Marginal Utility of Emissions:

June 10, 2022

From the paper, the marginal utility of emissions is given in equation (11). This equation gives us the following expression:

$$\frac{\eta}{\tilde{e}} = -\frac{d\phi(y)}{dy}\sum_{\ell=1}^{L}\omega_\ell\theta_\ell - \frac{d^2\phi(y)}{(dy)^2}|\varsigma|^2\tilde{e} + \frac{(1-\eta)}{\delta}[(\gamma_1+\gamma_2 y)\sum_{\ell=1}^{L}\omega_\ell\theta_\ell + \gamma_2|\varsigma|^2\tilde{e}]$$

The discounted value would be given by

$$\exp(-\delta t)\frac{\eta}{\tilde{e}} = \exp(-\delta t)\{-\frac{d\phi(y)}{dy}\sum_{\ell=1}^{L}\omega_\ell\theta_\ell - \frac{d^2\phi(y)}{(dy)^2}|\varsigma|^2\tilde{e} + \frac{(1-\eta)}{\delta}[(\gamma_1+\gamma_2 y)\sum_{\ell=1}^{L}\omega_\ell\theta_\ell + \gamma_2|\varsigma|^2\tilde{e}]\}$$

We can then integrate that up to get the aggregated marginal utility impact of reducing emissions by one unit

$$\int_0^\infty \exp(-\delta t)\frac{\eta}{\tilde{e}}dt = \int_0^\infty \exp(-\delta t)\{-\frac{d\phi(y)}{dy}\sum_{\ell=1}^{L}\omega_\ell\theta_\ell - \frac{d^2\phi(y)}{(dy)^2}|\varsigma|^2\tilde{e} + \frac{(1-\eta)}{\delta}[(\gamma_1+\gamma_2 y)\sum_{\ell=1}^{L}\omega_\ell\theta_\ell + \gamma_2|\varsigma|^2\tilde{e}]\}dt$$

In practive, the emissions pathways only go out 80 years, so we would get something like

$$\sum_{t=0}^{80} \exp(-\delta t)\frac{\eta}{\tilde{e}}dt = \sum_{t=0}^{80} \exp(-\delta t)\{-\frac{d\phi(y)}{dy}\sum_{\ell=1}^{L}\omega_\ell\theta_\ell - \frac{d^2\phi(y)}{(dy)^2}|\varsigma|^2\tilde{e} + \frac{(1-\eta)}{\delta}[(\gamma_1+\gamma_2 y)\sum_{\ell=1}^{L}\omega_\ell\theta_\ell + \gamma_2|\varsigma|^2\tilde{e}]\}dt$$

I think the discrete time approximation for $\exp(-\delta t)$ is $\frac{1}{1+\delta t}$.