

# Bulls and Cows Game - Technical Documentation

## Overview

This document provides technical documentation for two implementations of the Bulls and Cows game using information theory principles. The project consists of two main Python files: Bulls\_and\_Cows\_User\_Guessing.py and Bulls\_and\_Cows\_Computer\_Guessing.py.

## 1. User Game Implementation (Bulls\_and\_Cows\_User\_Guessing.py)

### Purpose

Implements a game where the computer generates a secret code and the user attempts to guess it, with entropy tracking to measure remaining uncertainty.

### Module Structure

#### 1.1 Functions

compute\_feedback(secret: str, guess: str) -> Tuple[int, int]

Calculates the feedback for a user's guess.

- Parameters:
  - secret: The secret 4-digit code
  - guess: The user's guess
- Returns: Tuple containing (bulls, cows)
- Algorithm:
  - Counts exact matches for bulls
  - Counts matching digits in wrong positions for cows
  - Uses set operations to avoid double counting

calculate\_current\_entropy(bulls: int, cows: int) -> float

Calculates remaining uncertainty after a guess.

- Parameters:
  - bulls: Number of correct digits in correct positions

- cows: Number of correct digits in wrong positions
- Returns: Current entropy in bits
- Algorithm:
  - Base entropy =  $4 * \log_2(10) \approx 13.29$  bits
  - Position certainty = bulls/4.0
  - Digit certainty = cows/8.0
  - Remaining uncertainty =  $1 - (\text{position\_certainty} + \text{digit\_certainty})$
  - Current entropy = base\_entropy \* remaining\_uncertainty

`is_valid_guess(guess: str) -> bool`

Validates user input.

- Parameters:
  - guess: User's input string
- Returns: Boolean indicating if guess is valid
- Validation Rules:
  - Must be 4 characters long
  - Must contain only digits
  - Must have unique digits

## Game Flow

1. Computer generates random 4-digit code
2. User makes guesses with feedback after each attempt
3. Entropy is calculated and displayed after each guess
4. Game ends on correct guess or after 20 attempts

## 2. Computer Solver Implementation (Bulls\_and\_Cows\_Computer\_Guessing.py)

### Purpose

Implements an optimal solver where the computer guesses the user's secret code using information theory for decision making.

### Module Structure

## 2.1 Functions

`compute_feedback(secret: str, guess: str) -> Tuple[int, int]`

Calculates feedback for a guess.

- Parameters:
  - secret: Current code being evaluated
  - guess: Computer's guess
- Returns: Tuple containing (bulls, cows)
- Algorithm:
  - Uses marking strategy to handle digit matching
  - Marks matched positions to avoid double counting
  - Separates bulls and cows calculation

`filter_codes(possible_codes: List[str], guess: str, bulls: int, cows: int) -> List[str]`

Filters possible codes based on feedback.

- Parameters:
  - possible\_codes: List of remaining possible codes
  - guess: Computer's last guess
  - bulls, cows: Feedback received
- Returns: Filtered list of possible codes
- Process:
  - Eliminates codes that wouldn't give same feedback
  - Maintains consistency with all previous feedback

`calculate_entropy(possible_codes: List[str], guess: str) -> float`

Calculates expected information gain for a guess.

- Parameters:
  - possible\_codes: Current list of possible codes
  - guess: Potential guess to evaluate
- Returns: Expected information gain in bits
- Algorithm:

- Uses Shannon entropy formula:  $-\sum P(x) * \log_2(P(x))$
- Calculates probability distribution of possible feedback
- Higher entropy indicates better guesses

`find_best_guess(possible_codes: List[str], all_codes: List[str]) -> Tuple[str, float]`

Determines optimal next guess.

- Parameters:
  - possible\_codes: Current list of possible codes
  - all\_codes: List of all valid guesses
- Returns: Tuple of (best\_guess, expected\_entropy)
- Optimization:
  - Evaluates entropy for each possible guess
  - Includes early stopping when maximum entropy is found
  - Considers all valid codes as potential guesses

User solving Game Strategy

1. Maintains set of possible codes
2. For each guess:
  - Calculates entropy for potential guesses
  - Selects guess with maximum expected information gain
  - Updates possible codes based on feedback
3. Continues until unique solution is found

Technical Details

Information Theory Application

User Game

- Initial entropy =  $4 * \log_2(10) \approx 13.29$  bits
- Entropy reduction based on feedback
- Linear uncertainty reduction model

### Computer Solving Game Strategy

- Uses Shannon entropy for decision making
- Probability-based information gain calculation
- Optimal guess selection through entropy maximization

### Performance Characteristics

- User game: Fixed memory usage,  $O(1)$  feedback calculation
- Computer solver:
  - Space complexity:  $O(n)$  where  $n$  is number of possible codes
  - Time complexity per guess:  $O(n * m)$  where  $m$  is number of all possible guesses
  - Typically solves in 5-6 guesses

### Error Handling

- Input validation for user guesses
- Feedback validation
- Detection of contradictory feedback
- Graceful handling of game termination