Homework 1

Qijun Han 12212635

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1 NE1.1

1.1 Problem a

$$G(s) = \frac{1}{s^2 + 2s + 6}$$

ODE:

$$y''(t) + 2y'(t) + 6y(t) = u(t)$$

let $x_1(t) = y(t), x_2(t) = y'(t)$, then we have
$\begin{cases} x_1'(t) = x_2(t) \\ x_2'(t) = -2x_2(t) - 6x_1(t) + u(t) \end{cases}$
so the state space representation is

$$\begin{aligned} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}' &= \begin{bmatrix} 0 & 1 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{1}$$

thus $A = \begin{bmatrix} 0 & 1 \\ -6 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$

1.2 Problem b

let $G_1(s) = \frac{1}{s^2+2s+6} = \frac{W(s)}{U(s)}$, $G_2(s) = s + 3 = \frac{Y(s)}{W(s)}$, then

$$G(s) = G_1(s)G_2(s)$$

ODE:

$$\begin{aligned} w''(t) + w'(t) + w(t) = u(t) \\ y(t) = w'(t) + 3w(t) \end{aligned} \tag{2}$$

let $x_1(t) = w(t), x_2(t) = w'(t)$, then we have

$$\begin{aligned} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}' &= \begin{bmatrix} 0 & 1 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{3}$$

and

$$y(t) = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

thus $A = \begin{bmatrix} 0 & 1 \\ -6 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$

1.3 Problem c

similar to (a), we have $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -8 & -4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$

1.4 Problem d

similar to (b), we have $A =$ [

2 Problem 2

3 Problem 3

4 Problem 4