CS6301 ML ASSIGNMENT

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BATCH - II

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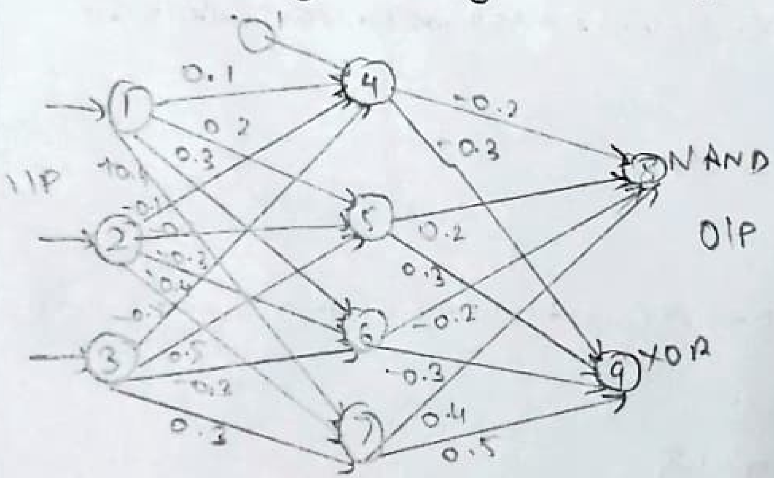
1. Multi Layer Perceptron (MLP)

	NAND	XOR
0 0 0	1	0
0 0 1	1	1
0 1 0	1	1
1 1 1	0	0
1 1 0	0	1

$\alpha = 0.8$

a)

x_1	x_2	x_3	Out NAND	Out XOR
1	1	0	0	1



	bias
4	0.2
5	0.5
6	-0.2
7	-0.5
8	0.1
9	0.2

S1: Forward propagation

- $I_j = O_j$ for I/P Layer

$O_1 = 1, \; O_2 = 1, \; O_3 = 0$

- $I_j = \sum_{i=1}^{m} x_i w_{ij} + x_0 + \theta_j$

$\Rightarrow x_4 \quad I_4$

$1 \times 0.1 + 1 \times (-0.1) + 0 \times (-0.5) + 1 \times 0.2$

$= 0.2$

$$O_4 = \frac{1}{1+e^{-I_4}} = \frac{1}{1+e^{-0.2}} = 0.55$$

$\Rightarrow x_5 \quad I_5 = 1\times 0.2 + 1\times(-0.2) + 0\times 0.5 + 1\times 0.5$

$= 0.5$

$O_5 = \dfrac{1}{1+e^{-0.5}} = 0.622$

$\Rightarrow x_6 \quad I_6 = 1\times 0.3 + 1\times(0.3) + 0\times(0.3) + 1\times(-0.2)$

$= -0.2$

$O_6 = \dfrac{1}{1+e^{0.2}} = 0.45$

$\Rightarrow x_7 \Rightarrow I_7 = 1\times 0.4 + 1\times(-0.4) + 0\times(0.3) + 1\times(-0.5)$

$= -0.5$

$O_7 = \dfrac{1}{1+e^{+0.5}} = \cancel{0.622}\ 0.378$

$x_8 \quad I_8 = 0.55\times(-0.2) + 0.622\times(0.2) + 0.45\times(-0.2) + 0.378\times(0.4) + 1\times 0.1$

$= 0.176$

$O_8 = \dfrac{1}{1+e^{-0.176}} = 0.544$

$x_9 \quad I_9 = 0.55\times(-0.3) + 0.622\times(0.3) + 0.45\times(-0.3) + 0.378\times 0.5 + 1\times 0.2$

$= 0.276$

$O_9 = \dfrac{1}{1+e^{-0.276}} = 0.568$

- $\text{Error}_8 = O_{des} - O_{est} = 0 - 0.544 = \overset{1-0.544}{|\cancel{-0.456}|} = \overset{0.544}{\cancel{0.456}}$

$\text{Error}_9 = O_{des} - O_{est} = 1 - 0.568 = 0.432$

S2: Backword propagation:

$\text{Err}_k = O_k(1-O_k)(O_{des}-O_k)$, op layer

$\text{Err}_j = O_j(1-O_j)\sum_k \text{Error}_k W_{jk}$, hid layer

O/P Layer

x_8 $Err_8 = (0.544)(0.456)(0.456) = 0.113$

x_9 $Err_9 = (0.568)(0.432)(0.432) = 0.106$

Hidden Layer

x_4 $Err_4 = (0.55)(0.45) \times (0.113 \times -0.2 + 0.106 \times -0.3)$

$= |-0.013| = 0.013$

x_5 $Err_5 = (0.622)(0.378) \times (0.113 \times 0.2 + 0.106 \times 0.3)$

$= 0.013$

x_6 $Err_6 = 0.45 \times 0.55 \times (0.113 \times -0.2 + 0.106 \times -0.3)$

$= |-0.013| = 0.013$

x_7 $Err_7 = 0.378 \times 0.622 \times (0.113 \times 0.4 + 0.106 \times 0.5)$

$= 0.023$

$\alpha = 0.8$

$\Delta w_{ij} = \alpha \times Error_j \times O_i$

$w_{ij} = w_{ij} + \Delta w_{ij}$

w_{ij}

$w_{14} = 0.1 + 0.8 \times 0.013 \times 1 = 0.11$

$w_{15} = 0.2 + 0.8 \times 0.013 \times 1 = 0.210$

$w_{16} = 0.3 + 0.8 \times 0.013 \times 1 = 0.310$

$w_{17} = 0.4 + 0.8 \times 0.023 \times 1 = 0.42$

$w_{24} = -0.1 + 0.8 \times 0.013 \times 1 = -0.09$

$w_{25} = -0.2 + 0.8 \times 0.013 \times 1 = -0.190$

$w_{26} = -0.3 + 0.8 \times 0.013 \times 1 = -0.290$

$w_{27} = -0.4 + 0.8 \times 0.023 \times 1 = -0.38$

w_{3j} = Original value

$w_{48} = -0.2 + 0.8 \times 0.113 \times 0.55 = -0.15$

$w_{49} = -0.3 + 0.8 \times 0.106 \times 0.55 = -0.253$

$w_{58} = 0.2 + 0.8 \times 0.113 \times 0.622 = 0.256$

$w_{59} = 0.3 + 0.8 \times 0.106 \times 0.622 = 0.353$

$w_{68} = -0.2 + 0.8 \times 0.113 \times \cancel{0.378}\ 0.45 = -0.16$

$w_{69} = -0.3 + 0.8 \times 0.106 \times \cancel{0.378}\ 0.45 = -0.262$

$w_{78} = 0.4 + 0.8 \times 0.113 \times 0.378 = 0.434$

$w_{79} = 0.5 + 0.8 \times 0.106 \times 0.378 = 0.532$

Update bias

$\theta_4 = 0.2 + 0.8 \times 0.013 = 0.21$

$\theta_5 = 0.5 + 0.8 \times 0.013 = 0.51$

$\theta_6 = -0.2 + 0.8 \times 0.013 = -0.19$

$\theta_7 = -0.5 + 0.8 \times 0.023 = -0.49$

$\theta_8 = 0.1 + 0.8 \times \cancel{0.456}\ 0.113 = 0.19$

$\theta_9 = 0.2 + 0.8 \times \cancel{0.432}\ 0.106 = 0.285$

Iteration 2:

x_4 $I_4 = 1 \times 0.11 + 1 \times -0.09 + 0 \times (-0.5) + 1 \times 0.21$

$= \cancel{1.23}\ \cancel{0.023}\ 0.23$

$O_4 = \cancel{\frac{1}{1+e^{-0.02}}} = \frac{1}{1+e^{-0.23}} = 0.557$

x_5 $I_5 = 1 \times 0.21 + 1 \times -0.19 + 0 \times 0.5 + 1 \times 0.51$

$= 0.53$

$O_5 = \frac{1}{1+e^{-0.53}} = 0.629$

x_6 $I_6 = 1 \times 0.31 + 1 \times -0.29 + 0 \times -0.3 + 1 \times (0.19)$

$= -0.17$

$O_6 = \frac{1}{1+e^{+0.17}} = 0.458$

$u_7 \Rightarrow I_7 = 1\times 0.42 + 1\times -0.38 + 0\times 0.3 + 1\times(-0.49)$

$= -0.45$

$O_7 = \frac{1}{1+e^{0.45}} = 0.389.$

$u_8 \quad I_8 = 0.557\times -0.15 + 0.629\times 0.256 + 0.458\times -0.16 +$

$0.389\times 0.434 + 1\times 0.19 = 0.363$

$O_8 = \frac{1}{1+e^{-0.363}} = 0.510$

$u_9 \quad I_9 = -0.253\times 0.557 + 0.629\times 0.353 + 0.458\times -0.262$

$0.389\times 0.532 + 1\times 0.285 = 0.453$

$O_9 = \frac{1}{1+e^{-0.453}} = 0.611$

- $Error_8 = 0 - 0.510 = |-0.510| = 0.510 < 0.544$

$Error_9 = 1 - 0.611 = 0.389 < 0.432$

$\Rightarrow$ We have reduced the error

from 0.544 to 0.510 for NAND

& 0.432 to 0.389 for XOR

in 2 Iterations

b)

x_1	x_2	x_3	Nand	XOR
0	0	0	1	0

Itr 1:
For hidden layer

$x_4 \Rightarrow I_4 = 0+0+0+0.2 = 0.2$ $\quad O_4 = 0.549$

$x_5 \Rightarrow I_5 = 0+0+0+0.5 = 0.5$ $\quad O_5 = 0.622$

$x_6 \Rightarrow I_6 = 0+0+0+(-0.2)) = -0.2$ $\quad O_6 = 0.450$

$x_7 \Rightarrow I_7 = 0+0+0-0.5 = -0.5$ $\quad O_7 = 0.377$

Output layer

$I_8 = 0.00754$ $\quad O_8 = 0.578$

$I_9 = 0.00754$ $\quad O_9 = 0.578$

error

$E_8 = 0.489$

$E_9 = -0.518$

Back propagation.

$\delta_8 = O_8(1-O_8)(E_8) = 0.120$

$\delta_9 = O_9(1-O_9)(E_9) = -0.129$

Hidden Layer

$\delta_4 = O_4(1-O_4)(w_{48}\delta_8 + w_{49}\delta_9)$

$= 3.6 \times 10^{-3} = 0.0036$

$\delta_5 = 0.0034$

$\delta_6 = O_6(1-O_6)(w_{68}\delta_8 + w_{69}\delta_9)$

$= 0.0036$

$\delta_7 = -0.0038$

Weight bias update

Weight's won't be updated as input is 0

$w_{48} = w_{ji} + \Delta w_j$

$w_{48} = 0.148$

$w_{49} = 0.294$

$w_{58} = 0.259$

$w_{59} = 0.364$

$w_{68} = 0.157$

$w_{69} = 0.346$

$w_{78} = 0.436$

$w_{79} = 0.462$

bias

$b_4 = 0.228$

$b_5 = -0.52$

$b_6 = 0.17$

$b_7 = 0.46$

$b_8 = 0.196$

$b_9 = 0.09$

Itr 2:

$I_4 = 0.505$ of

$$y_4 = \frac{1}{1+e^{-a_4}}$$

$a_4 = 0+0+0 + 0.228$

$y_4 = 0.505$

$y_5 = 0.627$

$y_6 = 0.542$

$y_7 = 0.613$

$a_8 = 0.589 + 0.196$

$y_8 = 0.686$

$a_9 = 0.015 + 0.09$

$y_9 = 0.526$

$\boxed{E_8 = 0.314 \quad E_9 = -0.526}$ prev Itr $\boxed{E_8 = 0.489 \quad E_9 = -0.518}$

$\therefore$ Errors reduced by back propagation

c) x_1 x_2 x_3 NAND XOR

1 1 1 0 0

$a_4 = (1)(0.1) - (0.1)(1) - (0.5)(1) + 0.2$
$= -0.3$

$a_5 = 1$ $O_4 = 0.425$

$a_6 = -0.5$ $O_5 = 0.731$

$a_7 = -0.2$ $O_6 = 0.377$

$O_7 = 0.450$

$a_8 = 0.2658$ $y_8 = 0.566$

$a_9 = 0.4037$ $y_9 = 0.599$

Error

$E_8 = -0.566$

$E_9 = -0.599$

Back propogation

$\delta_8 = -0.139$

$\delta_9 = -0.143$

$\delta_4 = O_4(1 - O_4)(\Sigma \delta_k w_{kj})$
$= 0.017$

$\delta_5 = -0.013$

$\delta_6 = 0.017$

$\delta_7 = -0.029$

Weight adjusting :

$w_{14} = 0.113$

$w_{24} = -0.087$

$w_{34} = -0.487$

$w_{15} = 0.09$

$w_{25} = -0.11$

$w_{35} = 0.49$

$w_{16} = 0.313$

$w_{26} = 0.013$

$w_{36} = 0.013$

$w_{17} = 0.023$

$w_{27} = 0.023$

$w_{37} = 0.023$

$w_{48} = -0.047$

$w_{49} = -0.048$

$w_{58} = -0.081$

$w_{59} = -0.083$

$w_{68} = -0.041$

$w_{69} = -0.043$

$w_{78} = -0.050$

$w_{79} = -0.051$

bias:

$b_j = a\, \delta_j + b_j$

$b_4 = 0.2136$

$b_5 = 0.4896$

$b_6 = -0.1864$

$b_7 = -0.5239$

$b_8 = -0.01$

$b_9 = 0.0856$

~~Iter~~ 2 :

$$y_4 = \frac{1}{1+e^{-a_4}}$$

$a_4 = 0.2526$

$a_5 = 0.4596$

$a_6 = -0.1474$

$a_7 = -0.4542$

$a_8 = -0.065$

$a_9 = 0.008$

$y_4 = 0.562$

$y_5 = 0.495$

$y_6 = 0.463$

$y_7 = 0.388$

$y_8 = 0.483$

$y_9 = 0.502$

Error R_2:

$E_8 = -0.489$ $E_8 = -0.566$

$E_9 = -0.502$ $E_9 = -0.599$

⇒ Error has been reduced

d)

x_1	x_2	x_3	NAND	XOR
0	0	1	1	1

$I_4 = -0.3$

$O_4 = 0.426$

$I_5 = 1$

$O_5 = 0.731$

$I_6 = -0.5$

$I_7 = -0.2$

$O_7 = 0.45$

$O_6 = 0.378$

$I_8 = 0.266$ $O_8 = 0.566$

$I_9 = 0.4034$ $O_9 = 0.599$

$E_8 = 0.434$ $E_9 = 0.401$

BACKWARD PROPAGATION :

$\delta_8 = O_8(1-O_8)(O_{des} - O_K) = 0.107$

$\delta_9 = 0.096$, $\delta_4 = O_4(1-O_4)\sum \delta_K W_{Kj}$

$\delta_4 = -0.012$, $\delta_5 = 0.010$, $\delta_6 = -0.012$, $\delta_7 = 0.022$

Updating weights & biases,

$W_{14} = 0.1$, $W_{15} = 0.2$, $W_{16} = 0.3$, $W_{17} = 0.4$

$W_{24} = -0.1$, $W_{25} = -0.1$, $W_{26} = -0.3$, $W_{27} = -0.4$

$W_{34} = -0.510$, $W_{35} = 0.508$, $W_{36} = 0.310$, $W_{37} = 0.318$

$W_{48} = -0.1636$, $W_{49} = -0.267$, $W_{58} = 0.262$, $W_{59} = 0.356$

$W_{68} = -0.168$, $W_{69} = -0.271$, $W_{78} = 0.439$, $W_{79} = 0.435$

$\theta_4 = 0.19$, $\theta_5 = 0.508$, $\theta_6 = -0.210$, $\theta_7 = -0.482$

$\theta_8 = 0.1 + (0.8 \times 0.107) = 0.186$

$\theta_9 = 0.2 + (0.8 \times 0.096) = 0.277$

Iteration 2

$I_5 = 1.016$	$O_5 = 0.734$
$I_4 = -0.32$	$O_4 = 0.421$
$I_6 = 0.1$	$O_6 = 0.525$
$I_7 = -0.164$	$O_7 = 0.459$
$I_8 = 0.4235$	$O_8 = 0.604$
$I_9 = 0.4833$	$O_9 = 0.619$
$Error_8 = \cancel{0.460}$ 0.396	$Error_9 = \cancel{0.435}$ 0.381

~~Back propagation~~

$\Rightarrow$ Error reduced by MLP successfully

e)

x_1	x_2	x_3	NAND	XOR
0	1	0	1	1

$I_4 = 0.1$	$O_4 = 0.525$
$I_5 = 0.3$	$O_5 = 0.622$
$I_6 = -0.5$	$O_6 = 0.378$
$I_7 = -0.9$	$O_7 = 0.289$
$I_8 = 0.1596$	$O_8 = 0.540$
$I_9 = 0.260$	$O_9 = 0.565$

$Error_8 = 0.460$ $Error_9 = 0.435$

BACK PROPAGATION:

$\delta_8 = O_8(1-O_8)(O_{des} - O_8) = 0.114$, $\delta_9 = 0.107$

$\delta_4 = -0.014$, $\delta_5 = 0.013$, $\delta_6 = 0.017$, $\delta_7 = 0.02$

Updating weights & biases

$W_{14} = 0.1$, $W_{15} = 0.2$, $W_{16} = 0.3$, $W_{17} = 0.4$

$W_{24} = -0.110$, $W_{25} = -0.192$, $W_{26} = -0.310$, $W_{27} = -0.382$

$W_{34} = -0.5$, $W_{35} = 0.5$, $W_{36} = -0.3$, $W_{37} = 0.3$

$W_{48} = -0.152$, $W_{49} = -0.255$, $W_{58} = 0.257$, $W_{59} = 0.353$

$W_{68} = -0.166$, $W_{69} = -0.268$, $W_{78} = 0.426$, $W_{79} = 0.425$

$\theta_4 = 0.189$, $\theta_5 = 0.510$, $\theta_6 = -0.19$, $\theta_7 = -0.484$

$\theta_8 = 0.191$, $\theta_9 = 0.286$

Iteration 2:

$I_4 = 0.079$ $O_4 = 0.520$

$I_5 = 0.318$ $O_5 = 0.579$

$I_6 = -0.5$ $O_6 = 0.3775$

$I_7 = -0.866$ $O_7 = 0.296$

$I_8 = 0.324$ $O_8 = 0.580$

$I_9 = 0.3824$ $O_9 = 0.594$

$Error_8 = 0.420$ $Error_9 = 0.406$

$\therefore$ Error reduced by MLP.

2. K Means Clustering (K = 2)

(0,0,0), (0,0,1), (0,1,0), (1,1,1), (1,1,0)

Choosing (0,0,0) & (1,1,1) as initial centroids.

Itr 1:

→ (0,0,1)

$D_1 = \sqrt{0^2+0^2+1^2} = 1$

$D_2 = \sqrt{(-1)^2+(-1)^2+0^2} = \sqrt{2}$ ⇒ Cluster 1

→ (0,1,0)

$D_1 = \sqrt{0^2+1^2+0^2} = 1$ ⇒ Cluster 1

$D_2 = \sqrt{(-1)^2+0^2+(-1)^2} = \sqrt{2}$

→ (1,1,0)

$D_1 = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$D_2 = \sqrt{0^2+0^2+(-1)^2} = 1$ ⇒ Cluster 2

Cluster 1 (0,0,0) (0,0,1) (0,1,0)

Cluster 2 (1,1,1) (1,1,0)

Updating Centroid 1 = $\left(\frac{0+0+0}{3}, \frac{0+0+1}{3}, \frac{0+1+0}{3}\right)$

$C_1 = (0, 0.33, 0.33)$, $C_2 = (1, 1, 0.5)$

Itr 2:

→ (0,0,0)

$D_1 = \sqrt{0^2+(-0.33)^2+(-0.33)^2} = 0.467$

$D_2 = \sqrt{(-1)^2+(-1)^2+(0.5)^2} = 1.5$ ⇒ Cluster 1

→ (0,0,1)

$D_1 = \sqrt{0^2+(-0.33)^2+(0.67)^2} = 0.747$ ⇒ Cluster 1

$D_2 = \sqrt{(-1)^2 + (-1)^2 + (0.5)^2} = 1.5$

For $(0,1,0)$,

$D_1 = \sqrt{0^2 + (0.67)^2 + (-0.33)^2} = 0.747$

$D_2 = \sqrt{(-1)^2 + 0^2 + (-0.5)^2} = 1.118$ $\Rightarrow$ Cluster 1

For $(1,1,1)$,

$D_1 = \sqrt{1^2 + (0.67)^2 + (0.67)^2} = 1.378$

$D_2 = \sqrt{0^2 + 0^2 + (0.5)^2} = 0.5$ $\Rightarrow$ Cluster 2

For $(1,1,0)$

$D_1 = \sqrt{1^2 + (0.67)^2 + (-0.33)^2} = 1.248$ $\Rightarrow$ Cluster 2

$D_2 = \sqrt{0^2 + 0^2 + (-0.5)^2} = 0.5$

Cluster 1 $(0,0,0)$ $(0,0,1)$ $(0,1,0)$

Cluster 2 $(1,1,1)$ $(1,1,0)$

clusters in both iterations are same

$\Rightarrow$ we have 2 clusters

of 3 datapoints and 2

datapoints.

3. SOFM, $\alpha = 0.8$, 3 inp 2 2 o/p

Training samples = (0,0,0), (0,0,1), (0,1,0), (1,1,1), (1,1,0)

Initial weight matrix

$$= \begin{bmatrix} -0.1 & 0.2 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

Iteration 1: (0,0,0)

$d^2(\text{unit } 1) = (0.1)^2 + (-0.2)^2 + (0.3)^2 = 0.14$

$d^2(\text{unit } 2) = (-0.5)^2 + (-0.2)^2 + (0.3)^2 = 0.38$

$\Rightarrow$ Unit 1 wins

Unit 1 = $[-0.1 \;\; 0.2 \;\; 0.3] + 0.8\,[0.1 \;\; 0.2 \;\; -0.3]$

$= [-0.02 \;\; 0.04 \;\; 0.06]$

Iteration 2: (0,0,1)

$d^2(\text{Unit } 1) = (0.02)^2 + (-0.04)^2 + (0.94)^2 = 0$

$d^2(\text{Unit } 2) = (-0.5)^2 + (-0.2)^2 + (0.7)^2 = 0.78$

$\Rightarrow$ Unit 2 wins

Unit 2 = $[0.5 \;\; 0.2 \;\; 0.3] + 0.8\,[-0.5 \;\; -0.2 \;\; 0.7]$

$= [0.1 \;\; 0.04 \;\; 0.86]$

Iteration 3: (0,1,0)

$d^2(\text{Unit } 1) = (0.02)^2 + (0.96)^2 + (-0.06)^2 = 0.926$

$d^2(\text{Unit } 2) = (-0.1)^2 + (0.96)^2 + (-0.86)^2 = 1.671$

$\Rightarrow$ Unit 1 wins

Unit 1 = [-0.02 0.04 0.06] + 0.8 [0.02 0.96 -0.06]

= [-0.004 0.808 0.012]

Iteration 4 (1,1,1)

d^2 (Unit 1) = $(1.004)^2 + (0.192)^2 + (0.988)^2 = 2.021$

d^2 (Unit 2) = $(0.9)^2 + (0.96)^2 + (0.14)^2 = 1.751$

⇒ Unit 2 wins

Unit 2 = [0.1 0.04 0.86] + 0.8 [0.9 0.96 0.14]

= [0.82 0.808 0.972]

Iteration 5 (1,1,0)

d^2 (Unit 1) = $(1.004)^2 + (0.192)^2 + (-0.012)^2 = 1.045$

d^2 (Unit 2) = $(0.18)^2 + (0.192)^2 + (-0.972)^2 = 1.014$

⇒ Unit 2 wins

Unit 2 = [0.82 0.808 0.972] + 0.8 [0.18 0.192 -0.972]

= [0.964 0.962 0.194]

⇒ Final weight matrix = $\begin{bmatrix} -0.004 & 0.808 & 0.012 \\ 0.964 & 0.962 & 0.194 \end{bmatrix}$

Mapping input to output units

(0,0,0) → Unit 1

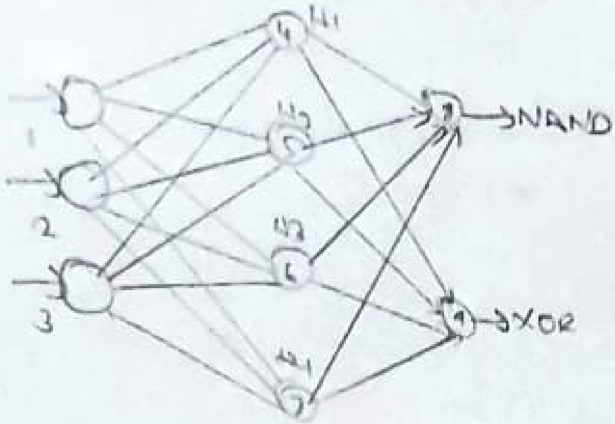
(0,0,1) → Unit 1

(0,1,0) → Unit 1

(1,1,1) → Unit 2

(1,1,0) → Unit 2

4.



RBFNN

$c_1 (0,0,0)$
$c_2 (0,0,1)$
$c_3 (0,1,0)$ $r^2=3$
$c_4 (1,1,1)$

$w_{48} = -0.1$ $w_{49} = 0.2$
$w_{58} = 0.3$ $w_{59} = -0.2$
$w_{68} = 0.1$ $w_{69} = -0.2$
$w_{78} = -0.5$ $w_{79} = 0.5$

IP 1:
For $x = (0,0,0)$

c_1 $d^2 = 0$

$\therefore H_1(x) = e^{-d^2/r^2} = e^{-0/3} = 1$

c_2 $d^2 = 1$

$H_2(x) = e^{-1/3} = 0.716$

$c_3 =$

$d^2 = (0-0)^2 + (0-1)^2 + (0-0)^2 = 1$

$H_3(x) = e^{-1/3} = 0.716$

c_4 $d^2 = 3$

$H_4(x) = e^{-3/3} = e^{-1} = 0.368$

For NAND

$\Sigma w_i H_i(x) = (-0.1)\times 1 + (0.3)\times 0.716 + (0.1)\times 0.716$
$+ (-0.5)\times 0.368$
$= 0.0024$

for XOR

$\sum w_i H_i(x) = 0.2(1) + (0.2)(0.716) + (-0.2 \times 0.716)$
$+ (0.5)(0.368) = 0.0976$

IP2:

for (0,0,1)

C_1

$d^2 = (0-0)^2 + (0-0)^2 + 1^2 = 1$

$H_1(x) = e^{-d^2/r^2} = e^{-1/3} = 0.716$

C_2

$d^2 = 0$

$H_2(x) = 1$

C_3

$d^2 = (0-1)^2 + (1-0)^2 = 2$

$H_3(x) = e^{-2/3} = 0.513$

C_4 $d^2 = 2$

$H_4(x) = e^{-2/3} = 0.513$

for NAND

$\sum w_i H_i(x) = (0.1) \times 0.716 + (0.3 \times 1) + (0.1 \times 0.513)$
$+ (-0.5) \times 0.513 = 0.232$

for XOR

$\sum w_i H_i(x) = (0.2 \times 0.716) + (-0.2 \times 1) + (-0.2 \times 0.513)$
$+ (0.5 \times 0.513) = 0.0971$

IP 3:

For (0,1,0)

C_1 $d^2 = 1$

$H_1(x) = e^{-1/3} = 0.716$

C_2 $d^2 = 2$

$H_2(x) = e^{-2/3} = 0.513$

C_3 $d^2 = 0$

$H_3(x) = 1$

C_4 $d^2 = 2$

$H_4(x) = e^{-2/3} = 0.513$

For NAND

$\Sigma x_i H_i(x) = (-0.1 \times 0.716) + (0.3 \times 0.513) + (0.1 \times 1) + (-0.5 \times 0.513)$
$= -0.0742$

For XOR

$\Sigma x_i H_i(x) = (0.2 \times 0.716) + (-0.2 \times 0.513) + (-0.2 \times 1)$
$+ (0.5 \times 0.513) = 0.0971$

IP 4:

For (1,1,1)

C_1 $d^2 = (1-0)^2 + (1-0)^2 + (1-0)^2 = 3$

$H_1(x) = e^{-3/3} = e^{-1} = 0.368$

C_2 $d^2 = (1-0)^2 + (1-0)^2 + (1-1)^2 = 2$

$H_2(x) = e^{-2/3} = 0.513$

C_3 $d^2 = (1-0)^2 + (1-1)^2 + (1-0)^2 = 2$

$H_3(x) = e^{-2/3} = 0.513$

C_4 $d^2 = 0$, $H_4(x) = 1$

For NAND

$\sum w_i H_i(x) = (-0.1) \times 0.368 + (0.3 \times 0.513) + (0.1 \times 0.513) + (-0.5) \times 1)$

$= \cancel{0.0} \; 0.3361$

For XOR

$\sum w_i H_i(x) = 0.2 \times 0.368 + (-0.2 \times 0.513) + (-0.2 \times 0.513) + (0.5 \times 1)$

$= 0.3684$,,

INPUT			$H_1(x)$	$H_2(x)$	$H_3(x)$	$H_4(x)$	$\sum_{j=1}^{m} w_j H_j(x)$		OUTPUT	
0	0	0	1.0	0.717	0.717	0.368	0.003	0.097	0	0
0	0	1	0.717	1.0	0.513	0.513	0.027	0.097	0	0
0	1	0	0.717	0.513	1.0	0.513	-0.074	0.097	0	0
1	1	1	0.368	0.513	0.513	1.0	0.332	0.368	1	1

5 Perceptron - XOR Gate

x EPOCH 1

x_1	x_2	x_3	Error	w_1	w_2	w_3	Oest	Odes
0	0	0	0	-0.1	-0.2	-0.1	0	0
0	0	1	1	-0.1	-0.2	0.7	0	1
0	1	0	1	-0.1	0.6	0.7	0	1
1	1	1	0	-0.1	0.6	0.7	1	1
1	1	0	-1	-0.9	-0.2	0.7	1	0

EPOCH 2

x_1	x_2	x_3	Error	w_1	w_2	w_3	Oest	Odes
0	0	0	0	-0.9	-0.2	0.7	0	0
0	0	1	0	-0.9	-0.2	0.7	1	1
0	1	0	1	-0.9	0.6	0.7	0	1
1	1	1	0	-0.9	0.6	0.7	1	1
1	1	0	0	-0.9	0.6	0.7	0	0

EPOCH 3

x_1	x_2	x_3	Error	w_1	w_2	w_3	Oest	Odes
0	0	0	0	-0.9	0.6	0.7	0	0
0	0	1	0	-0.9	0.6	0.7	1	1
0	1	0	0	-0.9	0.6	0.7	1	1
1	1	1	0	-0.9	0.6	0.7	1	1
1	1	0	0	-0.9	0.6	0.7	0	0

As it converges (XOR) because only
5 inputs given

Final weights: $w_1 = -0.9$, $w_2 = 0.6$, $w_3 = 0.7$