Mitigation model

1 State variables and law of motion

Total capital:

$$d\log K = (\mu_k + i - \frac{\kappa}{2}i^2 - \frac{\sigma_k^2}{2})dt + \sigma_k dW$$

Temperature anomaly:

$$dY = e(\theta_\ell dt + \varsigma dW)$$

$R\&D$ investment, X, leads to an increased arrival rate of a one time jump in green sector productivity:

$$d\log \mathcal{I}_g = -\zeta dt + \Psi_0(\frac{X}{\mathcal{I}_g})^{\Psi_1} dt - \frac{\sigma_g^2}{2}dt + \sigma_g dW$$

Here we use $\Psi_1 = 1/2$

Construct the following process:

$$\bar{Y}_t = \begin{cases} Y_t, & t \leq \tau \\ Y_t - Y_\tau + \bar{y}, & t > \tau \end{cases}$$

where τ is the date of the Poisson event.

Log damages:

$$\log N_t = \Gamma(\bar{Y}_t) + \iota_n \cdot Z_t$$

where

$$\Gamma(y) = \gamma_1 y + \frac{\gamma_2}{2}y^2 + \frac{\gamma_3^{(m)}}{2}\mathbb{I}_{y\geq\bar{y}}(y-\bar{y})^2, \quad m = 1, 2, \ldots, 20$$

Damage jump intensity

$$\mathcal{I}_d = \begin{cases} r_1\left(\exp(\frac{r_2}{2}(Y-\underline{y})^2) - 1\right), & Y \geq \underline{y} \\ 0, & Y < \underline{y} \end{cases}$$

2 Damage jump and technology jump

Suppose there are three technology states. Denote them as tech I, tech II and tech III. And the curvature in the "tail" of the damage function is only revealed when to decision-makers when a Poisson event is triggered. And technology change and damage curvature revelation are independent.

The following graph serves as a illustration for notations of different solutions:

$$\begin{tikzcd}
\boxed{\text{Pre-damage, tech I, } \Phi^{\mathrm{I}}} \arrow[r, "\text{Pre-damage}"] \arrow[d] & \boxed{\text{Pre-damage, tech II, } \Phi^{\mathrm{II}}} \arrow[d] \\
\boxed{\text{Post-damage, tech I, } \phi^{(m,\mathrm{I})}} \arrow[r, swap, "\text{Post-damage}"] & \boxed{\text{Post-damage, tech II, } \phi^{(m,\mathrm{II})}}
\end{tikzcd}$$

Figure 1: Damage jump and technology jump

3 Post-damage, tech II

At tech III, $\bar{\vartheta} = 0$. For $m = 1, 2, \ldots, 20$, value function $V^{(m)}(\log K, Y, \log N)$

$$\begin{aligned}
0 = \max_{i,e} \min_{\omega_\ell : \sum_{\ell=1}^L \omega_\ell = 1} & - \delta V^{(m)}(\log K, Y, \log N) + \delta \log \left(\alpha - i - \alpha\bar{\vartheta} \left[1 - \left(\frac{e}{\alpha \bar{\lambda} K} \right) \right]^\theta \right) - \delta \log N + \delta \log K \\
& + \frac{\partial V^{(m)}}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 V^{(m)}}{\partial \log K^2} \\
& + \frac{\partial V^{(m)}}{\partial Y} \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{1}{2} \frac{\partial^2 V^{(m)}}{\partial Y^2} |\varsigma|^2 e^2 \\
& + \frac{\partial V^{(m)}}{\partial \log N} \left(\left[\gamma_1 + \gamma_2 Y + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}} (Y - \bar{y}) \right] \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{\gamma_2 + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}}}{2} |\varsigma|^2 e^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 V^{(m)}}{\partial \log N^2} \left[\gamma_1 + \gamma_2 Y + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}} (Y - \bar{y}) \right]^2 |\varsigma|^2 e^2 \\
& + \xi_a \sum_{\ell=1}^L \omega_\ell \left(\log \omega_\ell - \log \pi_\ell \right)
\end{aligned}$$

We simplify out $\log N$ by expressing the value function as $V^{(m)} = v_d \log N + \phi^{(m)}(\log K, Y)$. Therefore, $v_d = -1$ and $\frac{\partial^2 V^{(m)}}{\partial \log N^2} = 0$. And $\phi^{(m)}$ solves the following HJB:

$$\begin{aligned}
0 = \max_{i,e} & - \delta \phi^{(m)}(\log K, Y) + \delta \log (\alpha - i) + \delta \log K \\
& + \frac{\partial \phi^{(m)}}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 \phi^{(m)}}{\partial \log K^2}
\end{aligned}$$

Denote solutions in this section are $V^{(m,\mathrm{III})}$ and $\phi^{(m,\mathrm{III})}$.

4 Post-damage, tech I

At tech I, $\bar{\vartheta}$ and $\bar{\lambda}$ pair use the following values:

Tech state	$\bar{\vartheta}$	$\bar{\lambda}$
tech I	0.04530	0.1206

And we add an additional state, $\log \mathcal{I}_g$ and an additional control variable $x = \frac{X}{K}$. X is the R&D investment and K is total capital.

At tech II, for $m = 1, 2, \ldots, 20$, value function $V^{(m)}(\log K, Y, \log \mathcal{I}_g, \log N)$ and suppose Poisson event for damage function is triggered.

$$
\begin{aligned}
0 = \max_{i,e,x} \min_{\omega_\ell : \sum_{\ell=1}^L \omega_\ell = 1} \min_{g>0} & - \delta V^{(m)}(\log K, Y, \log \mathcal{I}_g, \log N) + \delta \log \left(\alpha - i - \alpha \bar{\vartheta} \left[1 - \left(\frac{e}{\alpha \bar{\lambda} K} \right) \right]^\theta - x \right) \\
& - \delta \log N + \delta \log K \\
& + \frac{\partial V^{(m)}}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 V^{(m)}}{\partial \log K^2} \\
& + \frac{\partial V^{(m)}}{\partial Y} \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{1}{2} \frac{\partial^2 V^{(m)}}{\partial Y^2} |\varsigma|^2 e^2 \\
& + \frac{\partial V^{(m)}}{\partial \log \mathcal{I}_g} (-\zeta + \Psi_0 (x \frac{K}{\mathcal{I}_g})^{\Psi_1} - \frac{\sigma_g^2}{2}) + \frac{\sigma_g^2}{2} \frac{\partial^2 V^{(m)}}{\partial \log \mathcal{I}_g^2} \qquad \text{[additional state]} \\
& + \frac{\partial V^{(m)}}{\partial \log N} \left(\left[\gamma_1 + \gamma_2 Y + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}} (Y - \bar{y}) \right] \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{\gamma_2 + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}}}{2} |\varsigma|^2 e^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 V^{(m)}}{\partial \log N^2} \left[\gamma_1 + \gamma_2 Y + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}} (Y - \bar{y}) \right]^2 |\varsigma|^2 e^2 \\
& + \xi_a \sum_{\ell=1}^L \omega_\ell \left(\log \omega_\ell - \log \pi_\ell \right) \\
& + \xi_g \mathcal{I}_g \left(1 - g + g \log(g) \right) + \mathcal{I}_g g (V^{(m,\mathrm{III})} - V^{(m)}) \qquad \text{[robustness concern]}
\end{aligned}
$$

We simplify out $\log N$ by similarly expressing the value function as $V^{(m)} = \upsilon_d \log N + \phi^{(m)}(\log K, Y, \log \mathcal{I}_g)$.

Therefore, $\upsilon_d = -1$ and $\frac{\partial^2 V^{(m)}}{\partial \log N^2} = 0$. And $\phi^{(m)}(\log K, Y, \log \mathcal{I}_g)$ solves the following HJB:

$$
\begin{aligned}
0 = \max_{i,e,x} \min_{\omega_\ell : \sum_{\ell=1}^L \omega_\ell = 1} \min_{g>0} & - \delta \phi^{(m)}(\log K, Y, \log \mathcal{I}_g) + \delta \log \left(\alpha - i - \alpha \overline{\vartheta} \left[1 - \left(\frac{e}{\alpha \overline{\lambda} K} \right) \right]^\theta - x \right) + \delta \log K \\
& + \frac{\partial \phi^{(m)}}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 \phi^{(m)}}{\partial \log K^2} \\
& + \frac{\partial \phi^{(m)}}{\partial Y} \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{1}{2} \frac{\partial^2 \phi^{(m)}}{\partial Y^2} |\varsigma|^2 e^2 \\
& + \frac{\partial \phi^{(m)}}{\partial \log \mathcal{I}_g} (-\zeta + \Psi_0 (x \frac{K}{\mathcal{I}_g})^{\Psi_1} - \frac{\sigma_g^2}{2}) + \frac{\sigma_g^2}{2} \frac{\partial^2 \phi^{(m)}}{\partial \log \mathcal{I}_g^2} \\
& - \left(\left[\gamma_1 + \gamma_2 Y + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}} (Y - \bar{y}) \right] \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{\gamma_2 + \gamma_3^{(m)} \mathbb{I}_{Y > \bar{y}}}{2} |\varsigma|^2 e^2 \right) \\
& + \xi_a \sum_{\ell=1}^L \omega_\ell \left(\log \omega_\ell - \log \pi_\ell \right) \\
& + \xi_g \mathcal{I}_g \left(1 - g + g \log(g) \right) + \mathcal{I}_g g (\phi^{(m,\mathrm{III})} - \phi^{(m)})
\end{aligned}
$$

Denote post-damage tech II value functions solved from above HJBs as $V^{(m,\mathrm{II})}$ and $\phi^{(m,\mathrm{II})}$. We can solve for $V^{(m,\mathrm{I})}$ and $\phi^{(m,\mathrm{I})}$ by changing the values of $\bar{\vartheta}$ and $\bar{\lambda}$.

5 Pre-damage and pre technology jump (tech I) HJB

Denote $x = \frac{X}{K}$ as R& D invesment - total capital ratio, and there are two technology jumps.

Given pre-damage tech II value function $\mathcal{V}^{\mathrm{II}}(\log K, Y, \log \mathcal{I}_g, \log N)$, value function $\mathcal{V}(\log K, Y, \log \mathcal{I}_g, \log N)$

with 4 state variables solve for the following pre damage jump and pre technology jump HJB:

$$
\begin{aligned}
0 = \max_{i,e,x} \min_{\omega_\ell, \sum_{\ell=1}^{L}\omega_\ell = 1, g, g_m} \quad & - \delta \mathcal{V}(\log K, Y, \log \mathcal{I}_g, \log N) + \delta \log \left(\alpha - i - \alpha \overline{\vartheta} \left[1 - \left(\frac{e}{\alpha \overline{\lambda} K} \right) \right]^{\theta} - x \right) \\
& - \delta \log N + \delta \log K \\
& + \frac{\partial \mathcal{V}}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 \mathcal{V}}{\partial \log K^2} \\
& + \frac{\partial \mathcal{V}}{\partial Y} \sum_{\ell=1}^{L} \omega_\ell \theta_\ell e + \frac{1}{2} \frac{\partial^2 \mathcal{V}}{\partial Y^2} |\varsigma|^2 e^2 \\
& + \frac{\partial \mathcal{V}}{\partial \log \mathcal{I}_g} (-\zeta + \Psi_0 (x \frac{K}{\mathcal{I}_g})^{\Psi_1} - \frac{\sigma_g^2}{2}) + \frac{\sigma_g^2}{2} \frac{\partial^2 \mathcal{V}}{\partial \log \mathcal{I}_g^2} \\
& + \frac{\partial \mathcal{V}}{\partial \log N} \left([\gamma_1 + \gamma_2 Y] \sum_{\ell=1}^{L} \omega_\ell \theta_\ell e + \frac{\gamma_2}{2} |\varsigma|^2 e^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 \mathcal{V}}{\partial \log N^2} \left[\gamma_1 + \gamma_2 Y \right]^2 |\varsigma|^2 e^2 \\
& + \xi_a \sum_{\ell=1}^{L} \omega_\ell \left(\log \omega_\ell - \log \pi_\ell \right) \\
& + \xi_g \mathcal{I}_g \left(1 - g + g \log(g) \right) + \mathcal{I}_g g (\mathcal{V}^{\mathrm{II}} - \mathcal{V}) \\
& + \xi_d \mathcal{I}_d \sum_{m=1}^{M} \pi_d^m (1 - g_m + g_m \log(g_m)) + \mathcal{I}_d \sum_{m=1}^{M} \pi_d^m g_m (V^{(m,\mathrm{II})} - \mathcal{V})
\end{aligned}
$$

We simplify out $\log N$ by similarly expressing the pre-damage value function as $\mathcal{V} = \upsilon_d \log N + \Phi(\log K, Y, \log \mathcal{I}_g)$. Therefore, $\upsilon_d = -1$ and $\frac{\partial^2 \mathcal{V}}{\partial \log N^2} = 0$. And $\Phi(\log K, Y, \log \mathcal{I}_g)$ solves the following HJB:

$$
\begin{aligned}
0 = \max_{i,e,x} \min_{\omega_\ell, \sum_{\ell=1}^L \omega_\ell = 1, g, g_m} & - \delta \Phi(\log K, Y, \log \mathcal{I}_g) + \delta \log \left(\alpha - i - \alpha \overline{\vartheta} \left[1 - \left(\frac{e}{\alpha \overline{\lambda} K} \right) \right]^\theta - x \right) + \delta \log K \\
& + \frac{\partial \Phi}{\partial \log K} \left[\mu_k + i - \frac{\kappa}{2} i^2 - \frac{|\sigma_k|^2}{2} \right] + \frac{|\sigma_k|^2}{2} \frac{\partial^2 \Phi}{\partial \log K^2} \\
& + \frac{\partial \Phi}{\partial Y} \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{1}{2} \frac{\partial^2 \Phi}{\partial Y^2} |\varsigma|^2 e^2 \\
& + \frac{\partial V}{\partial \log \mathcal{I}_g} (-\zeta + \Psi_0 (x \frac{K}{\mathcal{I}_g})^{\Psi_1} - \frac{\sigma_g^2}{2}) + \frac{\sigma_g^2}{2} \frac{\partial^2 V}{\partial \log \mathcal{I}_g^2} \\
& - \left([\gamma_1 + \gamma_2 Y] \sum_{\ell=1}^L \omega_\ell \theta_\ell e + \frac{\gamma_2}{2} |\varsigma|^2 e^2 \right) \\
& + \xi_a \sum_{\ell=1}^L \omega_\ell \left(\log \omega_\ell - \log \pi_\ell \right) \\
& + \xi_g \mathcal{I}_g \left(1 - g + g \log(g) \right) + \mathcal{I}_g g (\Phi^{\mathrm{II}} - \Phi) \\
& + \xi_d \mathcal{I}_d \sum_{m=1}^M \pi_d g_m (1 - g_m + g_m \log(g_m)) + \mathcal{I}_d \sum_{m=1}^M \pi_d^m g_m (\phi^{(m,\mathrm{II})} - \Phi)
\end{aligned}
$$

Denote pre-damage tech I value functions solved from above HJBs as $\mathcal{V}^{\mathrm{I}}$ and Φ^{I}.

6 Uncertainty parameter configuration

- Smooth ambiguity: ξ_a
- Damage uncertainty, ξ_d,
- Technology jump uncertainty, ξ_g

	ξ_a	ξ_d	ξ_g
baseline	∞	∞	∞
case 1	2×10^{-4}	0.050	0.050
case 2	2×10^{-4}	0.025	0.025

7 Pathway comparisons, with 20 damage functions

7.1 pre damage jump, pre technology jump R& D investment

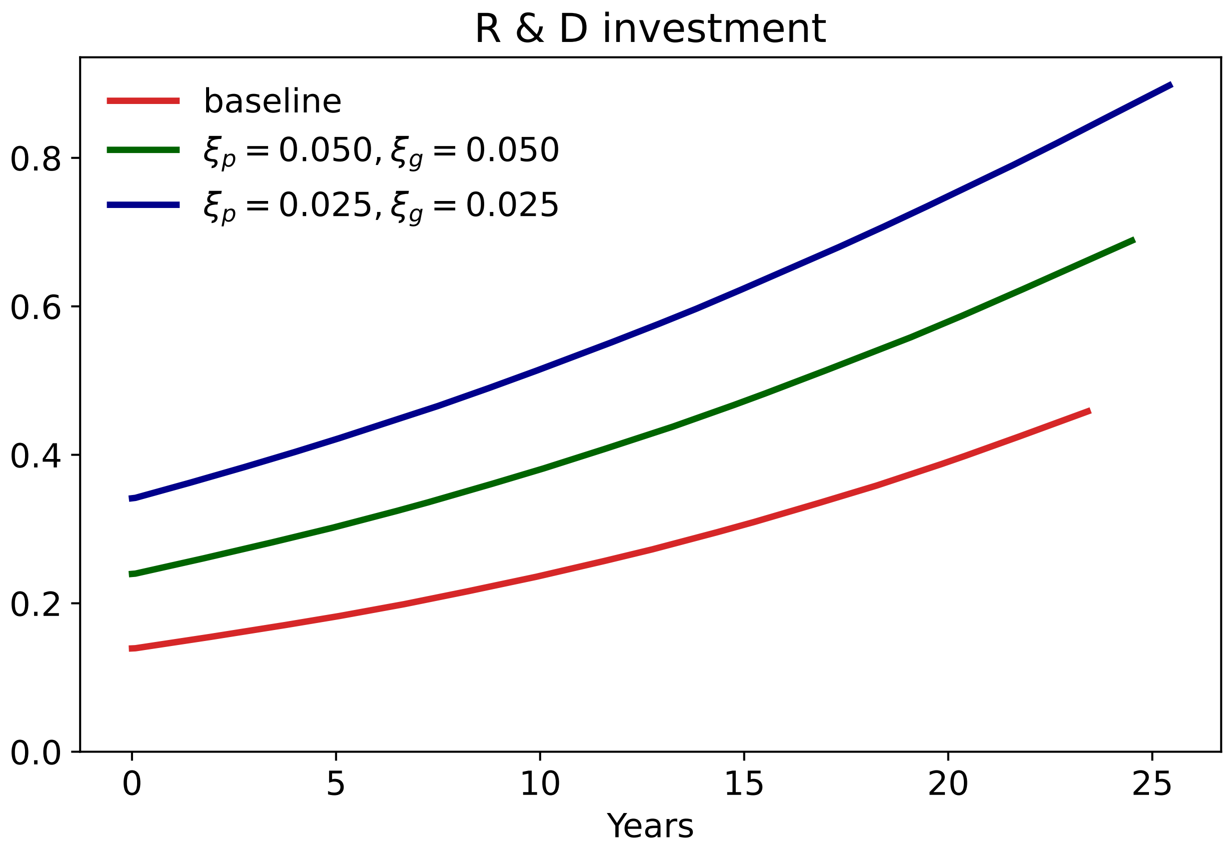


Figure 2: R & D investment, pathways stop when temperature anomaly hits 1.5^oC

7.2 pre damage jump, pre technology jump emission

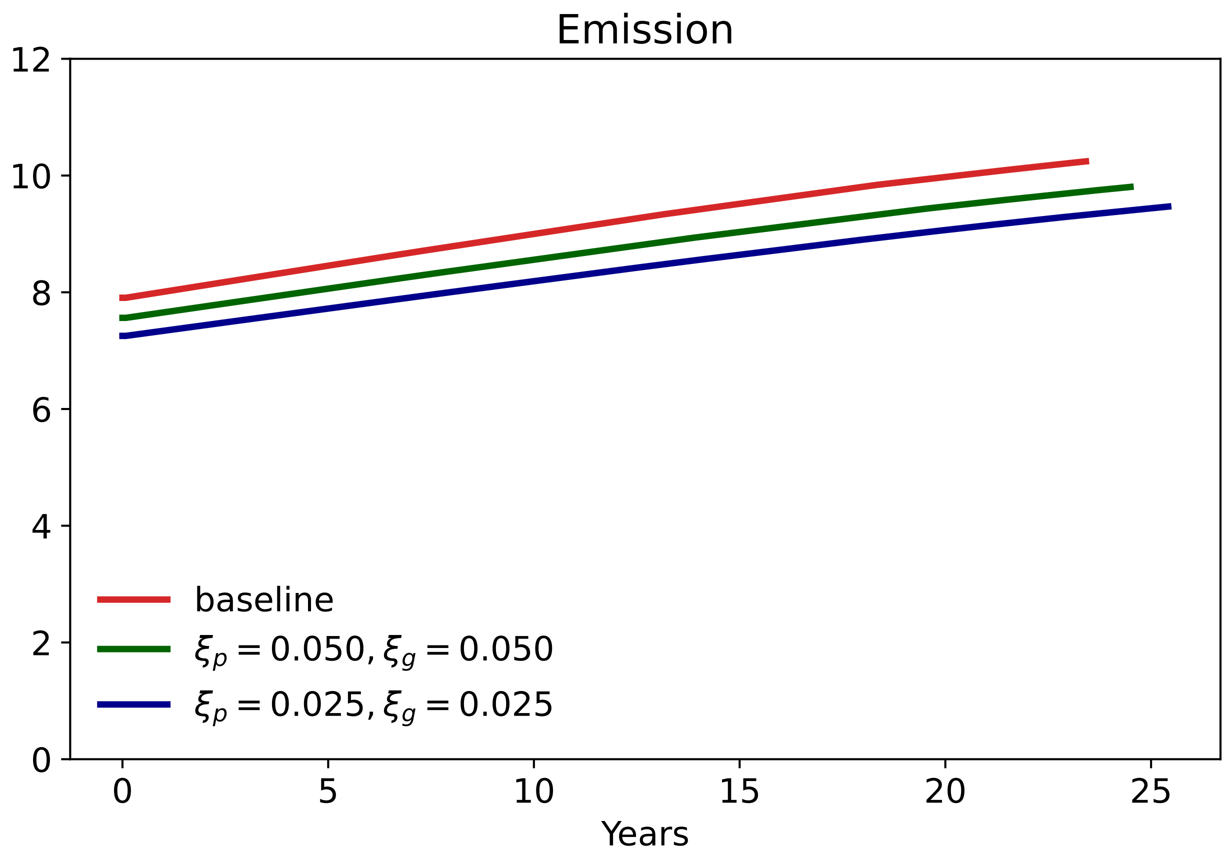


Figure 3: Emission, pathways stop when temperature anomaly hits 1.5^oC

7.3 pre damage jump, pre technology jump temperature anomaly

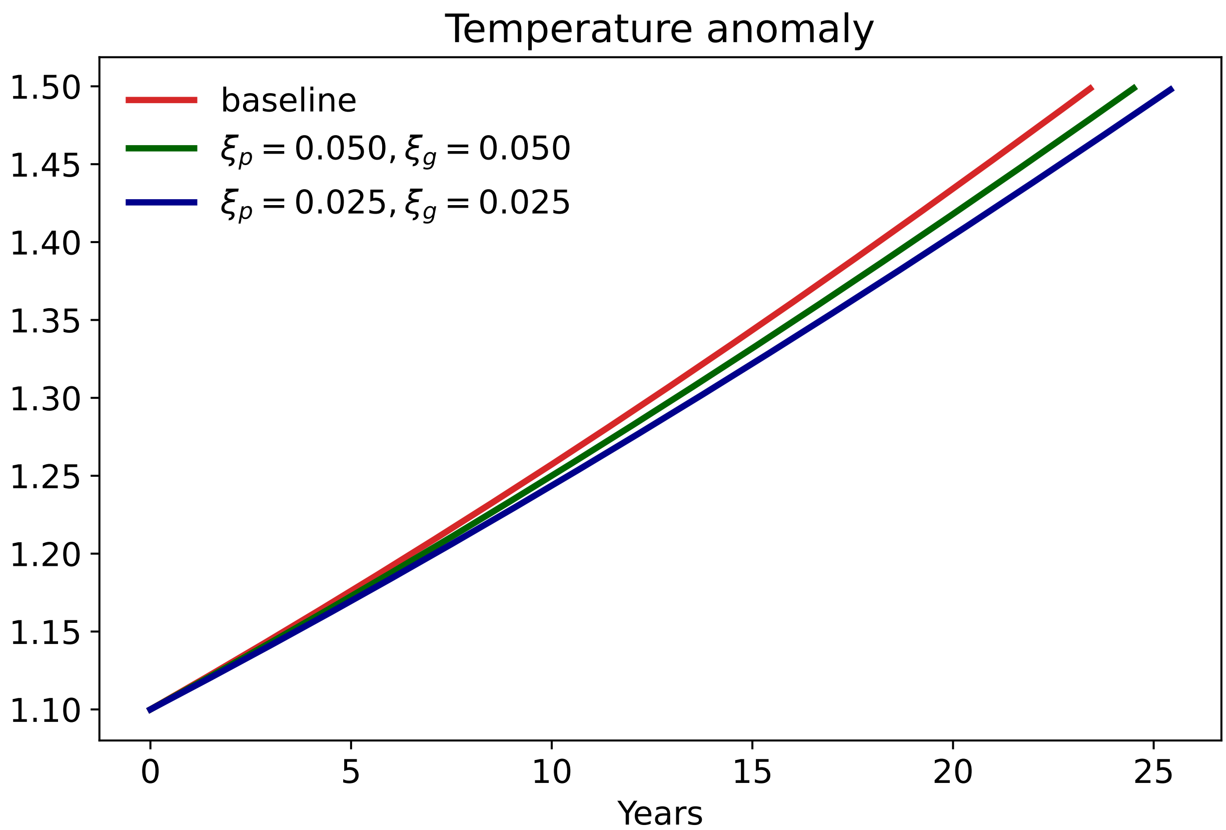


Figure 4: Temperature anomaly, pathways stop when temperature anomaly hits 1.5^oC

7.4 pre damage jump, pre technology jump technology jump intensity, $\mathcal{I}_g$

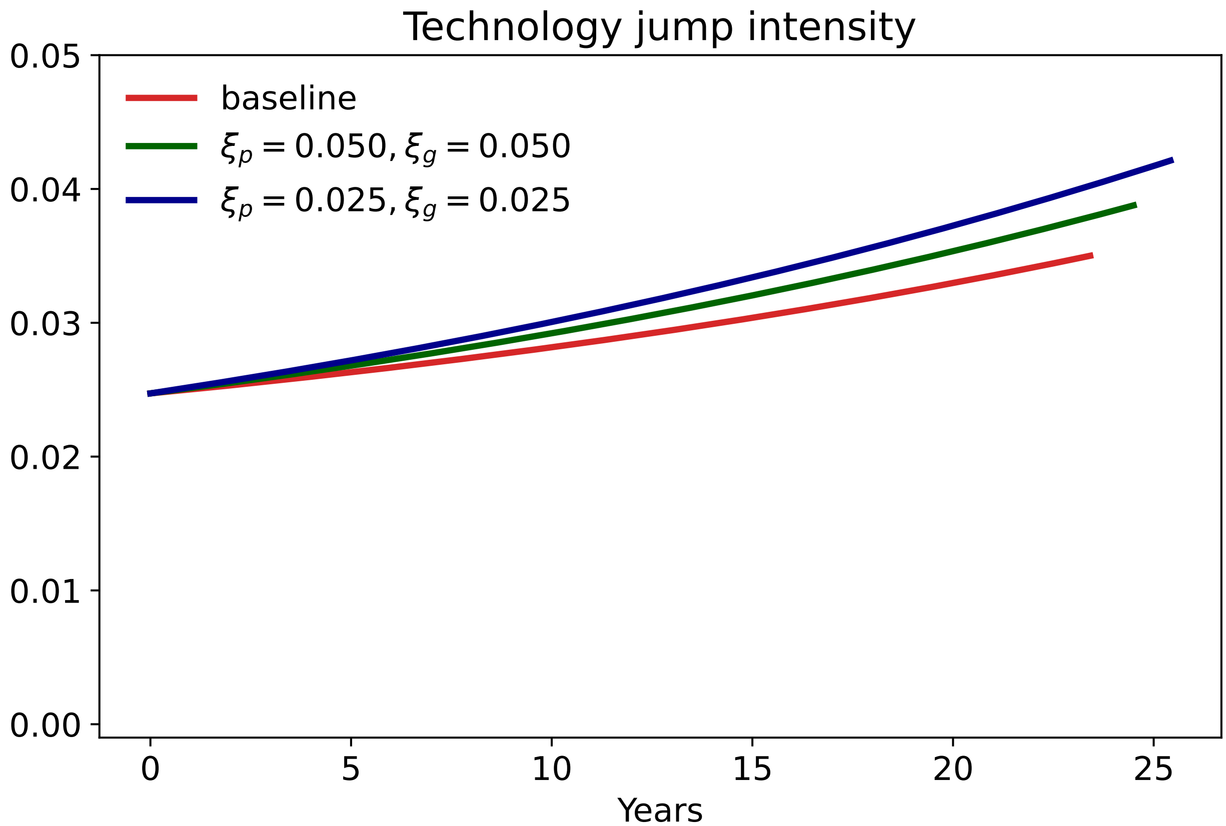
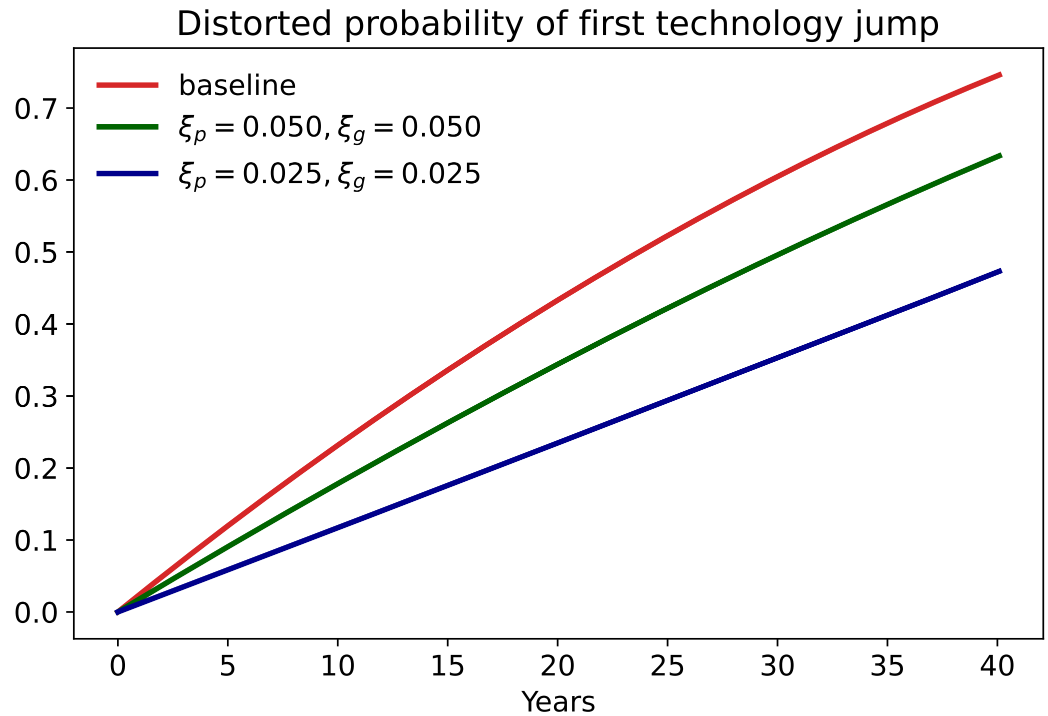
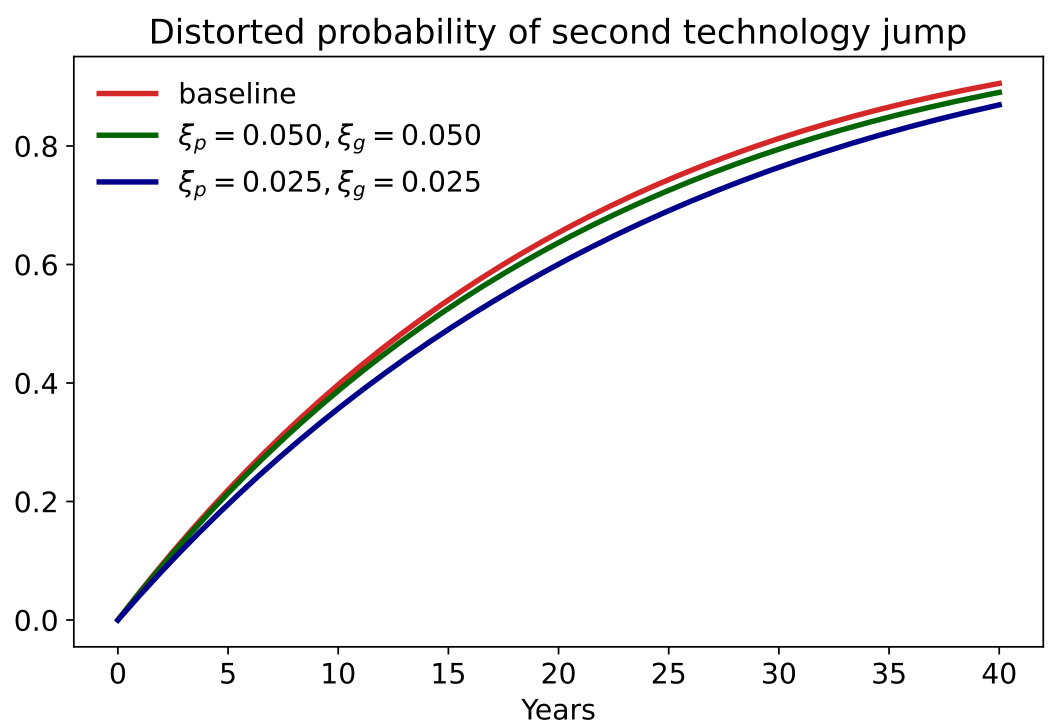


Figure 5: Technology jump intensity, $\mathcal{I}_g$, pathways stop when temperature anomaly hits 1.5^oC

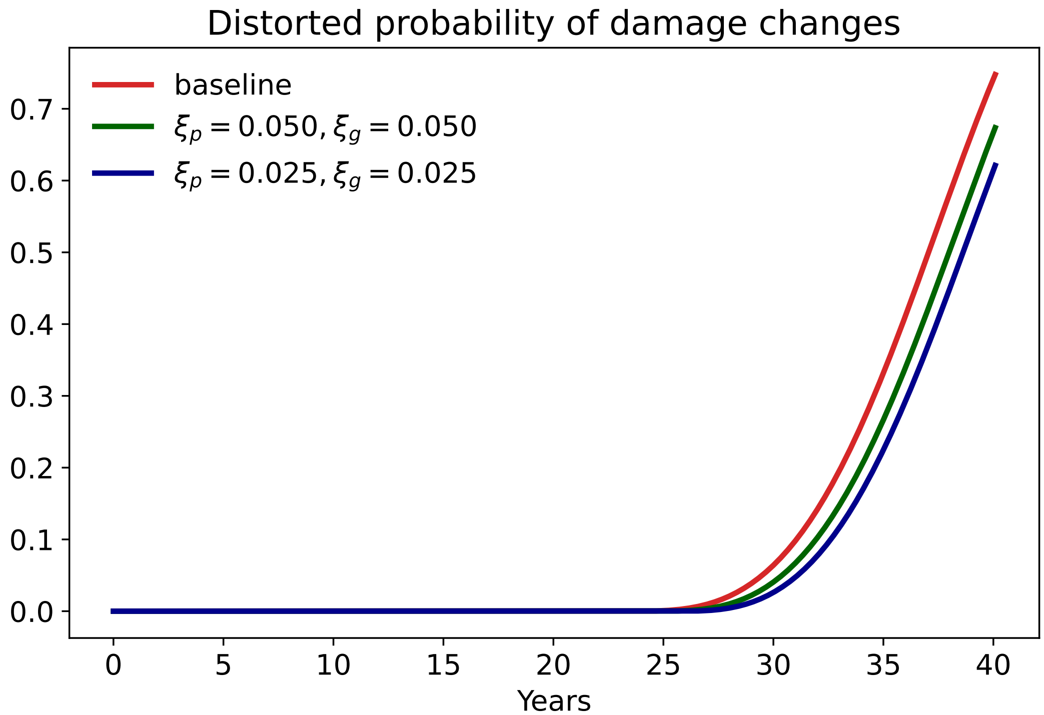
8 Jump probabilities and distorted probabilities

8.1 Distorted probability of Poisson events for technology changes with different uncertainty configurations





8.2 Distorted probability of Poisson events for damage with different uncertainty configurations



8.3 Probability distortion for damage function, with $\xi_a = 2 \times 10^{-4}$, $\xi_d = 0.05$ and $\xi_g = 0.05$

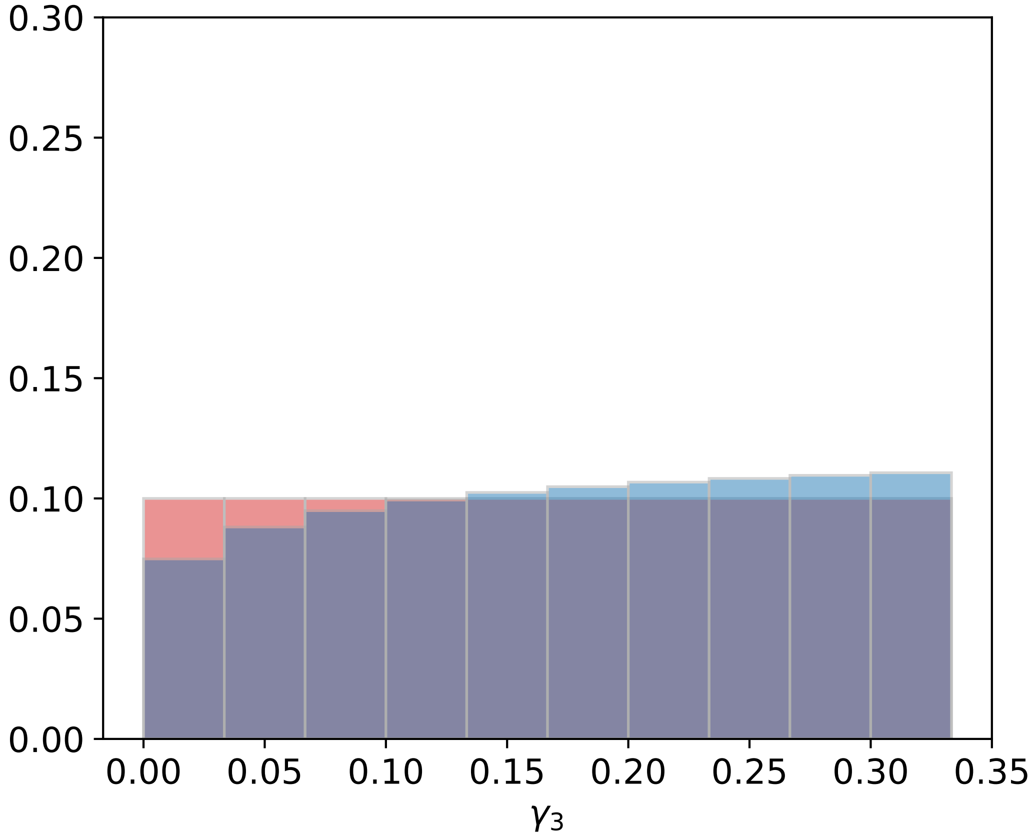


Figure 6: Red bars are for baseline probability, and blue bars are for distorted probability

8.4 Probability distortion for climate sensitivity models, with $\xi_a = 2 \times 10^{-4}$, $\xi_d = 0.05$ and $\xi_g = 0.05$

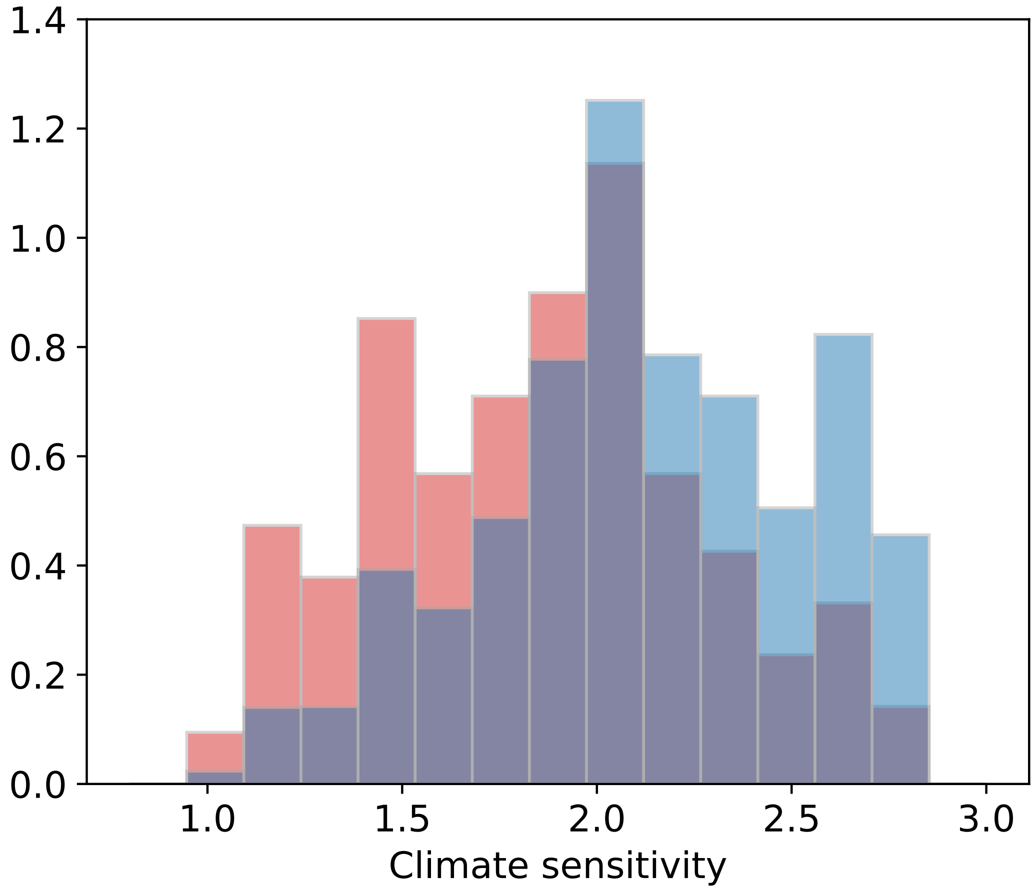


Figure 7: Red bars are for baseline density, and blue bars are for distorted density of climate sensitivity models