end{array}$$ **(½ mark)**

$\therefore \quad f(x)=(x-2)(x^2+3x+2)$ **(½ mark)**

$=(x-2)[x^2+x+2x+2]$ **(½ mark)**

$=(x-2)[x(x+1)+2(x+1)]$

$=(x-2)(x+2)(x+1)$ **(½ mark)**

19. $AC=BD$ ... (1)

$AC=AB+BC$ [B lies between A and C] ... (2)

(1 mark)

$BD=BC+CD$ [C lies between B and D] ... (3)

(1 mark)

Substituting (2) and (3) in (1), we get

$AB+BC=BC+CD$ **(½ mark)**

$AB=CD$ [Subtracting equals from equal]

(½ mark)

20. Since, AD is the bisector of $\angle BAC$ therefore, in ΔCAQ and ΔBAQ,

$\angle CAQ=\angle BAQ$ **(½ mark)**

Given that $\angle CQD=\angle BQD$ (Given)

$\Rightarrow$ $180°-\angle CQD=180°-\angle BQD$ **(½ mark)**

$\Rightarrow$ $\angle AQC=\angle AQB$ **(½ mark)**

$AQ=AQ$ (Common) **(½ mark)**

$\therefore$ $\Delta CAQ\cong\Delta BAQ$ (ASA Axiom) **(½ mark)**

$\therefore$ $CQ=BQ$ (C.P.C.T.) **(½ mark)**

21. From ΔPQT, we have given

$PQ = PT$ **(½ mark)**

Since, Angles opposite to equal sides

$\therefore \quad \angle PTQ = \angle PQT$ ....(1) **(½ mark)**

Now, In ΔPST and ΔPRQ,

We have

$PT = PQ$ and $\angle TPS = \angle QPR$ **(½ mark)**

$\therefore$ From (1)

$\angle PTQ = \angle PQT$

$\Rightarrow \quad \angle PTS = \angle PQR$

$\therefore \quad \Delta PST \cong \Delta PRQ$ (ASA Axiom) **(½ mark)**

and by C.P.C.T

$PS = PR$ **(½ mark)**

$\Rightarrow \quad \Delta PRS$ is isosceles. **(½ mark)**

22. Since, AD is a median in ΔABC which divides it into two triangles of equal areas. **(½ mark)**

$\therefore \quad \text{ar}(\Delta ABD) = \text{ar}(\Delta ACD)$ ....(1)

(½ mark)

Similarly, $\text{ar}(\Delta EBD) = \text{ar}(\Delta ECD)$ . ...(2)

(½ mark)

($\because$ ED is a median in ΔEBC)

Subtracting (2) from (1), we get

$\text{ar}(\Delta ABD) - \text{ar}(\Delta EBD)$ **(½ mark)**

$= \text{ar}(\Delta ACD) - \text{ar}(\Delta ECD)$ **(½ mark)**

$\Rightarrow \quad \text{ar}(\Delta ABE) = \text{ar}(\Delta ACE)$. **(½ mark)**

23. From ΔOAD and ΔOBC

we have given

$AD = BC$ **(½ mark)**

$\angle OAD = \angle OBC$ (Each = 90°) **(½ mark)**

$\angle AOD = \angle BOC$ (Vertically Opposite Angles)

(½ mark)

$\therefore$ By AAS rule $\Delta OAD \cong \Delta OBC$ **(½ mark)**

$\therefore$ By CPCT, $OA = OB$ **(½ mark)**

Thus CD bisects AB. **(½ mark)**

Or

ΔADB and ΔADC gives us that

$\angle ADB = \angle ADC$ (Each = 90°) **(½ mark)**

($\because$ AD is the perpendicular bisector of BC)

$\therefore \quad DB = DC$ **(½ mark)**

$AD = AD$ (Common) **(½ mark)**

$\therefore \quad \Delta ADB \cong \Delta ADC$ (By SAS Rule) **(1 mark)**

$\therefore \quad AB = AC$ (C.P.C.T) **(½ mark)**

Hence proved.

24. The sides are in the ratio 3 : 5 : 7.

So let the sides be $3x$, $5x$ and $7x$ respectively.

Now, perimeter = 300 m

$\Rightarrow \quad 3x + 5x + 7x = 300$ **(½ mark)**

$15x = 300$

$$x = \frac{300}{15} = 20$$ **(½ mark)**

So the sides are 60 m, 100 m and 140 m

a = 60 m

b = 100m **(½ mark)**

c = 140 m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{300}{2} = 150\,\text{m}$$ **(½ mark)**

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

(½ mark)

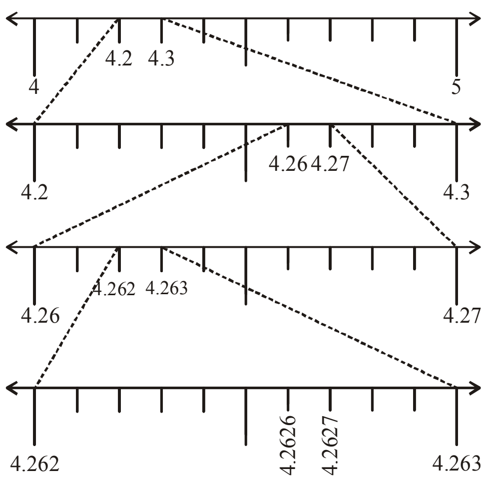
$$= \sqrt{150(150-60)(150-100)(150-140)}$$

$$= \sqrt{150 \times 90 \times 50 \times 10} = 1500\sqrt{3}\,\text{m}^2$$

(½ mark)

SECTION-D

25. $4.\overline{26} = 4.262626......$



(1 + 1 + 1 + 1 = 4 marks)

26. Let $x = 0.19666$ ...(i) **(½ mark)**

Multiplying both the sides by 100, we get

$100x = 19.666$ ...(ii) **(½ mark)**

Again, multiplying both the sides of (i) by 1000, we get

$1000x = 196.666$ ...(iii) **(½ mark)**

On subtracting (ii) from (iii), we obtain

$900x = 177$ **(½ mark)**

$x = \frac{177}{300}$ **(½ mark)**

$x = \frac{59}{300}$ **(½ mark)**

Yes, Anu is correct.

Values shown by Anu are: **(1 mark)**

(i) Knowledge

(ii) Curiosity

(iii) Truthfullness

27. Put $x + 1 = 0$ or $x = -1$ and $x + 2 = 0$ or $x = -2$ in $p(x)$ **(½ mark)**

Then, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$ **(½ mark)**

$\Rightarrow -1 + 3 + 2\alpha + \beta = 0$

$\Rightarrow \beta = -2\alpha - 2$ .... (1) **(½ mark)**

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow -8 + 12 + 4\alpha + \beta = 0$ **(½ mark)**

$\Rightarrow \beta = -4\alpha - 4$ .... (2) **(½ mark)**

By equalising both of the above equation

$-2\alpha - 2 = -4\alpha - 4$ **(½ mark)**

$\Rightarrow 2\alpha = -2 \Rightarrow \alpha = -1$

$\alpha = -1$ put in eq. (1) **(½ mark)**

$\Rightarrow \beta = -2(-1) - 2 = 2 - 2 = 0.$

Hence $\alpha = -1, \beta = 0$ **(½ mark)**

Or

Let $f(x) = x^3 - px^2 - qx + 24.$

Since, $(x + 3)$ is a factor of $f(x)$, so by factor theorem, $f(-3) = 0$ **(½ mark)**

$\therefore f(-3) = (-3)^3 - p(-3)^2 - q(-3) + 24 = 0$

$\therefore -27 - 9p + 3q + 24 = 0$ **(½ mark)**

$\therefore -3p + q - 1 = 0$ .... (1) **(½ mark)**

Similarly, if $(x - 4)$ is a factor of $f(x)$, then $f(4) = 0$

$\therefore (4)^3 - p(4)^2 - q(4) + 24 = 0$

$\therefore 64 - 16p - 4q + 24 = 0$

$\therefore -4p - q + 22 = 0$ .... (2) **(½ mark)**

Solving *eq*. (1) and (2)

$$\begin{array}{r} -3p + q - 1 = 0 \\ -4p - q + 22 = 0 \\ \hline -7p + 21 = 0 \\ \hline \end{array}$$

(½ mark)

$\therefore p = 3$ **(½ mark)**

Substituting, p = 3 in eq. (1) we get

$-3(3) + q - 1 = 0$ **(½ mark)**

$\therefore q = 10 \quad \therefore p = 3$ and $q = 10$ **(½ mark)**

28. (*i*) We have $(999)^3 = (1000 - 1)^3$ **(½ mark)**

$= (1000)^3 - (1)^3 - 3(1000)(1)(1000 - 1)$ **(½ mark)**

$= 1000000000 - 1 - 2997000 = 997002999$ **(1 mark)**

(*ii*) Consider $95 \times 96 = (90 + 5) \times (90 + 6)$ **(½ mark)**

$= (90)^2 + (5 + 6)(90) + (5)(6)$ **(½ mark)**

$= 8100 + 990 + 30 = 9120.$ **(1 mark)**

29. $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$

When $p(x)$ is divided by $(x + 1)$, remainder

$= p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b$ **(½ mark)**

$= 1 + 2 + 3 + a + b = a + b + 6$

$\Rightarrow a + b + 6 = 19$

$\Rightarrow a + b = 13$ ...(i) **(½ mark)**

When $p(x)$ is divided by $(x - 1)$, remainder

$= p(1) = 1^4 - 2 \times 1^3 + 3 \times 1^2 - a \times 1 + b$ **(½ mark)**

$= 1 - 2 + 3 - a + b$

$\Rightarrow -a + b + 2 = 5$

$\Rightarrow -a + b = 3$ ...(ii) **(½ mark)**

Adding (i) and (ii), we get

$2b = 16 \Rightarrow b = 8$ **(½ mark)**

Susstituting b = 8 in equation (i), we get

$a + 8 = 13 \quad \Rightarrow a = 5$ **(½ mark)**

$a = 5, b = 8$

Hence, $p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$ **(½ mark)**

When $p(x)$ is divided by $(x + 2)$, remainder,

$= p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$

$= 16 + 16 + 12 + 10 + 8 = 62$ **(½ mark)**

30. If a straight line ℓ falls on two straight lines m and n such that the sum of the interior angles on one side of ℓ is two right angles, then by Euclid's fifth postulate the lines will not meet on this side of ℓ. Next, we know that the sum of the interior angles on the other side of line ℓ will also be two right angles. Therefore, they will not meet on the other side also. So, the lines m and n never meet and are, therefore, parallel. **(4 marks)**

31. Given a ΔABC which is isosceles with $AB = AC$. Side BA is produced to D such that $AD = AB$.

To Prove : $\angle BCD$ is a right angle.

Proof : Since, ΔABC is an isosceles

$\therefore$ $\angle ABC = \angle ACB$ ....(1) **(½ mark)**

$AC = AD$

($\because AB = AC$ and $AD = AB$)

$\therefore$ In ΔACD,

$\angle CDA = \angle ACD$ **(½ mark)**

(Angles opposite to equal sides of a triangle are equal)

$\angle CDB = \angle ACD$ ....(2) **(½ mark)**

By adding (1) and (2), we get

$\angle ABC + \angle CDB = \angle ACB + \angle ACD$

(½ mark)

$\Rightarrow$ $\angle ABC + \angle CDB = \angle BCD$ ....(3) **(½ mark)**

Now, In ΔBCD,

$\angle BCD + \angle DBC + \angle CDB = 180°$

(By angle sum property)

$\Rightarrow$ $\angle BCD + \angle ABC + \angle CDB = 180°$ **(½ mark)**

$\Rightarrow$ $\angle BCD + \angle BCD = 180°$ (Using (3))

$\Rightarrow$ $2\angle BCD = 180°$ **(½ mark)**

$\Rightarrow$ $\angle BCD = 90°$

$\Rightarrow$ $\angle BCD$ is a right angle. **(½ mark)**

32. As we know the exterior angle is equal to the sum of the two interior opposite angles

$\therefore$ $\angle QRT = \angle RQS + \angle QSR$

$\Rightarrow$ $65° = 28° + \angle QSR$

$\Rightarrow$ $\angle QSR = 65° - 28° = 37°$ **(½ mark)**

Also given $PQ \perp SP$

$\therefore$ $\angle QPS = 90°$ **(½ mark)**

Also $PQ \parallel SR$ gives

$\angle QPS + \angle PSR = 180°$ **(½ mark)**

($\because$ The sum of consecutive interior angles on the same side of the transversal is 180°)

$\angle PSR = 90°$ **(½ mark)**

$\Rightarrow$ $\angle PSQ + \angle QSR = 90° \Rightarrow y + 37° = 90°$

$\Rightarrow$ $y = 90° - 37° = 53°$ **(½ mark)**

Now, from ΔPQS, we have

$\angle PQS + \angle QSP + \angle QPS = 180°$ **(½ mark)**

(By angle sum property of a triangle)

$\Rightarrow$ $x + y + 90° = 180°$

$\Rightarrow$ $x + 53° + 90° = 180°$ **(½ mark)**

$\Rightarrow$ $x = 180° - 143° = 37°$. **(½ mark)**

33. Let in ΔPQR, exterior $\angle PRS = 115°$ and $\angle P = 35°$

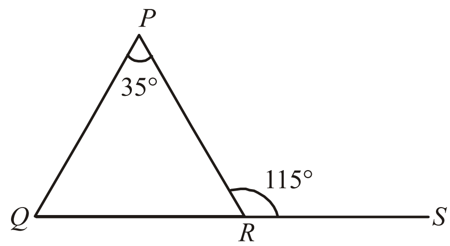
We know that, **(½ mark)**

$\angle PRS = \angle P + \angle Q$ (Exterior Angle Theorem)

(½ mark)

$\Rightarrow$ $115° = 35° + \angle Q$ **(½ mark)**

$\Rightarrow$ $\angle Q = 115° - 35° = 80°$ **(½ mark)**



(½ mark)

Again, in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$ **(½ mark)**

($\because$ The sum of the three angles of a triangle is 180°)

$\Rightarrow$ $35° + 80° + \angle R = 180°$ **(½ mark)**

$\Rightarrow$ $\angle R = 180° - 115°$

$\Rightarrow$ $\angle R = 65°$. **(½ mark)**

Or

$\angle PRE + \angle PRQ = 180°$ (Linear Pair Axiom)

(½ mark)

$\Rightarrow$ $\angle PRE = 180° - \angle R$ ...(1)

(½ mark)

$\Rightarrow \quad \angle PQF + \angle PQR = 180°$ **(½ mark)**

(Linear Pair Axiom)

$\Rightarrow \quad \angle PQF + \angle Q = 180°$ ...(2)

(½ mark)

Adding (1) and (2), we have

$\angle PRE + \angle PQF = 360° - (\angle Q + \angle R)$ ...(3)

(½ mark)

In ΔPQR,

$\angle Q + \angle R = 180° - \angle P$ ...(4) **(½ mark)**

From (3) and (4),

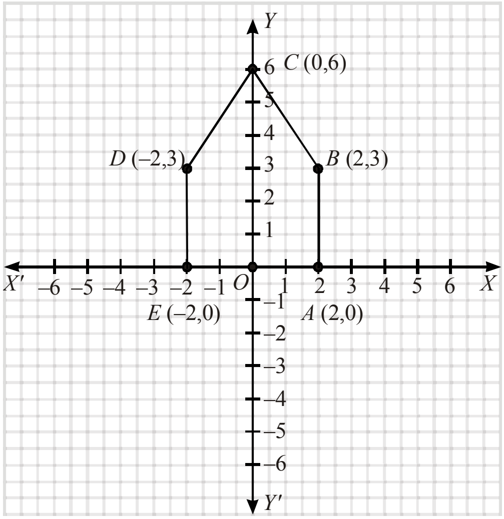
$\angle PRE + \angle PQF = 360° - (180° - \angle P)$

(½ mark)

$= 180° + \angle P > 180°$ ($\because \angle P$ is positive)

(½ mark)

34. Let the given points are $A(2, 0)$, $B(2, 3)$, $C(0, 6)$, $D(-2, 3)$ and $E(-2, 0)$. After plotting and joining, we get the figure.



(3 marks)

After plotting A, B, D and E and joining, we get a 'Pentagon.' **(1 mark)**