FORWARD PASS AND BACKPROPAGATION

FORWARD PASS:

Forward passing is the process of passing input data through the neural network to produce an output. The data passes from the input layer through hidden layers and ends at the output layer. During this process, the network calculates the activations for each layer.

Consider a single layer (l)in the network. The output z(l)z^{(l)}z(l) for layer l is:

z(l)= **W^(l(a^(l-1) + b^(l)z(l)=W(l)a(l−1)+b(l)**

Where:

- W(l) is the weight matrix of layer l.
- a(l−1) is the activation from the previous layer (or input features for the first layer).
- b(l) is the bias vector for layer l.

BACKPROPAGATION:

Backpropagation is the process of adjusting the weights and biases to minimize the error in the network's output. It involves propagating the error backward from the output to the input, allowing the network to learn. The goal is to minimize the loss function.

the MSE loss function is: **L(y',y)=(1/2)(y-y,)^2**

The error at the output layer:

$$\delta^{(L)} = \frac{\partial L}{\partial a^{(L)}} \cdot f'(z^{(L)})$$

$\delta^{(L)}$ - error for output layer

Weight Update:

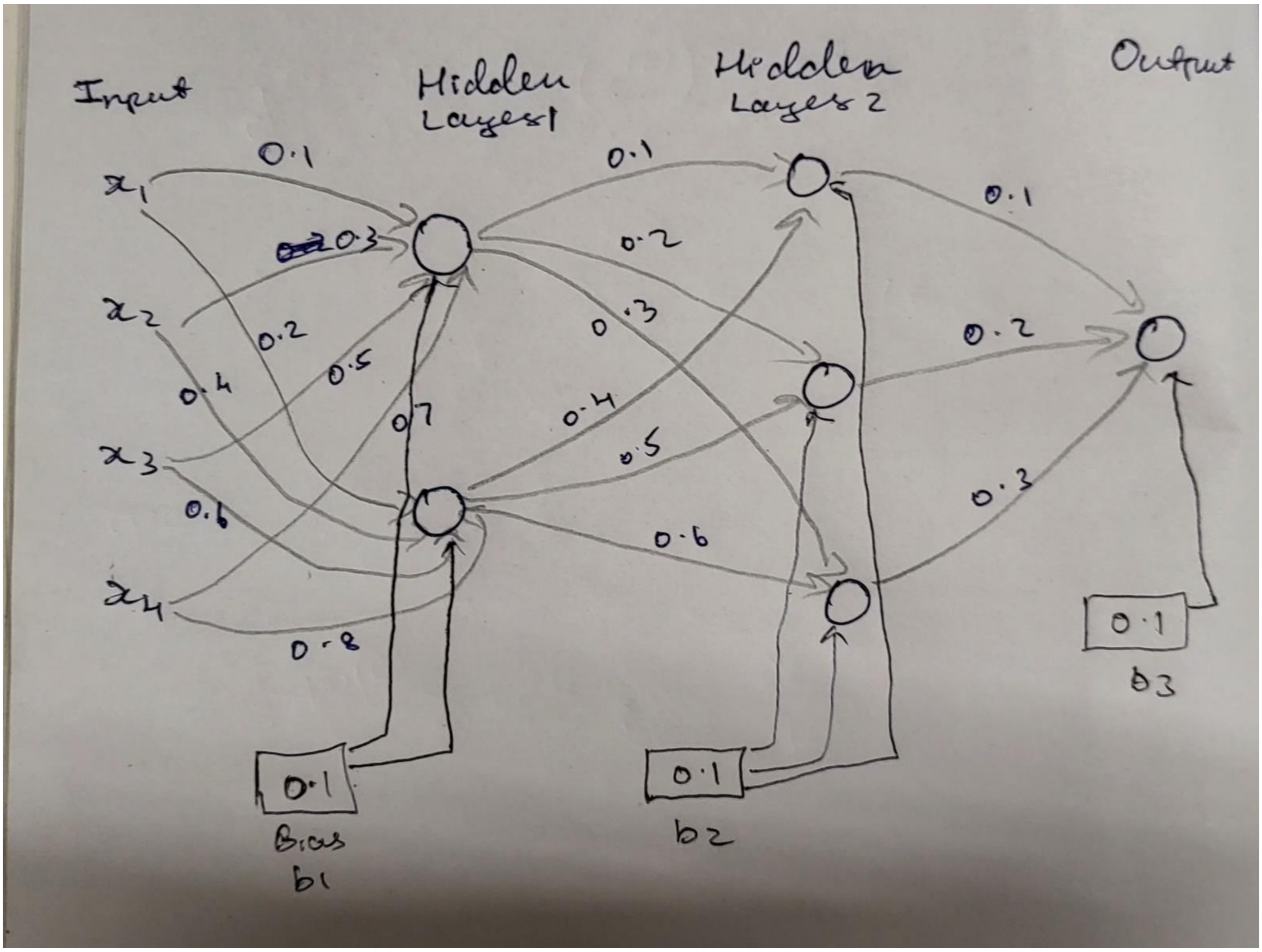
<u>Weight</u> <u>Update</u>

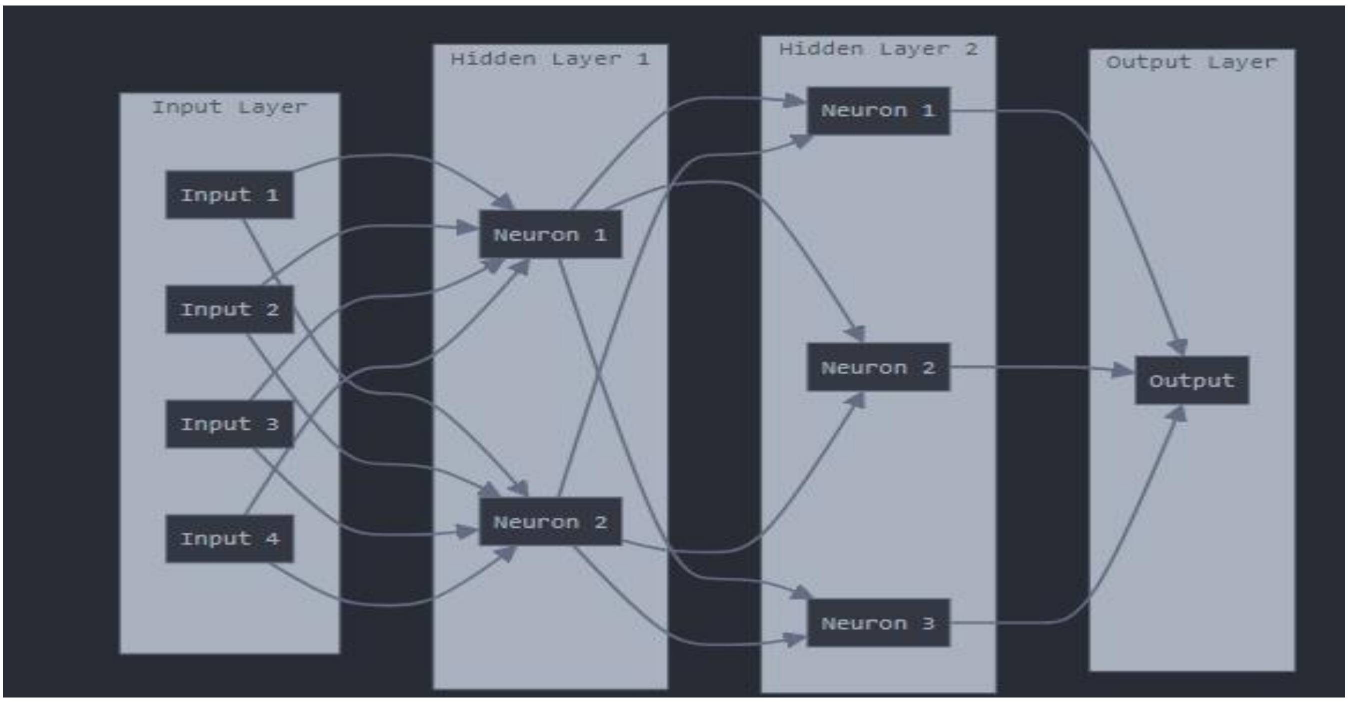
$$W^{(l)} = W^{(l)} - \alpha \cdot \frac{\partial L}{\partial W^{(l)}}$$

Here I am going to create a neural network with 4 inputs and 2 hidden layers.

Hidden layer 1 will have 2 neurons and Hidden ayer 2 will have 3 neurons.

The Structure of neural network will be:





FORWARD PASS Calculation

Layer 1 (Input to Hidden Layer 1)

W_1 = [[0.1, 0.2], [0.3, 0.4], [0.5, 0.6], [0.7, 0.8]] b_1 = [0.1, 0.1]

Layer 2 (Hidden Layer 1 to Hidden Layer 2)

W_2 = [[0.1, 0.2, 0.3], [0.4, 0.5, 0.6]] b_2 = [0.1, 0.1, 0.1]

Layer 3 (Hidden Layer 2 to Output)

W_3 = [[0.1], [0.2], [0.3]] b_3 = [0.1]

Input values

x = [1, 2, 3, 4]

Target output

y = 1

Learning rate

$\alpha = 0.1$

Iteration 1: Forward Pass

Hidden Layer 1

$z_1 = W_1^T x + b_1 = [0.1(1) + 0.3(2) + 0.5(3) + 0.7(4) + 0.1, 0.2(1) + 0.4(2) + 0.6(3) + 0.8(4) + 0.1] = [5.1, 6.1]$

$a_1 = \sigma(z_1) = [0.9940, 0.9977]$

Hidden Layer 2

$z_2 = W_2^T a_1 + b_2 = [0.1(0.9940) + 0.4(0.9977) + 0.1, 0.2(0.9940) + 0.5(0.9977) + 0.1, 0.3(0.9940) + 0.6(0.9977) + 0.1] = [0.5981, 0.7970, 0.9960]$

$a_2 = \sigma(z_2) = [0.6453, 0.6892, 0.7302]$

Output Layer

$z_3 = W_3^T a_2 + b_3 = 0.1(0.6453) + 0.2(0.6892) + 0.3(0.7302) + 0.1 = 0.4714$

$a_3 = \sigma(z_3) = 0.6157$

Backward Pass Calculation:

Output Layer

$\delta_3 = a_3 - y = 0.6157 - 1 = -0.3843$ $\nabla W_3 = a_2\delta_3 = [-0.2480, -0.2649, -0.2807]$ $\nabla b_3 = \delta_3 = -0.3843$

Hidden Layer 2

$\delta_2 = (W_3\delta_3)\ (a_2\ (1 - a_2)) = [-0.0087, -0.0142, -0.0203]$ $\nabla W_2 = a_1\delta_2^T$ $\nabla b_2 = \delta_2$

Hidden Layer 1

$\delta_1 = (W_2\delta_2)\ \ (a_1\ \ (1 - a_1)) = [-0.0001, -0.0000]$ $\nabla W_1 = x\delta_1^T$ $\nabla b_1 = \delta_1$

Weight Updation

Hidden layer 2 – output:

$\nabla W_3 = a_2\delta_3 = [-0.2401, -0.2568, -0.2722]$

$W_3new[0] = 0.1248 - 0.1 * (-0.2401) = 0.1488$

$W_3new[1] = 0.2265 - 0.1 * (-0.2568) = 0.2522$

$W_3new[2] = 0.3281 - 0.1 * (-0.2722) = 0.3553$

Hidden Layer 1 to Hidden Layer 2:

$\nabla W_2 = a_1\delta_2^T = [0.9940, 0.9977] * [-0.0087, -0.0141, -0.0201]^T = [\ [-0.0086, -0.0140, -0.0200], [-0.0087, -0.0141, -0.0201]\ ]$

$W_2new[0][0] = 0.1009 - 0.1 * (-0.0086) = 0.1018$

$W_2new[0][1] = 0.2014 - 0.1 * (-0.0140) = 0.2028$

$Wnew[0][2] = 0.3020 - 0.1 * (-0.0200) = 0.3040$

$W_2new[1][0] = 0.4000 - 0.1 * (-0.0087) = 0.4009$

$W_2new[1][1] = 0.5014 - 0.1 * (-0.0141) = 0.5028$

$W_2new[1][2] = 0.6020 - 0.1 * (-0.0201) = 0.6040$

Input to Hidden Layer1:

$\nabla W_1 = x\delta_1^T$ = [1, 2, 3, 4] * [-0.0001, -0.0000]^T = [[-0.0001, -0.0000], [-0.0002, -0.0000], [-0.0003, -0.0000], [-0.0004, -0.0000]]

W_1new[0][0] = 0.1000 - 0.1 * (-0.0001) = 0.1000

W_1new[0][1] = 0.2000 - 0.1 * (-0.0000) = 0.2000

Wnew[1][0] = 0.3000 - 0.1 * (-0.0002) = 0.3000

W_1new[1][1] = 0.4000 - 0.1 * (-0.0000) = 0.4000

W_1new[2][0] = 0.5000 - 0.1 * (-0.0003) = 0.5000

Wnew[2][1] = 0.6000 - 0.1 * (-0.0000) = 0.6000

W_1new[3][0] = 0.7000 - 0.1 * (-0.0004) = 0.7000

Wnew[3][1] = 0.8000 - 0.1 * (-0.0000) = 0.8000

ITERATION 2:

Forward Pass

Hidden Layer 1

$z_1 = W_1^T x + b_1$ = [0.1(1) + 0.3(2) + 0.5(3) + 0.7(4) + 0.1, 0.2(1) + 0.4(2) + 0.6(3) + 0.8(4) + 0.1] = [5.1, 6.1]

$a_1 = \sigma(z_1)$ = [0.9940, 0.9977]

Hidden Layer 2

$z_2 = W_2^T a_1 + b_2$ = [0.1009(0.9940) + 0.4000(0.9977) + 0.1009, 0.2014(0.9940) + 0.5014(0.9977) + 0.1014, 0.3020(0.9940) + 0.6020(0.9977) + 0.1020] = [0.6009, 0.8028, 1.0048]

$a_2 = \sigma(z_2)$ = [0.6458, 0.6906, 0.7321]

Output Layer

$z_3 = W_3^T a_2 + b_3$ = 0.1248(0.6458) + 0.2265(0.6906) + 0.3281(0.7321) + 0.1384 = 0.5246

$a_3 = \sigma(z_3)$ = 0.6282

Backward Pass

Output Layer

$\delta_3 = a_3 - y = 0.6282 - 1 = -0.3718$ $\nabla W_3 = a_2\delta_3 = [-0.2401, -0.2568, -0.2722]$ $\nabla b_3 = \delta_3 = -0.3718$

Hidden Layer 2

$\delta_2 = (W_3\delta_3)\ \ (a_2\,(1 - a_2)) = [-0.0087, -0.0141, -0.0201]$ $\nabla W_2 = a_1\delta_2^T$ $\nabla b_2 = \delta_2$

Hidden Layer 1

$\delta_1 = (W_2\delta_2)\ \ (a_1\ \ (1 - a_1)) = [-0.0001, -0.0000]$ $\nabla W_1 = x\delta_1^T$ $\nabla b_1 = \delta_1$