Digital Signal Processing Laboratory

LabSheet-01

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Discrete Time Sinusoids and Exponentials

A. Properties of Sinusoids

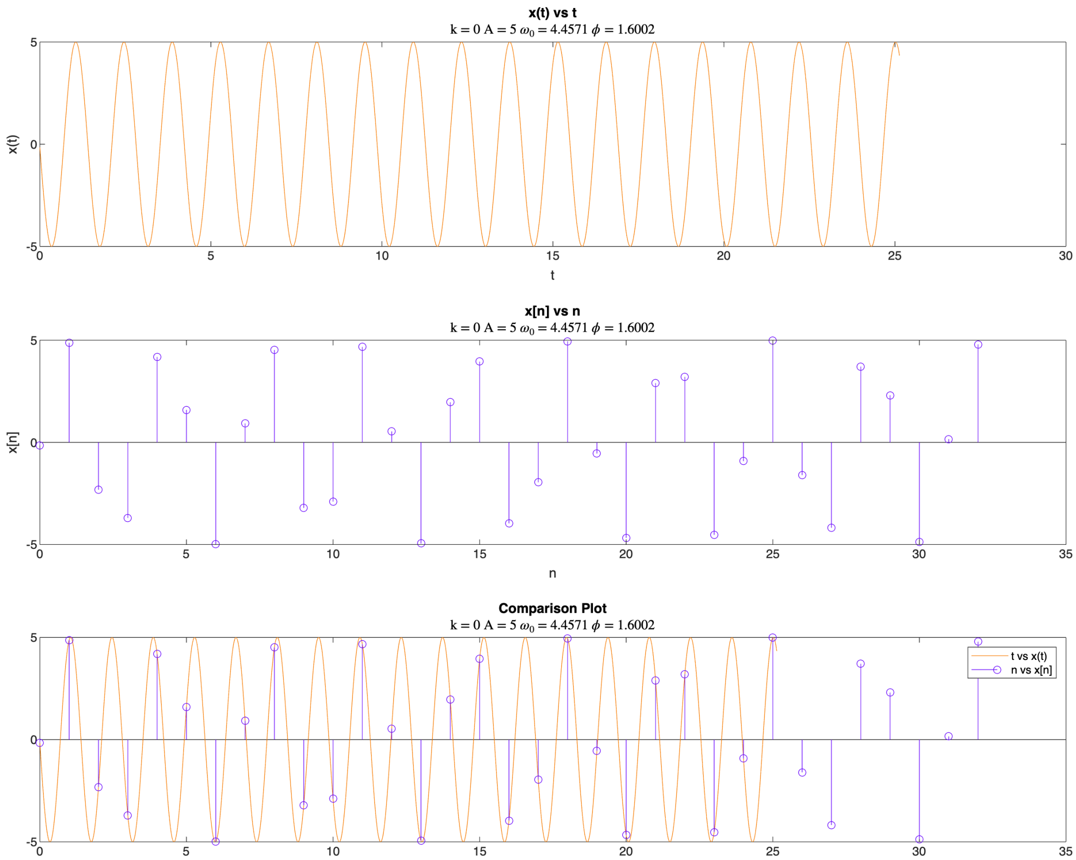
I. Frequencies Separated by Interval 2 pi

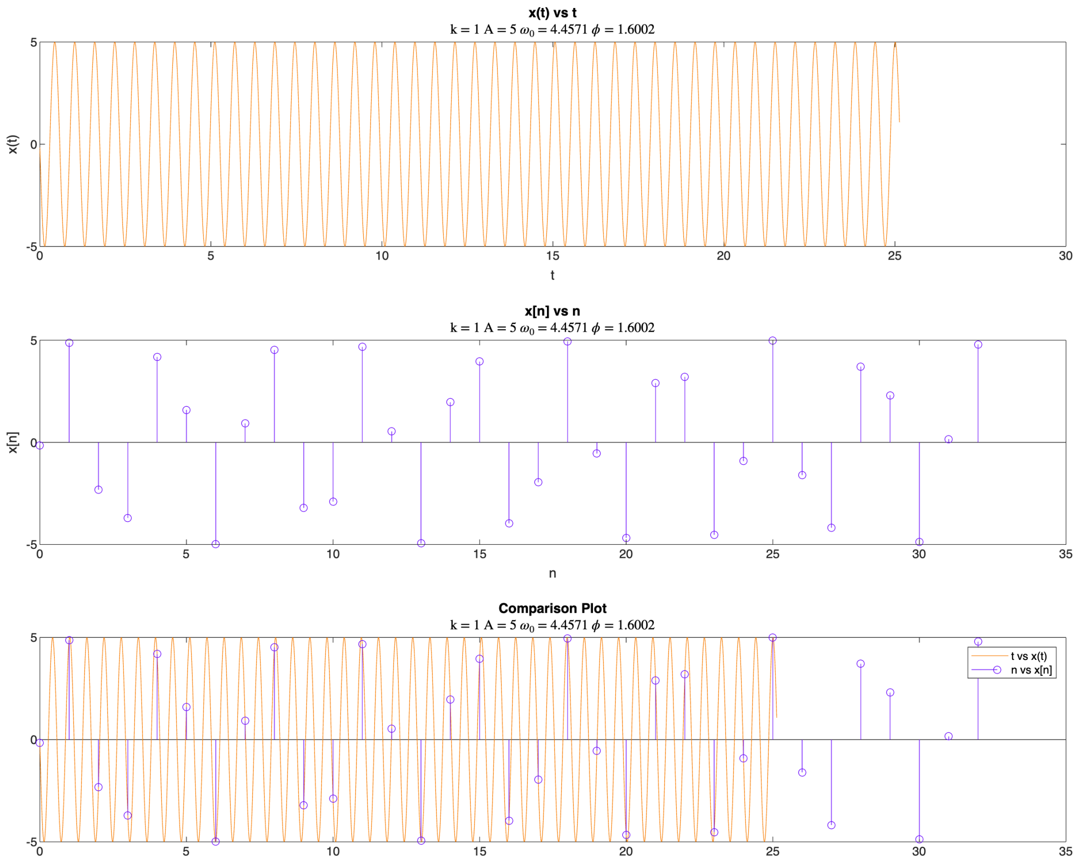
$x(t) = A * cos((\omega_0 + (2 * \pi * k)) * t + \phi)$

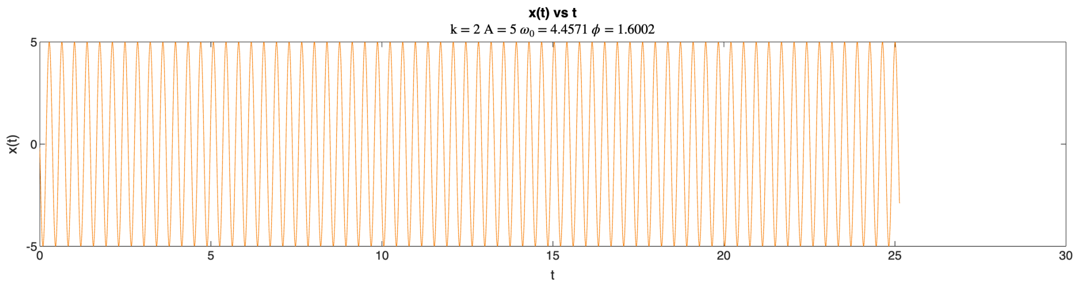
$x[n] = A * cos[(\omega_0 + (2 * \pi * k)) * n + \phi]$

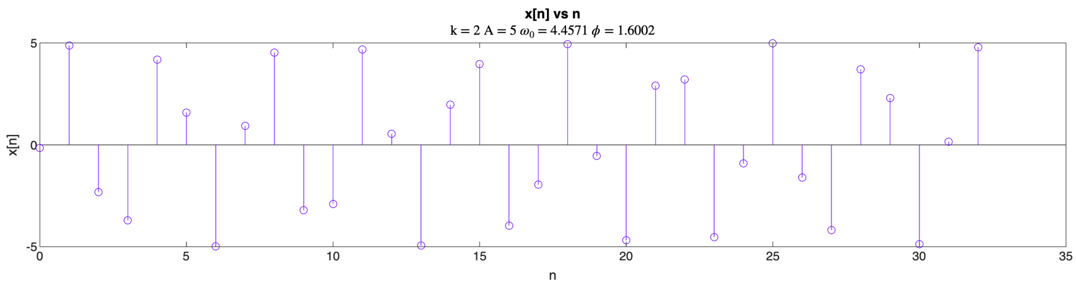
```
% Colour Constants
COLOUR_VIOLET = "#6E0FFE";
COLOUR_ORANGE = "#F87E05";

% Initialization of Required Variables
a = 5;
w0 = rand() * 2 * pi;
phi = -pi + rand() * 2 * pi;

% t for Continuous Time
t = 0:0.0001:8*pi;

% n for Discrete Time
n = 0:1:32;

% Given k = 0, 1, 2, 3, 4
for k = 0:4
    % Computing x(t)
    x_t = a * cos((w0 + (2 * pi * k)) * t + phi);

    % New Figure for Plotting
    figure;

    % 1st Subplot, x(t)
    subplot(3,1,1);
    plot(t, x_t, Color = COLOUR_ORANGE);
    xlabel('t');
```

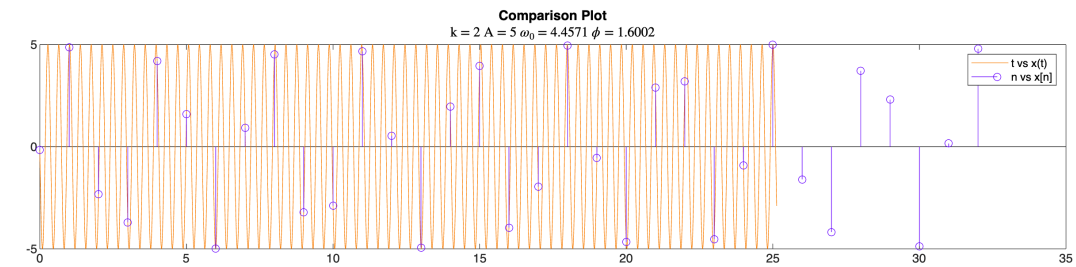
```
    ylabel('x(t)');
    %xlim([-1, 8*pi + 1]);
    %ylim([-6, 6])
    title('x(t) vs t');
    subtitle(['k = ' num2str(k) ' A = ' num2str(a) ' $\omega_0$ = '
num2str(w0) ' $\phi$ = ' num2str(phi)], Interpreter = 'latex');

    % Increasing Subplot size by 3 (Subplot Scales down by 3)
    figProps = gcf;
    figProps.Position(3:4) = figProps.Position(3:4) * 3;

    % 2nd Subplot, x[n]
    x_n = a * cos((w0 + (2 * pi * k)) * n + phi);
    subplot(3,1,2);
    stem(n, x_n, Color = COLOUR_VIOLET);
    xlabel('n');
    ylabel('x[n]');
    %ylim([-6,6])
    %xlim([-1,33]);
    title('x[n] vs n');
    subtitle(['k = ' num2str(k) ' A = ' num2str(a) ' $\omega_0$ = '
num2str(w0) ' $\phi$ = ' num2str(phi)], Interpreter = 'latex');

    % 3rd Subplot, x(t) & x[n]
    % First Plot x(t), Hold Figure and Plot x[n] (Sketches in Same Plot)
    subplot(3,1,3);
    plot(t, x_t, Color = COLOUR_ORANGE);
    hold on;
    stem(n, x_n, Color = COLOUR_VIOLET);
    title("Comparison Plot");
    legend({'t vs x(t)' 'n vs x[n]'})
    subtitle(['k = ' num2str(k) ' A = ' num2str(a) ' $\omega_0$ = '
num2str(w0) ' $\phi$ = ' num2str(phi)], Interpreter = 'latex');

end
```

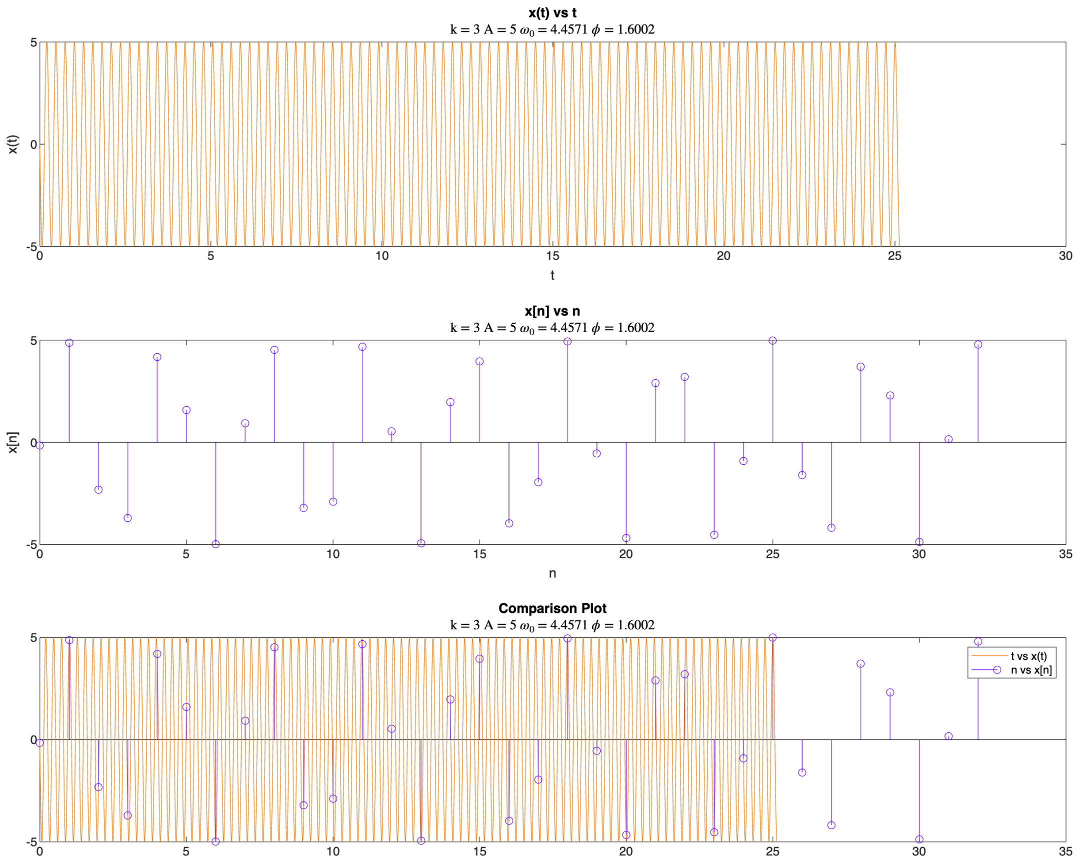

x(t) vs t
k = 0 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x(t)
t
x[n] vs n
k = 0 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x[n]
n
Comparison Plot
k = 0 A = 5 ω_0 = 4.4571 ϕ = 1.6002
t vs x(t)
n vs x[n]


x(t) vs t
k = 1 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x(t)
t
x[n] vs n
k = 1 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x[n]
n
Comparison Plot
k = 1 A = 5 ω_0 = 4.4571 ϕ = 1.6002
t vs x(t)
n vs x[n]

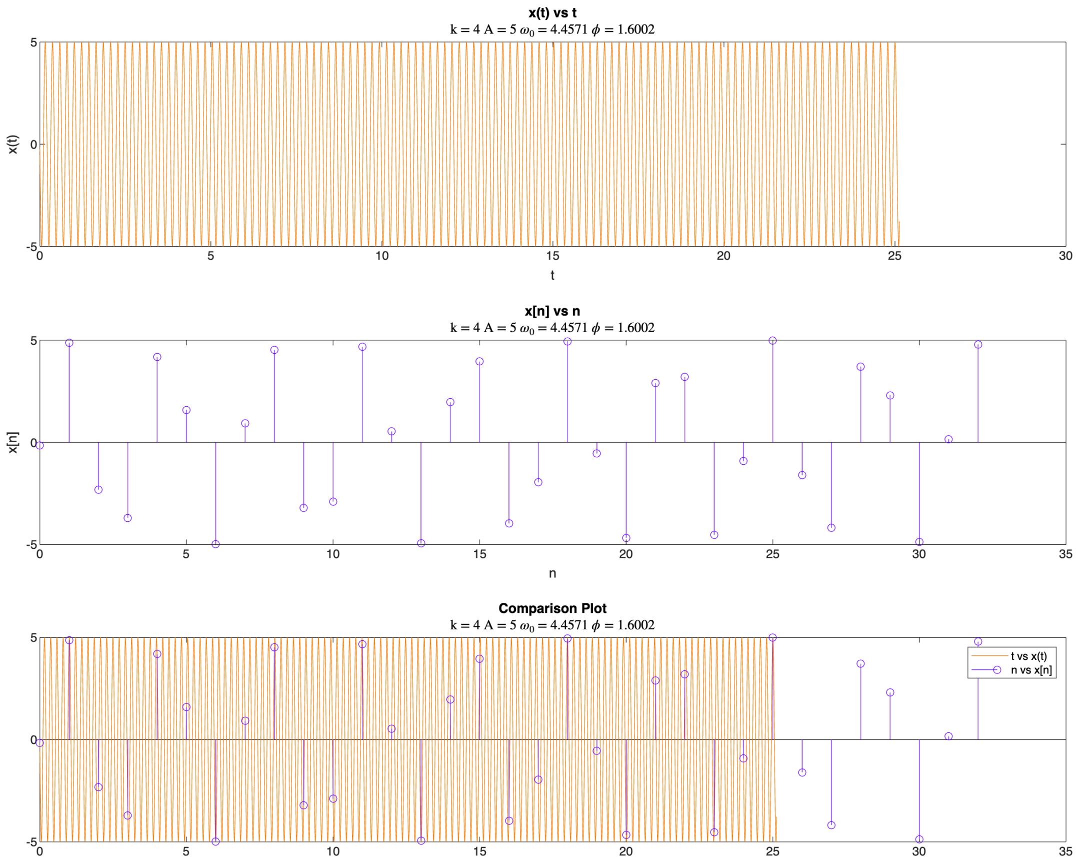

x(t) vs t
k = 2 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x(t)
t


x[n] vs n
k = 2 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x[n]
n


Comparison Plot
k = 2 A = 5 ω_0 = 4.4571 ϕ = 1.6002
t vs x(t)
n vs x[n]



x(t) vs t
k = 3 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x(t)
t
x[n] vs n
k = 3 A = 5 ω_0 = 4.4571 ϕ = 1.6002
x[n]
n
Comparison Plot
k = 3 A = 5 ω_0 = 4.4571 ϕ = 1.6002
t vs x(t)
n vs x[n]



Observation:

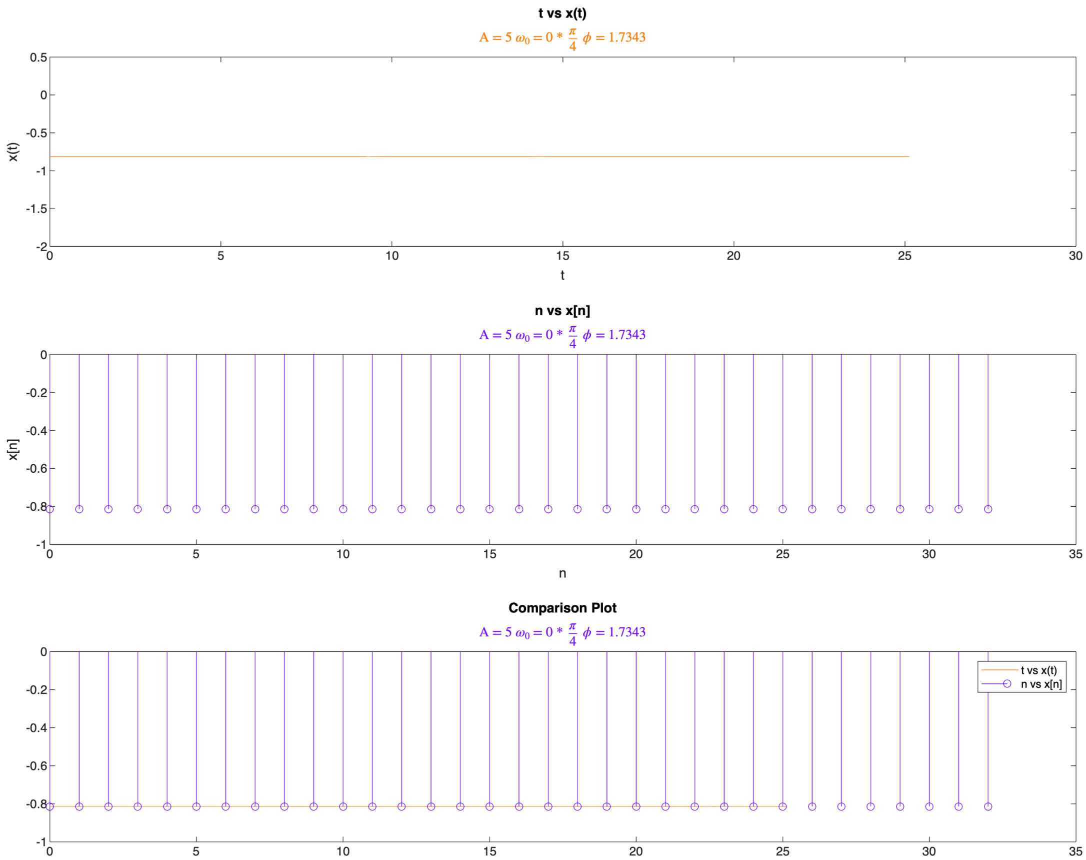
For continuous time plots, as k increases, angular frequency increases which in turn makes the graph look compressed like a spring.

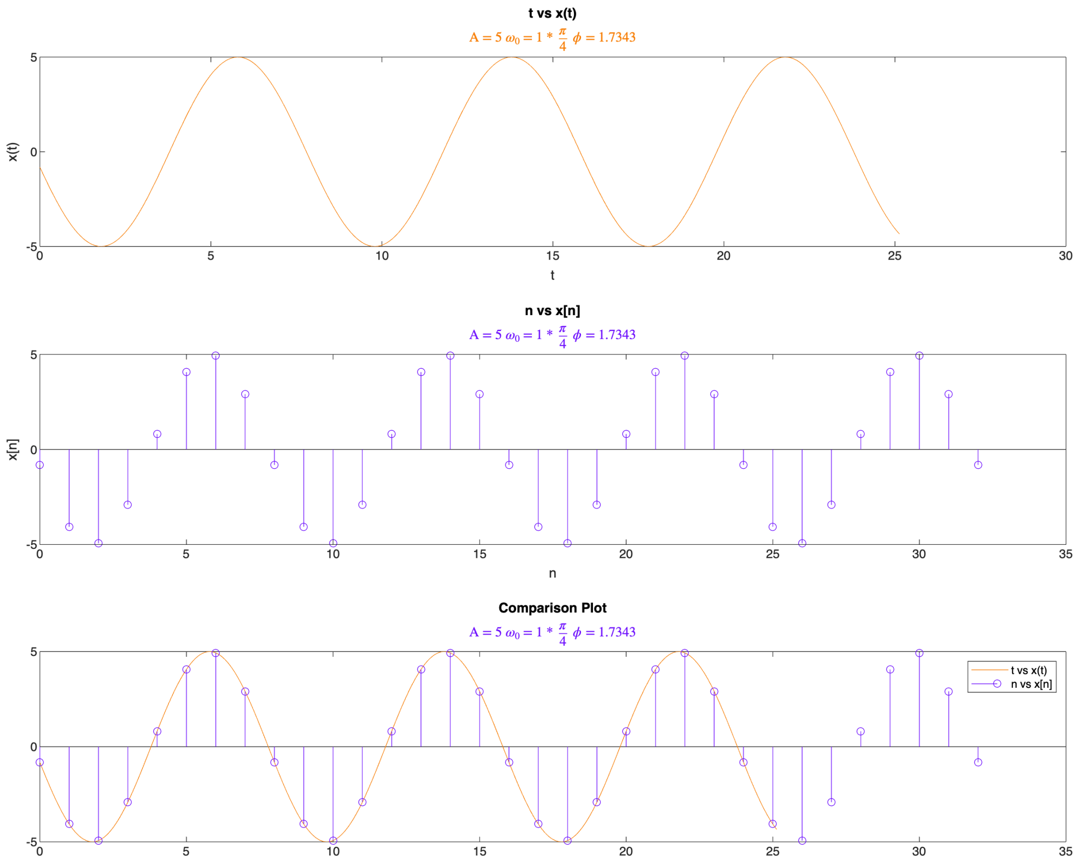
For discrete time plots, frequencies as integral multiple of 2π added to ω_0 gives the same discrete time plot as for the original ω_0.

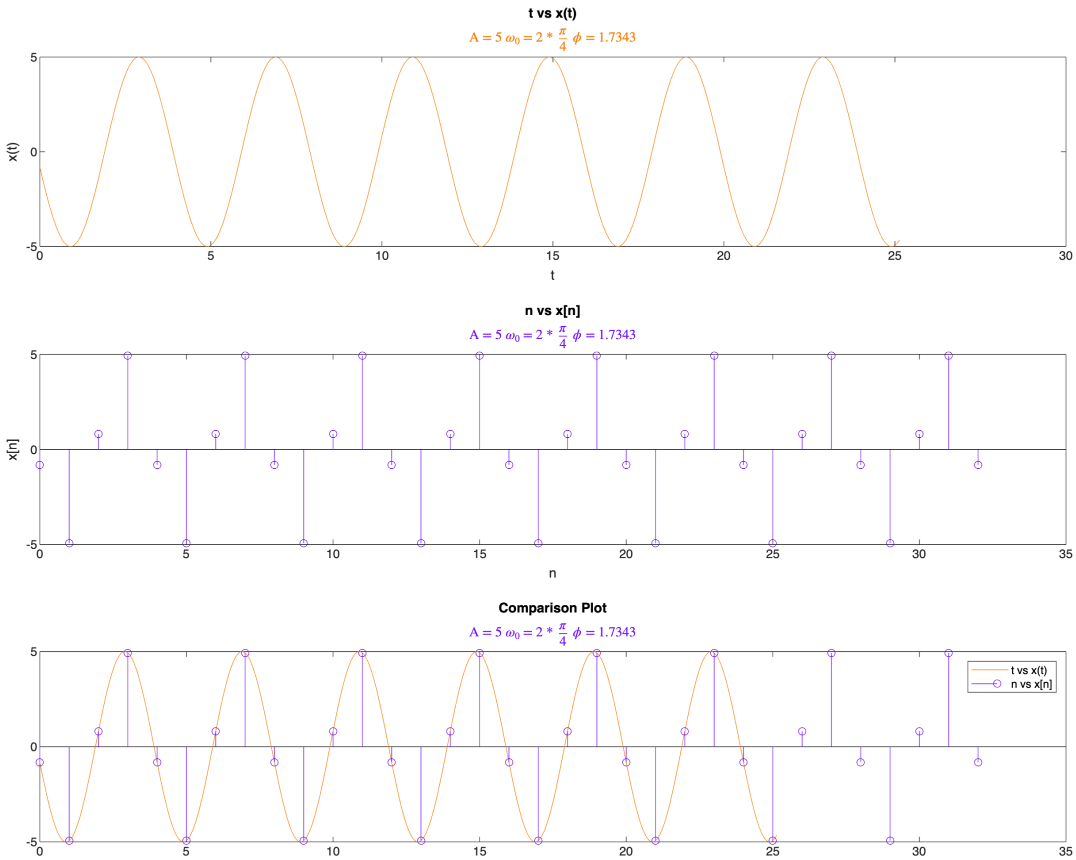
II. Frequencies within $[0, 2\pi]$

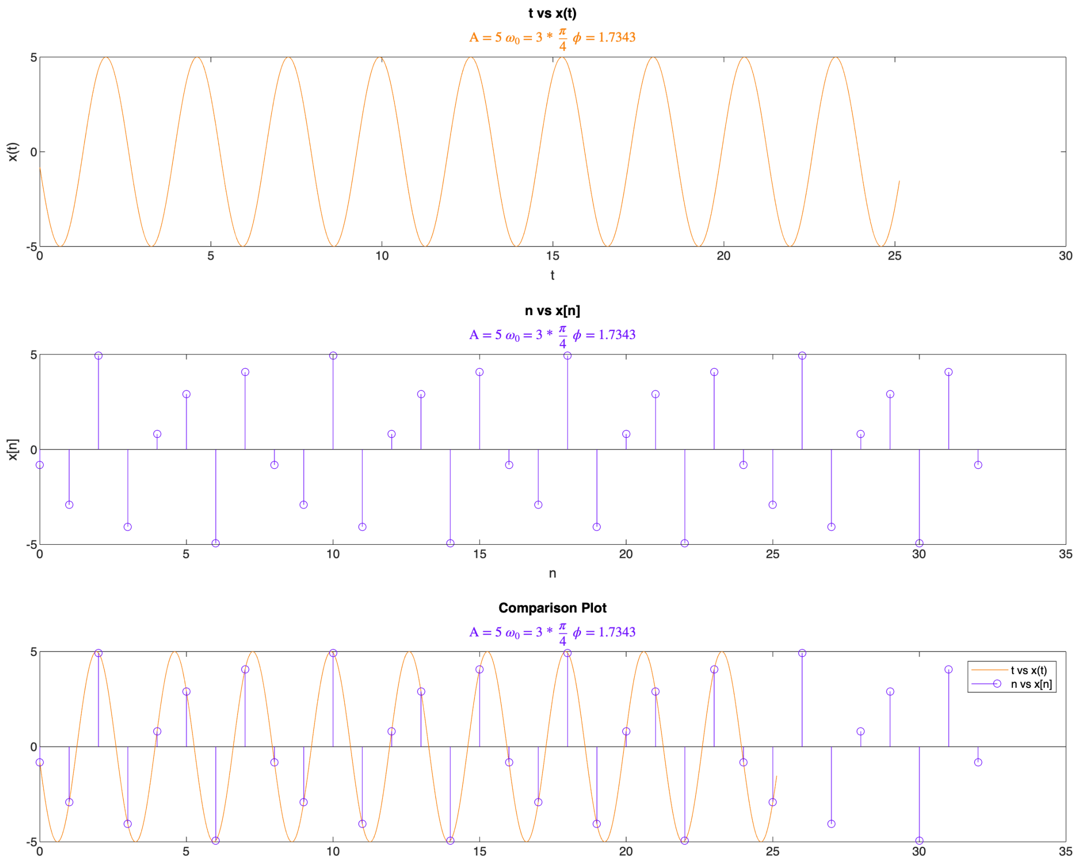
```
% Initialization of Required Variables
a = 5;
phi = rand() * 2 * pi;

% t for Continuous Time
t = 0:0.0001:8*pi;

% n for Discrete Time
n = 0:1:32;
```

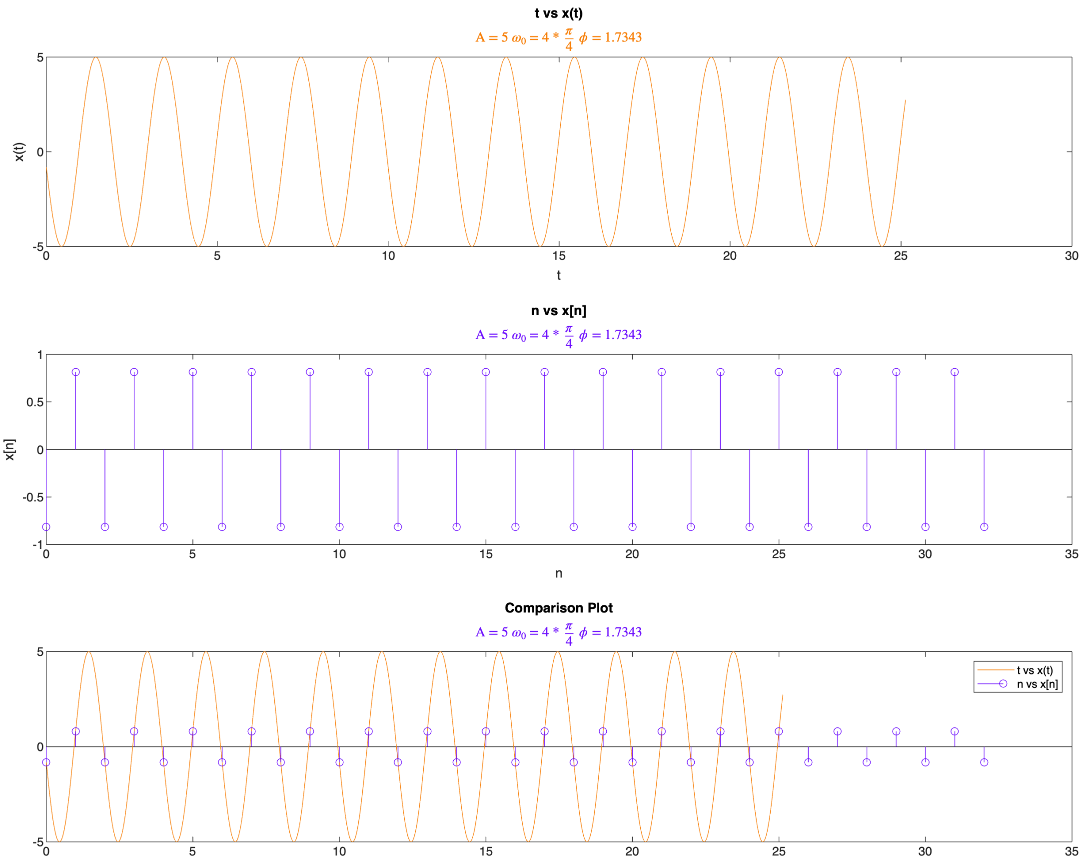
```
for k = 0:8
    w0 = k * pi / 4;
    % Computing x_t and x_n
    x_t = a * cos((w0 * t) + phi);
    x_n = a * cos((w0 * n) + phi);

    % New Figure
    figure;

    %1st Subplot x(t)
    subplot(3,1,1);
    plot(t, x_t, Color = COLOUR_ORANGE);
    xlabel('t');
    ylabel('x(t)');
    %xlim([-1, 8*pi + 1]);
    %ylim([-6, 6])
    title('t vs x(t)');
    subtitle(['A = ' num2str(a) ' $\omega_0$ = ' num2str(k) ' * $\frac{\pi}
{4}$ ' ' $\phi$ = ' num2str(phi)], Interpreter = 'latex', Color =
COLOUR_ORANGE);

    figProps = gcf;
    figProps.Position(3:4) = figProps.Position(3:4) * 3;

    %2nd Subplot x[n]
    subplot(3,1,2);
    stem(n, x_n, Color = COLOUR_VIOLET);
    xlabel('n');
    ylabel('x[n]');
    %ylim([-6,6])
    %xlim([-1,33]);
    title('n vs x[n]');
    subtitle(['A = ' num2str(a) ' $\omega_0$ = ' num2str(k) ' * $\frac{\pi}
{4}$ ' ' $\phi$ = ' num2str(phi)], Interpreter = 'latex', Color =
COLOUR_VIOLET);

    %3rd Subplot x(t) & x[n]
    subplot(3,1,3);
    plot(t, x_t, Color = COLOUR_ORANGE);
    hold on;
    stem(n, x_n, Color = COLOUR_VIOLET);
    title("Comparison Plot");
    legend({'t vs x(t)' 'n vs x[n]'})
    subtitle(['A = ' num2str(a) ' $\omega_0$ = ' num2str(k) ' * $\frac{\pi}
{4}$ ' ' $\phi$ = ' num2str(phi)], Interpreter = 'latex', Color =
COLOUR_VIOLET);
end
```

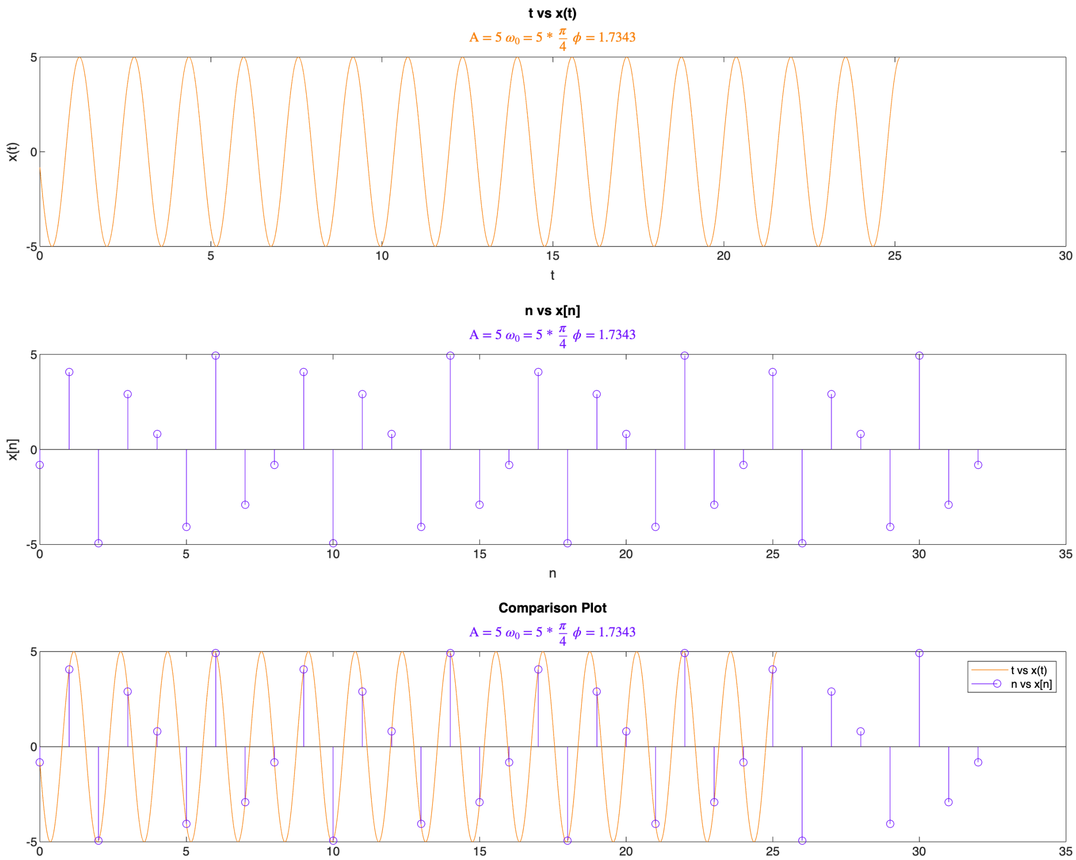

t vs x(t)
$A = 5\ \omega_0 = 0 * \frac{\pi}{4}\ \phi = 1.7343$
x(t)
t
n vs x[n]
$A = 5\ \omega_0 = 0 * \frac{\pi}{4}\ \phi = 1.7343$
x[n]
n
Comparison Plot
$A = 5\ \omega_0 = 0 * \frac{\pi}{4}\ \phi = 1.7343$
t vs x(t)
n vs x[n]


t vs x(t)
A = 5 ω0 = 1 * π/4 ϕ = 1.7343
x(t)
t
n vs x[n]
A = 5 ω0 = 1 * π/4 ϕ = 1.7343
x[n]
n
Comparison Plot
A = 5 ω0 = 1 * π/4 ϕ = 1.7343
t vs x(t)
n vs x[n]


t vs x(t)
$A = 5\ \omega_0 = 2 * \frac{\pi}{4}\ \phi = 1.7343$
x(t)
t
n vs x[n]
$A = 5\ \omega_0 = 2 * \frac{\pi}{4}\ \phi = 1.7343$
x[n]
n
Comparison Plot
$A = 5\ \omega_0 = 2 * \frac{\pi}{4}\ \phi = 1.7343$
t vs x(t)
n vs x[n]


t vs x(t)
$A = 5 \ \omega_0 = 3 * \frac{\pi}{4} \ \phi = 1.7343$
x(t)
t
n vs x[n]
$A = 5 \ \omega_0 = 3 * \frac{\pi}{4} \ \phi = 1.7343$
x[n]
n
Comparison Plot
$A = 5 \ \omega_0 = 3 * \frac{\pi}{4} \ \phi = 1.7343$
t vs x(t)
n vs x[n]

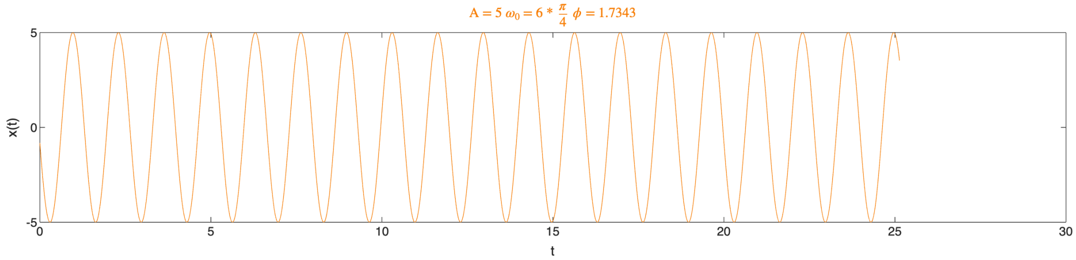

t vs x(t)
A = 5 ω0 = 4 * π/4 ϕ = 1.7343
x(t)
t
n vs x[n]
A = 5 ω0 = 4 * π/4 ϕ = 1.7343
x[n]
n
Comparison Plot
A = 5 ω0 = 4 * π/4 ϕ = 1.7343
t vs x(t)
n vs x[n]



t vs x(t)
$A = 5$ $\omega_0 = 5 * \frac{\pi}{4}$ $\phi = 1.7343$
x(t)
t
n vs x[n]
$A = 5$ $\omega_0 = 5 * \frac{\pi}{4}$ $\phi = 1.7343$
x[n]
n
Comparison Plot
$A = 5$ $\omega_0 = 5 * \frac{\pi}{4}$ $\phi = 1.7343$
t vs x(t)
n vs x[n]

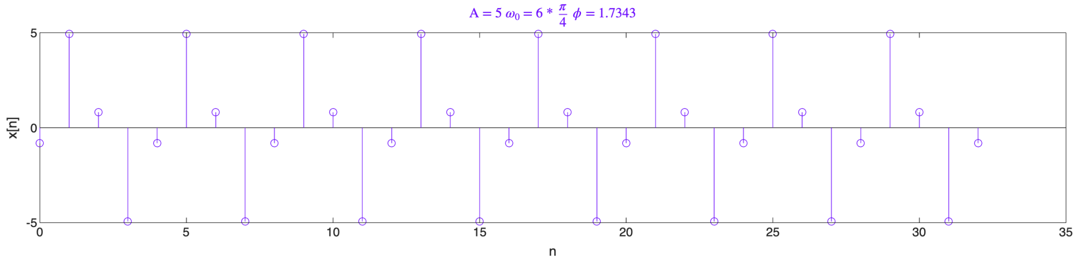
t vs x(t)

$A = 5\ \omega_0 = 6 * \frac{\pi}{4}\ \phi = 1.7343$



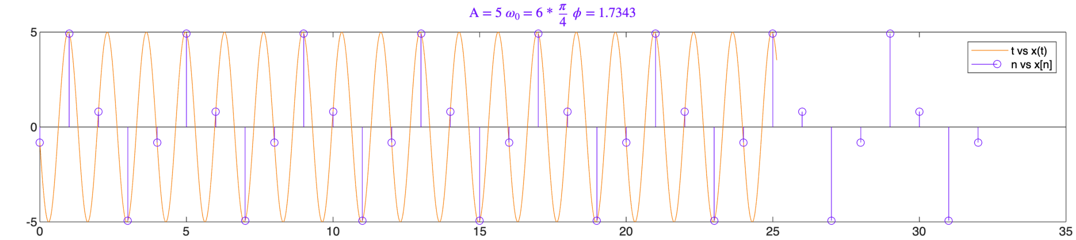
n vs x[n]

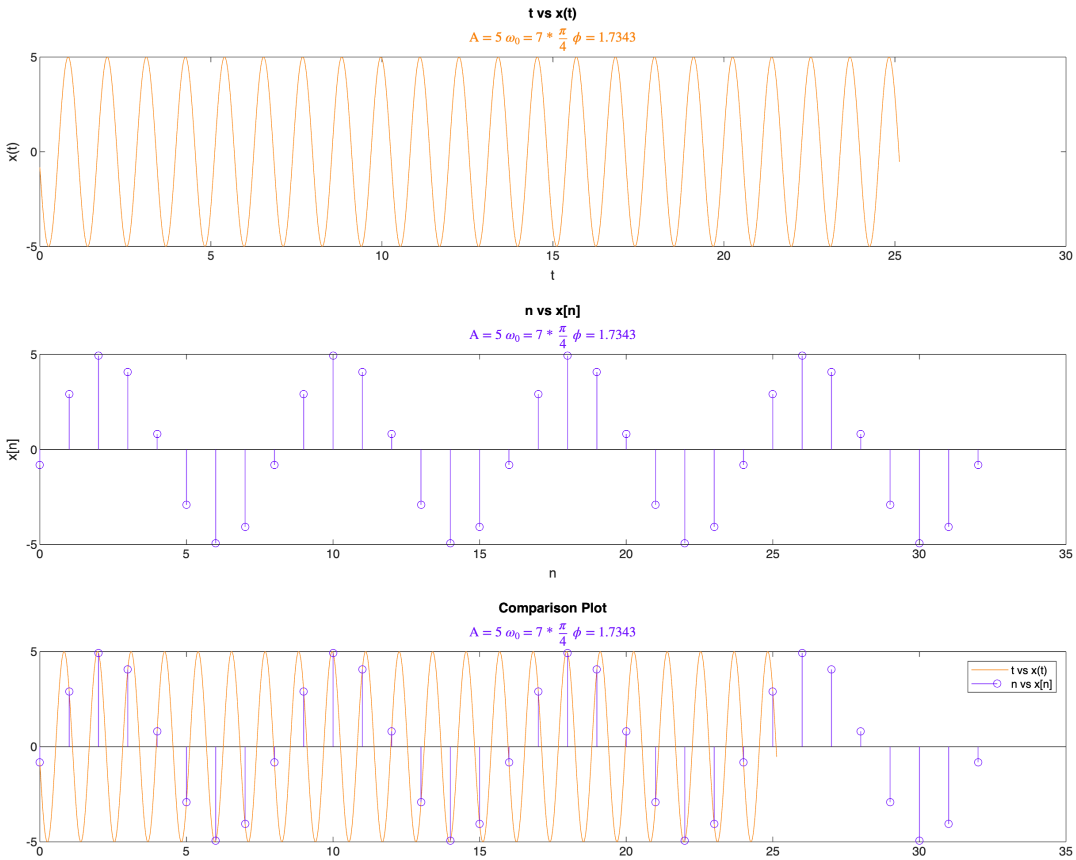
$A = 5\ \omega_0 = 6 * \frac{\pi}{4}\ \phi = 1.7343$

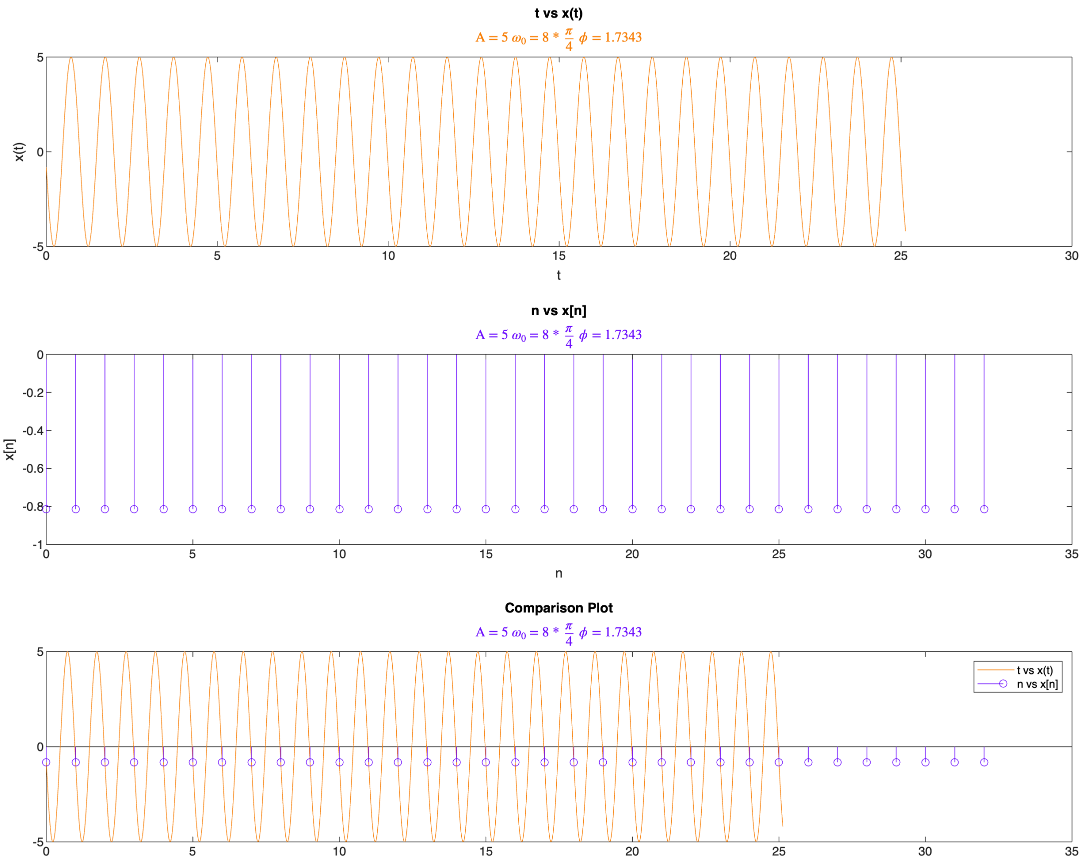


Comparison Plot

$A = 5\ \omega_0 = 6 * \frac{\pi}{4}\ \phi = 1.7343$




t vs x(t)
$A = 5\ \omega_0 = 7 * \frac{\pi}{4}\ \phi = 1.7343$
x(t)
t
n vs x[n]
$A = 5\ \omega_0 = 7 * \frac{\pi}{4}\ \phi = 1.7343$
x[n]
n
Comparison Plot
$A = 5\ \omega_0 = 7 * \frac{\pi}{4}\ \phi = 1.7343$
t vs x(t)
n vs x[n]



Observation:

For discrete time sinusoid $x_n = A\cos\,(\omega_0 n + \phi)$ as ω_0 increases from $[0, \pi]$, x_n oscillates progressively and more rapidly. However as ω_0 increases from $[\pi, 2\pi]$ oscillation decreases and rapidity decreases.

III. Periodicity

Continuous Sinusoids:

(1) $x_t = A\cos\,(\frac{\pi}{4}t + \phi)$

(2) $x_t = A\cos\,(\frac{3\pi}{8}t + \phi)$

(3) $x_t = A\cos\,(t + \phi)$

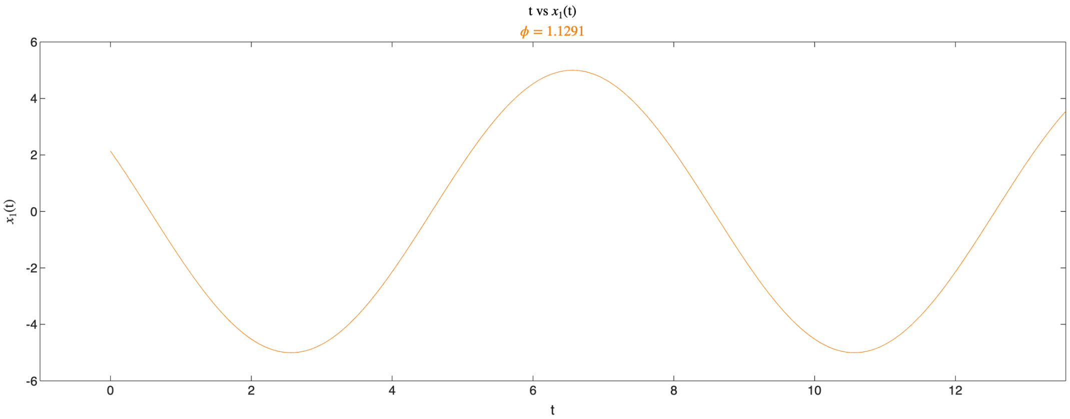
Discrete Sinusoids:

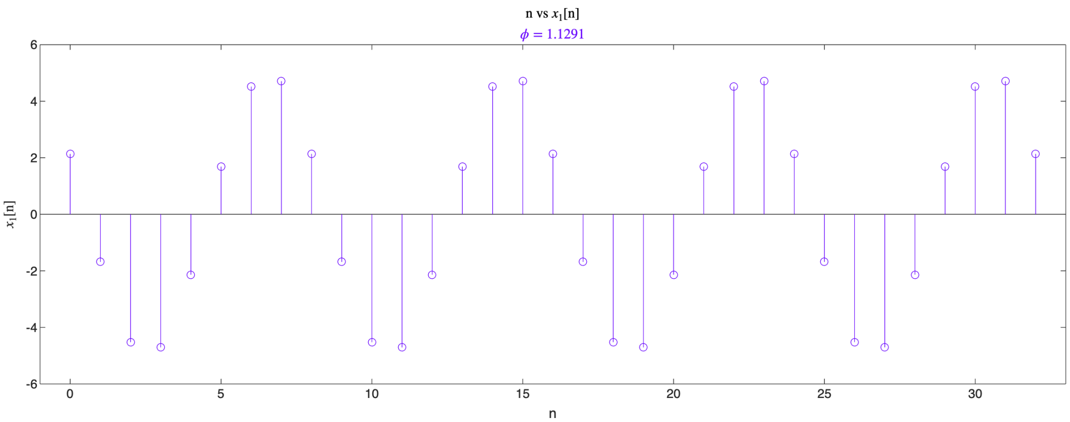
(1) $x[n] = A\cos\left[\frac{\pi}{4}n + \phi\right]$

(2) $x[n] = A\cos\left[\frac{3\pi}{8}n + \phi\right]$

(3) $x[n] = A\cos\left[n + \phi\right]$

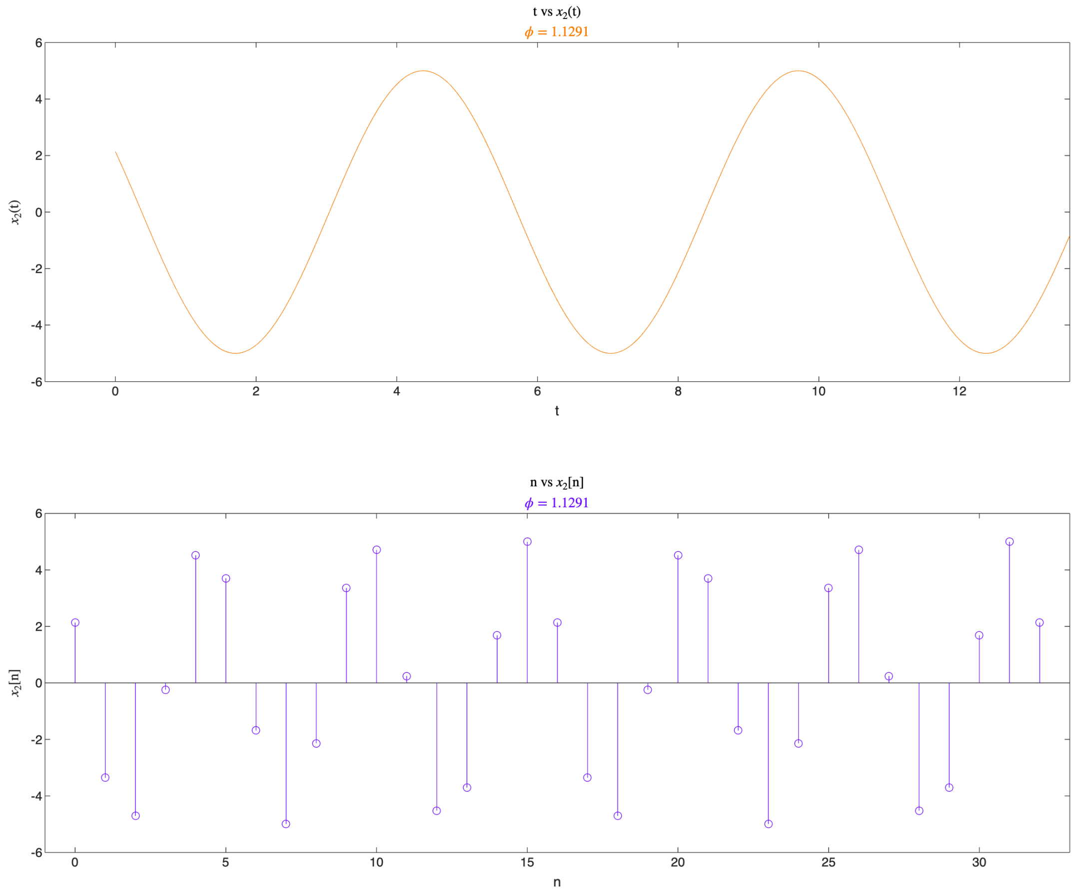
```
% Initialization of required variables
a = 5;
phi = -pi + rand() * 2 * pi;
t = 0:0.0001:8*pi;
n = 0:1:32;

% Computing of x(t) as given above
x1_t = a * cos(((pi/4) * t) + phi);
x2_t = a * cos(((3 * pi/8) * t) + phi);
x3_t = a * cos(t + phi);

% Computing of x[n] as given above
x1_n = a * cos(((pi/4) * n) + phi);
x2_n = a * cos(((3 * pi/8) * n) + phi);
x3_n = a * cos(n + phi);

% Plotting x1_t, x2_t, x3_t, x1_n, x2_n, x3_n

figure;
subplot(2,1,1);
plot(t, x1_t, Color = COLOUR_ORANGE);
xlabel("t");
ylabel("$x_1$(t)", Interpreter = "latex");
xlim([-1, 4*pi + 1]);
ylim([-6, 6])
title("t vs $x_1$(t)", Interpreter = "latex");
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_ORANGE, Interpreter =
'latex');

figProps = gcf;
figProps.Position(3:4) = figProps.Position(3:4) * 2;

subplot(2,1,2);
stem(n, x1_n, Color = COLOUR_VIOLET);
xlabel("n");
ylabel("$x_1$[n]", Interpreter = "latex");
ylim([-6,6])
xlim([-1,33]);
title("n vs $x_1$[n]", Interpreter = "latex");
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_VIOLET, Interpreter =
'latex');
```





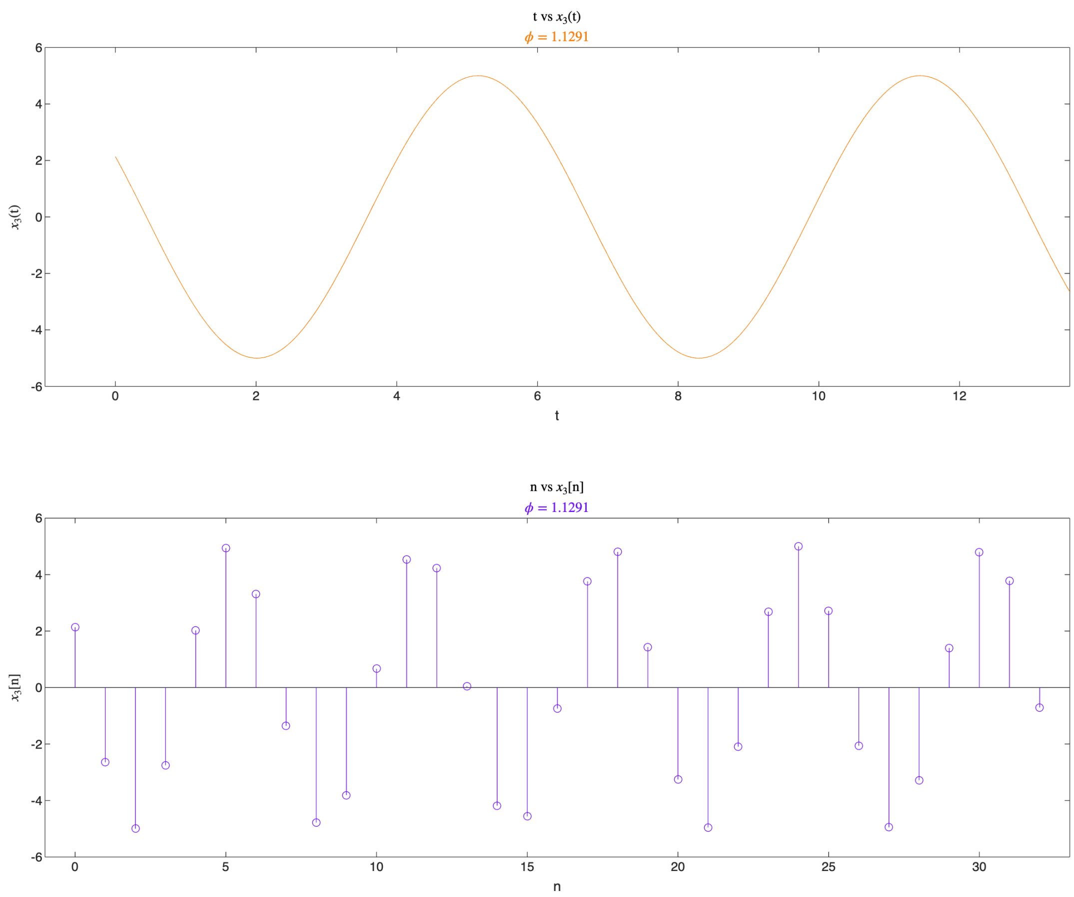

```
figure;
subplot(2,1,1);
plot(t, x2_t, Color = COLOUR_ORANGE);
xlabel("t");
ylabel("$x_2$(t)", Interpreter = "latex");
xlim([-1, 4*pi + 1]);
ylim([-6, 6])
title("t vs $x_2$(t)", Interpreter = "latex");
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_ORANGE, Interpreter =
'latex');

figProps = gcf;
figProps.Position(3:4) = figProps.Position(3:4) * 2;

subplot(2,1,2);
stem(n, x2_n, Color = COLOUR_VIOLET);
xlabel("n");
ylabel("$x_2$[n]", Interpreter = "latex");
ylim([-6,6])
xlim([-1,33]);
title("n vs $x_2$[n]", Interpreter = "latex");
```

```
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_VIOLET, Interpreter =
'latex');
```




```
figure;
subplot(2,1,1);
plot(t, x3_t, Color = COLOUR_ORANGE);
xlabel("t");
ylabel("$x_3$(t)", Interpreter = "latex");
xlim([-1, 4*pi + 1]);
ylim([-6, 6])
title("t vs $x_3$(t)", Interpreter = "latex");
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_ORANGE, Interpreter =
'latex');

figProps = gcf;
figProps.Position(3:4) = figProps.Position(3:4) * 2;

subplot(2,1,2);
stem(n, x3_n, Color = COLOUR_VIOLET);
xlabel("n");
ylabel("$x_3$[n]", Interpreter = "latex");
```

```
ylim([-6,6])
xlim([-1,33]);
title("n vs $x_3$[n]", Interpreter = "latex");
subtitle(['$\phi$ = ' num2str(phi)], Color = COLOUR_VIOLET, Interpreter = 'latex');
```



Observation:

Continuous time sinusoids are always periodic irrespective of what is the value of ω_0 .

Thus Periods for Continuous time sinusoids given above are

(1) T_0 = 8

(2) T_0 = 16/3

(3) T_0 = 2π

Which is clearly concluded in the graph.

Discrete time sinusoids are periodic only if ω_0 is an integral multiple of $\frac{2\pi}{N}$.

Thus N for Discerete time sinusoids given above are

(1) N = 8, T_0 = 8

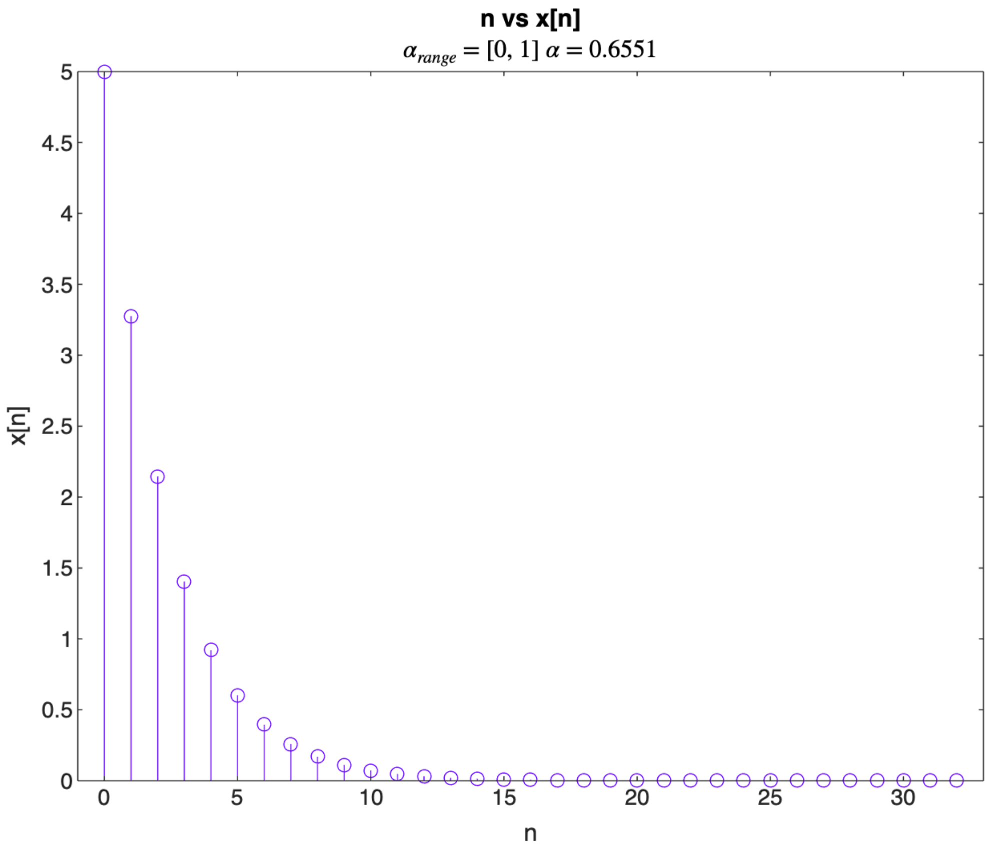
(2) N = 16, T_0 = 16/3

(3) N = (Not Any Integer) thus it is not periodic.

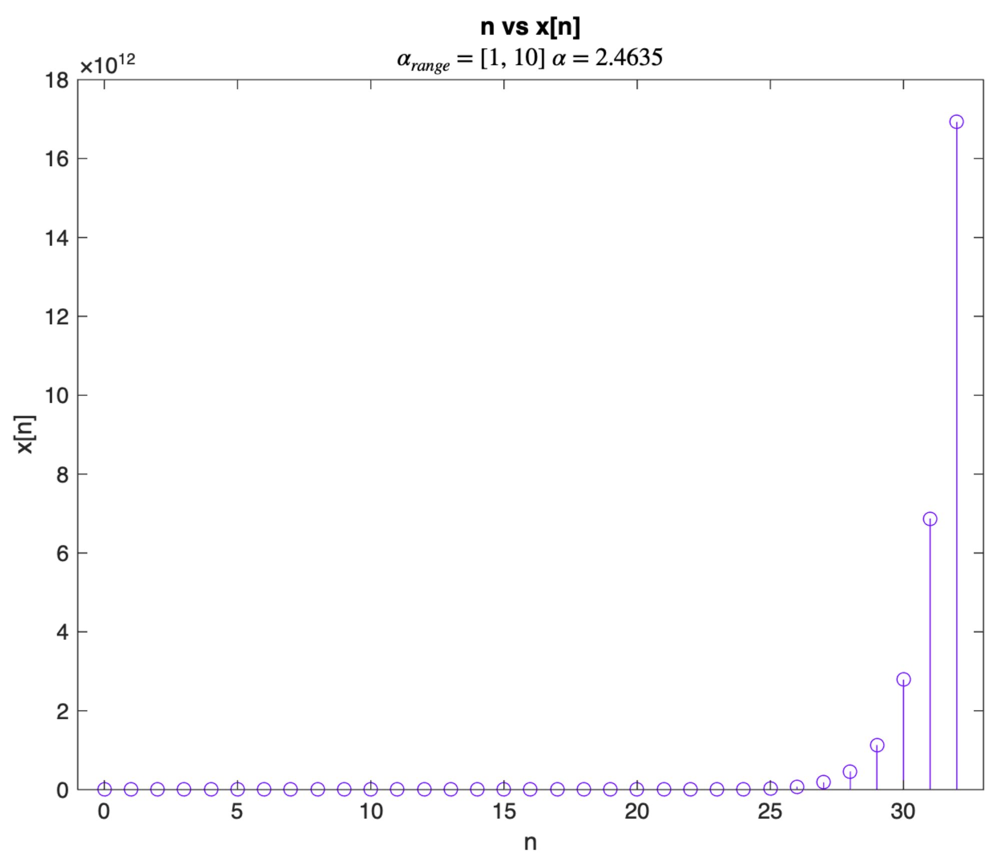
Which is concluded by the last graph.

B. Exponential Sequences

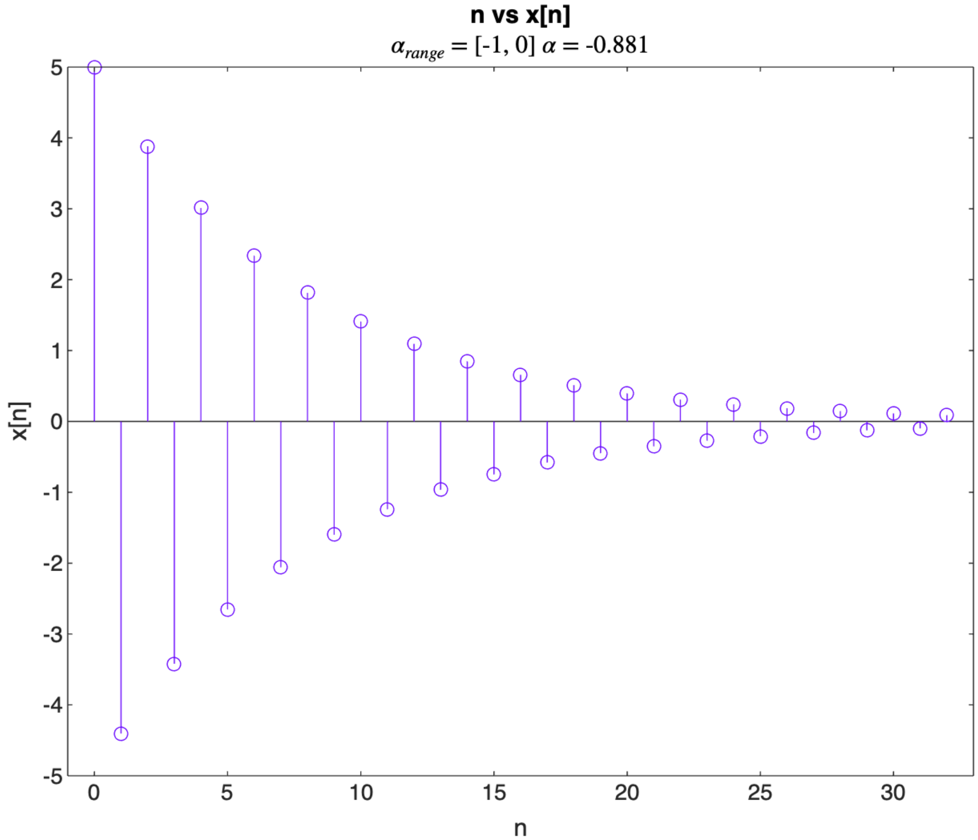
```
% Initialization of Required Variables
a = 5;
n = 0:1:32;

alphaRange = [0, 1; 1, 10; -1, 0; -10, -1];

% Alpha generation and plotting
for i = 1:size(alphaRange, 1)
    alpha = alphaRange(i, 1) + ((alphaRange(i, 2) - alphaRange(i, 1)) *
rand());

    figure;
    x_n = a * (alpha .^ n);
    stem(n, x_n, Color = COLOUR_VIOLET);
    xlabel('n');
    ylabel('x[n]');
    xlim([-1,33]);
    title('n vs x[n]');
    subtitle(['$\alpha_{range}$ = [' num2str(alphaRange(i, 1)) ', '
num2str(alphaRange(i, 2)) '] $\alpha$ = ' num2str(alpha)], 'Interpreter',
'latex');
end
```

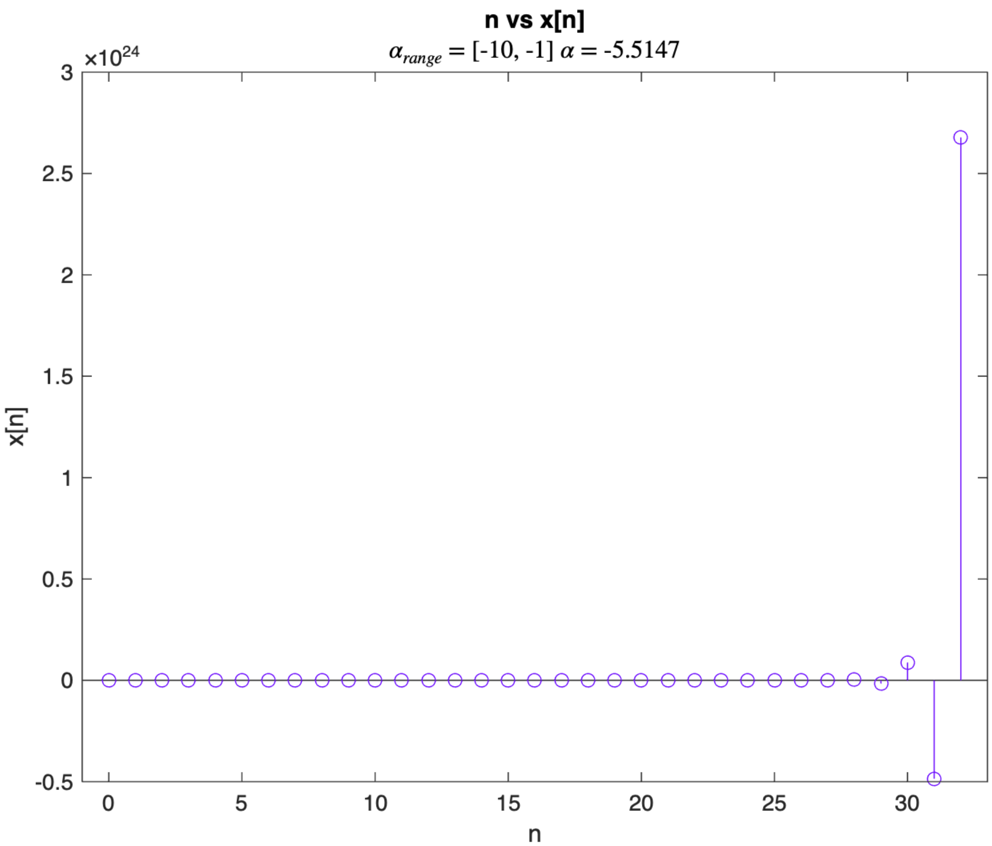


n vs x[n]
$\alpha_{range} = [0, 1]$ $\alpha = 0.6551$
x[n]
n



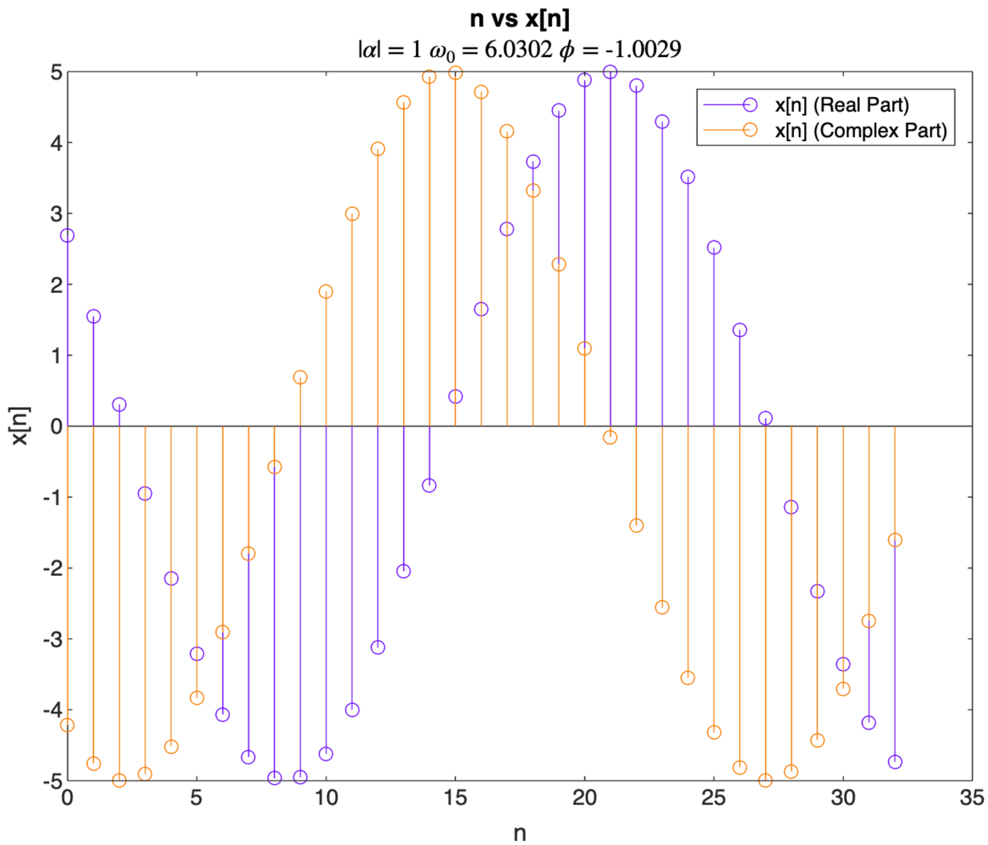
n vs x[n]
$\alpha_{range} = [1, 10]$ $\alpha = 2.4635$
$\times 10^{12}$
x[n]
n



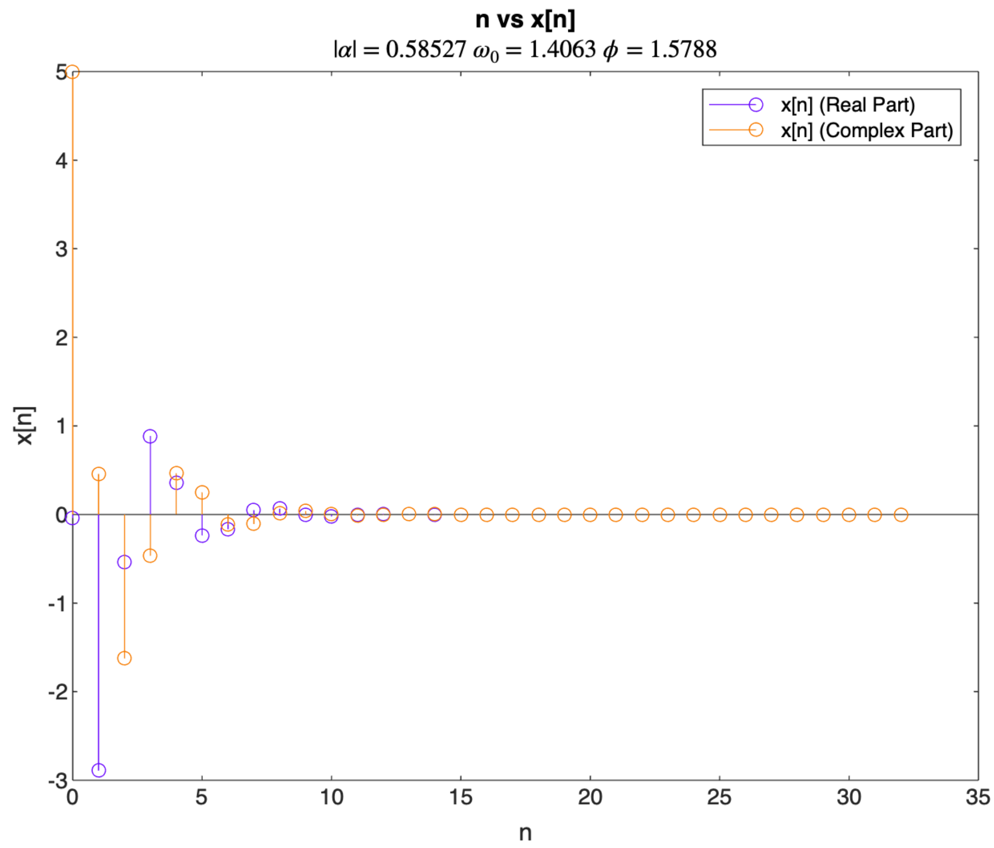


C. Complex Sinusoids and Exponentials

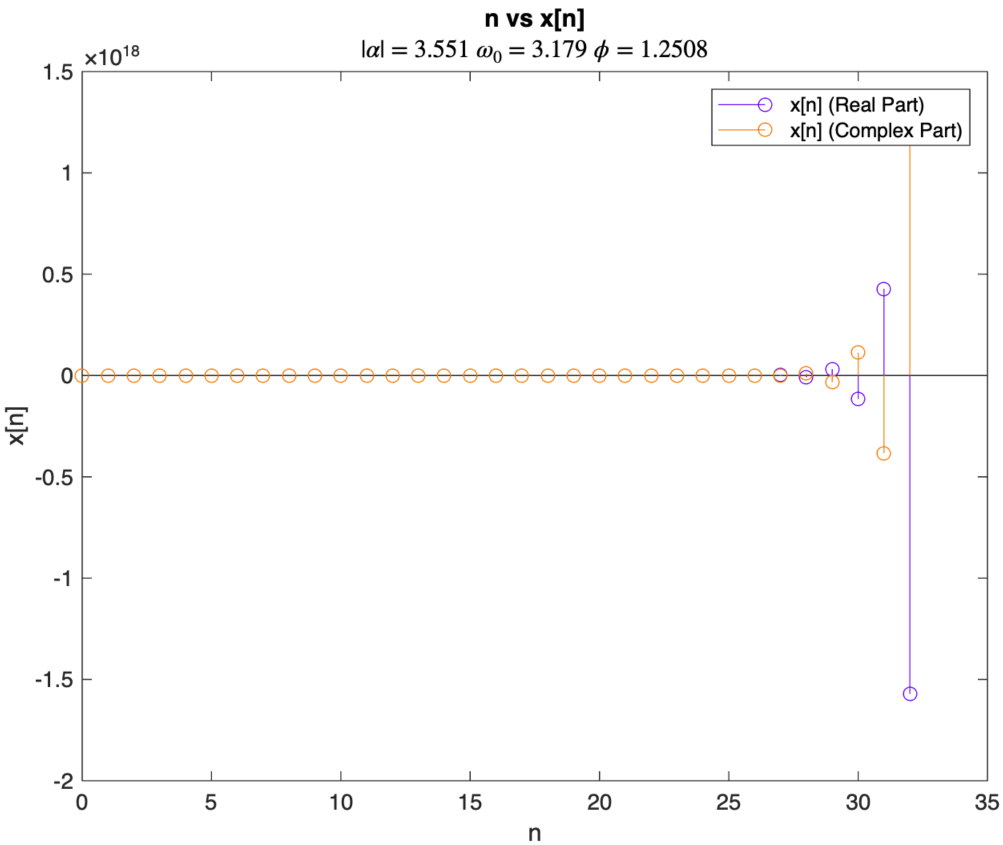
```matlab
% Function for sketching Real and Complex parts of exponential based on the
% given alpha value
function sketchDecomposedExponential(alpha)
    modA = 5;
    w0 = rand() * 2 * pi;
    phi = -pi + rand() * 2 * pi;
    n = 0:1:32;

    figure;
    x_nReal = modA * (alpha .^ n) .* cos(((w0 * n) + phi));
    x_nComplex = modA * (alpha .^ n) .* sin(((w0 * n) + phi));

    stem(n, x_nReal, Color = "#6E0FFE");
    hold on;
    stem(n, x_nComplex, Color = "#F87E05");
    xlabel('n');
    ylabel('x[n]');
    legend({'x[n] (Real Part)', 'x[n] (Complex Part)'});
    title('n vs x[n]');
    subtitle(['|$\alpha$| = ' num2str(alpha) ' $\omega_0$ = ' num2str(w0) '
$\phi$ = ' num2str(phi)], Interpreter = 'latex');

end

modAlpha = 1;

sketchDecomposedExponential(modAlpha);
```




```
modAlpha = 0 + (1-0) * rand();

sketchDecomposedExponential(modAlpha);
```




```
modAlpha = 1 + (10 - 0) * rand();

sketchDecomposedExponential(modAlpha);
```



n vs x[n]
$|\alpha| = 3.551$ $\omega_0 = 3.179$ $\phi = 1.2508$
×10^18
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x[n]
0
5
10
15
20
25
30
35
n
x[n] (Real Part)
x[n] (Complex Part)