LDA Steps:

1) $m_i = \begin{bmatrix} \mu_{w_i 1} \\ \vdots \\ \mu_{w_i n} \end{bmatrix}$ $\quad$ $i = 1, 2, \ldots,$ no of classes
$n = $ # features

2) $\Sigma_i = \frac{1}{N_i - 1} \Sigma (x - m_i)(x - m_i)^T$ $\quad$ N_i = # samples of i^{th} class,
m = overall mean

$$S_W = \sum_{i=1}^{c} (N_i - 1) \Sigma_i$$

$$S_B = \sum_{i=1}^{c} N_i (m_i - m)(m_i - m)^T$$

3) Calculate eigenvalue & eigenvector for $S_W^{-1} S_B$

4) Normalize the highest value eigenvector and project it $y = w^T x$.

ML-Endsem-21S6:

PART B - 15) $C_1 = \{(1,1), (2,1), (1,2), (2,2)\}$
$C_2 = \{(4,4), (4,5), (5,4), (5,5)\}$

$$\mu_1 = \begin{bmatrix} 6/4 \\ 6/4 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}, \quad \mu_2 = \begin{bmatrix} 18/4 \\ 18/4 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 4.5 \end{bmatrix}$$

Global mean, $\mu = \begin{bmatrix} 24/8 \\ 24/8 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$$\Sigma_1 = \frac{1}{4-1} \left(\begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} \begin{bmatrix} -0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \right)$$

$$\Rightarrow \Sigma_1 = \frac{1}{3}\left(\begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix} + \begin{bmatrix} 0.25 & -0.25 \\ -0.25 & 0.25 \end{bmatrix} + \begin{bmatrix} 0.25 & -0.25 \\ -0.25 & 0.25 \end{bmatrix} + \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix} \right)$$

$$\Rightarrow \Sigma_1 = \frac{1}{3}\left(\begin{bmatrix} 1.00 & 0 \\ 0 & 1.00 \end{bmatrix} \right)$$

Now, $$\Sigma_2 = \frac{1}{4-1}\left(\begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} \begin{bmatrix} -0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \right)$$

$$= \frac{1}{3}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$\therefore \quad S_w = \sum_i (N_i - 1)\Sigma_i = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{c} N_i (\mu_i - \mu)(\mu_i - \mu)^T$$

$$= 4\left(\begin{bmatrix} -1.5 \\ -1.5 \end{bmatrix} \begin{bmatrix} -1.5 & -1.5 \end{bmatrix} \right) + 4\left(\begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix} \begin{bmatrix} 1.5 & 1.5 \end{bmatrix} \right)$$

$$= 4\left(\begin{bmatrix} 2.25 & 2.25 \\ 2.25 & 2.25 \end{bmatrix} + \begin{bmatrix} 2.25 & 2.25 \\ 2.25 & 2.25 \end{bmatrix} \right)$$

$$S_B = \begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix}$$

Now, $S_w^{-1} S_B = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix}$

$$= \frac{2}{4}\begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix}$$

Eigen vectors and eigenvalues:

$$\begin{vmatrix} 9-\lambda & 9 \\ 9 & 9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (9-\lambda)^2 - 9 = 0$$

$$\Rightarrow 81 + \lambda^2 - 18\lambda - 81 = 0$$

$$\Rightarrow \lambda = 18 \text{ (or) } \lambda = 0 .$$

Highest eigenvalue: 18,

$\Rightarrow$ Eigen vector: $\begin{bmatrix} -9 & 9 \\ 9 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\left.\begin{aligned} &\Rightarrow -9x + 9y = 0 \\ &\Rightarrow 9x - 9y = 0 \end{aligned}\right\} \Rightarrow 9x = 9y \quad \therefore x = y$$

$\Rightarrow$ Eigen vec. (normalized) $= \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$

Projection: $y = w^T * \underbrace{(x-\mu)}_{\text{global mean}} = \begin{bmatrix} 0.707 & 0.707 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{cases} C_1 = \{-2.828, -2.121, -2.121, -1.414\}, \\ C_2 = \{1.414, 2.121, 2.121, 2.828\}. \end{cases}$$

$\hookrightarrow$ New datapoints wrt global mean as origin and in the direction of $\begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$ //

ML- Endsem - 2136

PART B - 16) $C_1 = \{(1,1), (2,1), (1,2), (2,2)\}$

$C_2 = \{(4,4), (4,5), (5,4), (5,5)\}$

$$\mu_{global} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Scatter matrix $= \sum_i (x_i - \mu)(x_i - \mu)^T$

$$= \begin{bmatrix} -2 \\ -2 \end{bmatrix} \begin{bmatrix} -2 & -2 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} \begin{bmatrix} -1 & -2 \end{bmatrix}$$

$$+ \begin{bmatrix} -2 \\ -1 \end{bmatrix} \begin{bmatrix} -2 & -1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$S = \begin{bmatrix} 20 & 18 \\ 18 & 20 \end{bmatrix}$$

Finding eigenvalues, $\begin{vmatrix} 20-\lambda & 18 \\ 18 & 20-\lambda \end{vmatrix} = 0$

$$(20-\lambda)^2 = 18^2$$

$\therefore \lambda = -2$ (or) $+38$

Eigenvec for $\lambda = 38$

$$\begin{bmatrix} -18 & 18 \\ 18 & -18 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} \Rightarrow -18x + 18y = 0 \\ \Rightarrow 18x - 18y = 0 \end{array}\right\} \Rightarrow 18x = 18y \quad \therefore x = y.$$

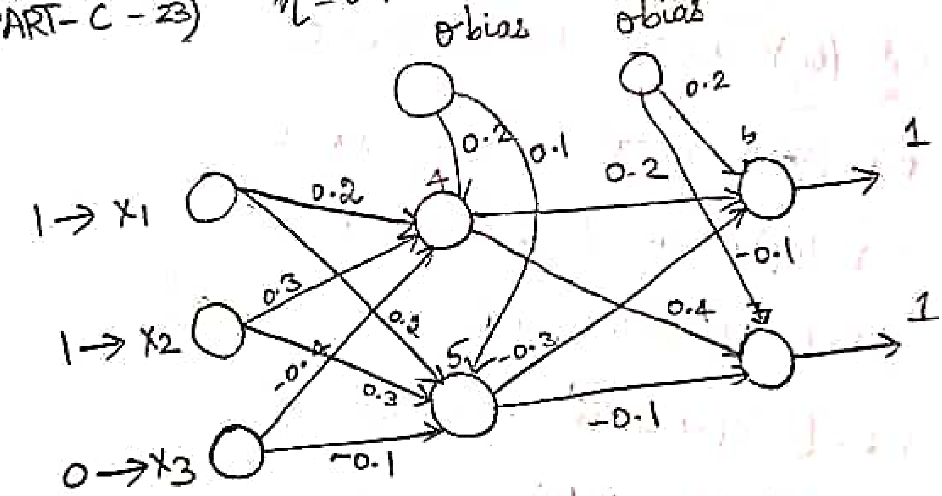
$$\text{Normalized eig vec} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

$$\therefore \text{Projection} = \begin{bmatrix} 0.707 & 0.707 \end{bmatrix}^{*} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left.\begin{array}{l} \Rightarrow C_1 = \{-2.828, -2.121, -2.121, -1.414\}, \\ C_2 = \{1.414, 2.121, 2.121, 2.828\} // \end{array}\right\} \text{In the dir. of } \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

ML-Endsem-21S6

PART-C-23) $\eta = 0.1$



Use: sigmoid activation.

Itr1: Forward prop:

$h_4 = 1 \times 0.2 + 1 \times 0.3 + 0 \times (-0.4) + 0.2 = 0.7$

$a_4 = g(h_4) = 0.66818$

$h_5 = 0.1 + 0.2 \times 1 + 0.3 \times 1 + (-0.1) \times 0 = 0.6$

$a_5 = g(h_5) = 0.64565$

$$h_6 = 0.2 + 0.2\,(0.66818) + (-0.3)\,(0.64565) = 0.139941$$

$$a_6 = g(h_6) = 0.53492$$

$$h_7 = -0.1 + 0.4\,(0.66818) + (-0.1)\,(0.64565) = 0.102707$$

$$a_7 = g(h_7) = 0.52565$$

Back prop:

$$\delta_o(6) = (0.53492 - 1)\,(0.53492)\,(1 - 0.53492) = -0.11570$$

$$\delta_o(7) = (0.52565 - 1)\,(0.52565)\,(1 - 0.52565) = -0.11827$$

$$\delta_h(4) = (0.66818)\,(1 - 0.66818)\begin{pmatrix}(0.2)(-0.11570) + \\ (0.4)(-0.11827)\end{pmatrix} = -0.015619$$

$$\delta_h(5) = (0.64565)\,(1 - 0.64565)\begin{pmatrix}(-0.3)(-0.11570) + \\ (-0.1)(-0.11827)\end{pmatrix} = +0.01064$$

Update:

$$w_{46} = 0.2 - (0.1)(-0.11570)(0.66818) = 0.20773$$

$$w_{47} = 0.4 - (0.1)(-0.11827)(0.66818) = 0.40790$$

$$w_{56} = -0.3 - (0.1)(-0.11570)(0.64565) = -0.29253$$

$$w_{57} = -0.1 - (0.1)(-0.11827)(0.64565) = -0.09236$$

$$w_{14} = 0.2 - (0.1)(-0.015619)(1) = 0.20156$$

$$w_{15} = 0.2 - (0.1)(0.01064)(1) = 0.19893$$

$$w_{24} = 0.3 - (0.1)(-0.01561 9)(1) = 0.30156$$

$$w_{25} = 0.3 - (0.1)(0.01064)(1) = 0.29893$$

$$w_{34} = -0.4 - (0.1)(-0.015619)(0) = -0.4$$

$$w_{35} = -0.1 - (0.1)(0.01064)(0) = -0.1 \;//$$

ML-EndSem-21S6

PART B - 14)

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$\sigma = \sqrt{\frac{1}{p}\sum_{i=1}^{p}(\mu-\mu_i)^2}$

$\Rightarrow \sigma = 1$

$\therefore 2\sigma^2 = 2$

Given: 4 centers: (0,0) (0,1) (1,0) (1,1) $\quad e^{\frac{-r^2}{2\sigma^2}}$

		(0,0)	(0,1)	(1,0)	(1,1)				
x_1	x_2	r_1^2	r_2^2	r_3^2	r_4^2	ϕ_1	ϕ_2	ϕ_3	ϕ_4
0	0	0	1	1	2	1	0.606	0.606	0.368
0	1	1	0	2	1	0.606	1	0.368	0.606
1	0	1	2	0	1	0.606	0.368	1	0.606
1	1	2	1	1	0	0.368	0.606	0.606	1

Given: weights; 1, -1, -1 and 1

x_1	x_2	Σnet	activation
0	0	0.156	0
0	1	-0.156	1
1	0	-0.156	1
1	1	0.156	0

(Let activation fn be

$$f(x) = \begin{cases} 1, & x < 0 \\ 0, & x \geq 0 \end{cases}$$

ML-EndSem-21S6

PART-B - 11) Find-S: Initial: $h_0 = \{\phi, \phi, \phi, \phi, \phi\}$

(i) +ve class: $h_1 = \{$<pg, yes, good, Male, Married>$\}$

(ii) +ve class: $h_2 = \{$<pg, yes, good, ?, ?>$\}$

(iii) -ve class: $h_3 = \{$<pg, yes, good, ?, ?>$\}$

(iv) +ve class: $h_4 = \{$<pg, yes, good, ?, ?>$\}$

ML- EndSem-22S6

PART-B : 11) Candidate Elimination:

Initial: $S_0 = \{\langle \phi, \phi, \phi, \phi, \phi, \phi \rangle\}$

$G_0 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

(i) +ve class:

$G_1 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

$S_1 = \{\langle \text{Above 90, excellent, good, yes, fast, yes} \rangle\}$

(ii) +ve class:

$G_2 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

$S_2 = \{\langle \text{Above 90}, ?, \text{good, yes, fast, yes} \rangle\}$

(iii) -ve class:

$$G_3 = \left\{ \begin{array}{l} \langle \text{Above 90}, ?, ?, ?, ?, ? \rangle, \langle ?, ?, \text{good}, ?, ?, ? \rangle, \\ \langle ?, ?, ?, \text{yes}, ?, ? \rangle, \langle ?, ?, ?, ?, \text{fast}, ? \rangle, \\ \langle ?, ?, ?, ?, ?, \text{yes} \rangle, \langle ?, \text{excellent}, ?, ?, ?, ? \rangle \end{array} \right\}$$

$S_3 = \{\langle \text{Above 90}, ?, \text{good, yes, fast, yes} \rangle\}$

(iv) -ve class:

$$G_4 = \left\{ \begin{array}{l} \langle ?, \text{excellent}, ?, ?, ?, ? \rangle, \langle \text{Above 90}, ?, ?, \text{no}, ?, ? \rangle, \\ \langle ?, ?, ?, ?, ?, \text{yes} \rangle, \langle \text{Above 90}, ?, \text{good}, ?, ?, ? \rangle, \\ \langle \text{Above 90}, ?, ?, \text{yes}, ?, ? \rangle, \langle ?, ?, \text{good}, \text{no}, ?, ? \rangle \\ \langle ?, ?, ?, ?, \text{fast}, ? \rangle, \end{array} \right\}$$

$S_4 = \{\langle \text{Above 90}, ?, \text{good, yes, fast, yes} \rangle\}$

(v) -ve class:

$G_5 = G_4$

$S_5 = S_4$.

ML- EndSem-22S6

PART-B - 21) $y = mx + c$,

$$m = \frac{SS_{xy}}{SS_{xx}}, \quad SS_{xy} = \Sigma xy - \frac{\Sigma x \, \Sigma y}{n}$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

Normalize x: 11 14 17 20 23

y: 11 19 19 14 ?

x	y	x^2	xy
11	11	121	121
14	19	196	266
17	19	289	323
20	14	400	280

$\Sigma x = 62$ $\Sigma y = 63$ $\Sigma x^2 = 1006$ $\Sigma xy = 990$

$$\therefore \quad m = \frac{SS_{xy}}{SS_{xx}} = \frac{990 - \frac{(62)(63)}{4}}{1006 - \frac{(62)^2}{4}} = \frac{13.5}{45} = 0.3$$

$$\Rightarrow c = \bar{y} - m\bar{x}$$
$$= 15.75 - (0.3)(15.5) = 11.1$$

$$\therefore y = 0.3x + 11.1$$
$$\Rightarrow y(23) = 0.3(23) + 11.1 = 18$$

ML-EndSem-22SS

PART-B - 2(ii)

(x-2000)	y	xy	x^2
5	12	60	25
6	19	114	36
7	29	203	49
8	37	296	64
9	45	405	81

$\Sigma x = 35$ $\Sigma y = 142$ $\Sigma xy = 1078$ $\Sigma x^2 = 255$

$\bar{x} = 7$ $\bar{y} = 28.4$

$$m = \frac{SS_{xy}}{SS_{xx}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{1078 - \frac{(35)(142)}{5}}{255 - \frac{(35)^2}{5}}$$

$$\therefore m = \frac{84}{10} = 8.4$$

$$\therefore c = \bar{y} - m\bar{x} = 28.4 - 8.4(7) = -30.4$$

$$\therefore y = 8.4(x-2000) + (-30.4)$$

For 2012: $y = 8.4 * 12 - 30.4 = \underline{\underline{70.4}}$

ML-EndSem-22S5

PART-B: 11)(ii) Candidate Elimination:

Initial: $S_0 = \{<\phi, \phi, \phi>\}$

$G_0 = \{<?, ?, ?>\}$

(i) -ve class:

$S_0 = \{<\phi, \phi, \phi>\}$

$G_0 = \{$<Small, ?, ?>, <?, Blue, ?>, <?, ?, Triangle>$\}$

(ii) -ve class:

$S_1 = \{<\phi, \phi, \phi>\}$

$G_1 = \{$<Small, Blue, ?>, <Small, ?, Circle>, <?, Blue, ?>, <Big, ?, Tri>, <?, Blue, Tri>$\}$

(iii) +ve class:

$S_2 = \{$<Small, Red, Circle>$\}$

$G_2 = \{<Small, ?, Circle>\}$

(iv) -ve class:

$S_3 = \{<Small, Red, Circle>\}$

$G_3 = \{<Small, ?, Circle>\}$

(v) +ve class:

$S_4 = \{<Small, ?, Circle>\}$

$G_4 = \{<Small, ?, Circle>\}$

ML- EndSem- 21 S5

PART-B: 13) $P(Skips) = \frac{4}{6} = \frac{2}{3}$, $P(Reads) = \frac{2}{6} = \frac{1}{3}$

For Skips:

$P(known \mid Skips) = \frac{3+1}{4+2} = \frac{2}{3}$

$P(New \mid Skips) = \frac{1+1}{4+2} = \frac{1}{3}$

$P(short \mid Skips) = \frac{0+1}{4+2} = \frac{1}{6}$

$P(Work \mid Skips) = \frac{1+1}{4+2} = \frac{1}{3}$

$P(Skips \mid X) = \frac{P(X \mid Skips) * P(Skips)}{P(X)}$

$= \frac{\frac{2}{3} * \frac{1}{3} * \frac{1}{6} * \frac{1}{3} * \frac{2}{3}}{P(X)}$

$= \frac{0.00823}{P(X)}$

For Reads:

$P(known \mid Reads) = \frac{1+1}{2+2} = \frac{1}{2}$

$P(New \mid Reads) = \frac{2+1}{2+2} = \frac{3}{4}$

$P(short \mid Reads) = \frac{2+1}{2+2} = \frac{3}{4}$

$P(Work \mid Reads) = \frac{1+1}{2+2} = \frac{1}{2}$

$\frac{P(X \mid Reads) * P(Reads)}{P(X)}$

$= \frac{\frac{1}{2} * \frac{3}{4} * \frac{3}{4} * \frac{1}{2} * \frac{1}{3}}{P(X)}$

$= \frac{0.046875}{P(X)}$ ✓

$\boxed{\text{READS}}$

Sridhar Book Example - 6.3

(i) Using ID3

$$H(\text{Job offer}) = -\frac{7}{10}\log\frac{7}{10} - \frac{3}{10}\log\frac{3}{10} = 0.88129$$

$$H(\text{Joboffer} \mid \text{CGPA}) = (0.4)(0.811278) = 0.32451$$

CGPA \ Job offer →	Yes	No	Entropy
<8	0	2	0
≥8	4	0	0
≥9	3	1	0.811278

$$\text{Gain}_{CGPA} = 0.88129 - 0.32451 = 0.556778$$

$$H(\text{Joboffer} \mid \text{Int}) = (0.6)(0.65002) + (0.4)(1) = 0.790013$$

Int \ JO →	Yes	No	Entropy
Yes	5	1	0.650022
No	2	2	1

$$\text{Gain}_{Int} = 0.88129 - 0.790013 = 0.091276$$

$$H(JO \mid PK) = (0.5)(0.72198) + (0.3)(0.91829) = 0.63647$$

PK \ JO →	Yes	No	Entropy
Good	2	0	0
VGood	4	1	0.721928
Avg	1	2	0.91829

$Gain_{PK} = 0.88129 - 0.63647 = 0.24481$

$H(JO \mid CS) = (0.5)(0.721928) = 0.360964$

CS\JO → ↓	Yes	No	Entropy
Poor	0	2	0
Mod.	3	0	0
Good	4	1	0.721928

$Gain_{CS} = 0.88129 - 0.360964 = 0.520326$

ROOT NODE :: CGPA

New Data:

~~CGPA~~ Int.	PK	CS	JO
Yes	VG	G	Y
No	Avg	P	N
Yes	G	M	Y
No	VG	G	Y

$H(Job\ offer) = 0.811278$

$H(JO \mid Int) = (0.5)1 = 0.5$

Int\JO → ↓	Yes	No	Entropy
Yes	2	0	0
No	1	1	1

$Gain_{Int} = 0.311278$

$H(JO|PK) = 0$

PK \ JO → ↓	Yes	No	Entropy
Avg	0	1	0
Good	1	0	0
VGood	2	0	0

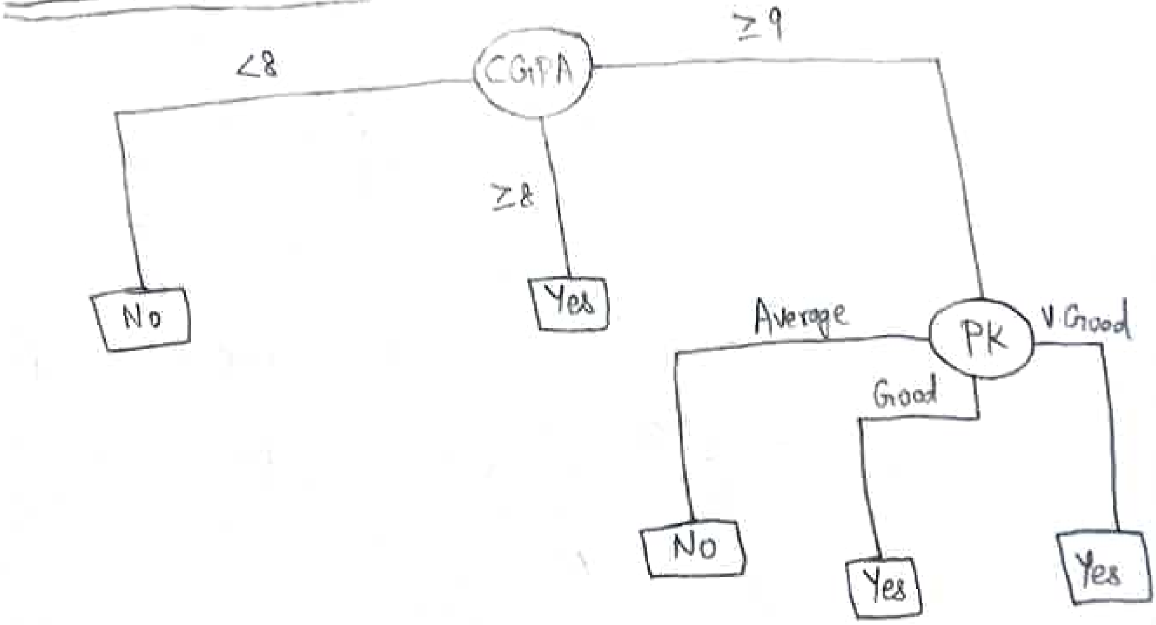
$Gain_{PK} = 0.811278$

$H(JO|CS) = 0$

CS \ JO → ↓	Yes	No	Entropy
P	0	1	0
M	1	0	0
G	2	0	0

$G_{CS} = 0.811278$

<u>Final Decision Tree:</u>



(ii) Using C4.5

$Gain_{CGPA} = 0.556778$

$Info\_split = -0.2 \log 0.2 - 0.4 \log 0.4 - 0.4 \log 0.4 = 1.521928$

$Gain Ratio_{CGPA} = 0.365837$

$Gain_{Int} = 0.091276$

$Info\_split = -0.6 \log 0.6 - 0.4 \log 0.4 = 0.97095$

$Gain Ratio_{Int.} = 0.094006$

$Gain_{PK} = 0.24481$

$Info\_split = -0.2 \log 0.2 - 0.5 \log 0.5 - 0.3 \log 0.3 = 1.48547$

$Gain Ratio_{PK} = 0.164802$

$Gain_{CS} = 0.52036$

$Info\_split = -0.2 \log 0.2 - 0.3 \log 0.3 - 0.5 \log 0.5 = 1.48547$

$Gain Ratio_{CS} = 0.3503$

ROOT NODE: CGPA

$Gain_{Int} = 0.311278$

$Info\_split = -0.5 \log 0.5 - 0.5 \log 0.5 = 1$

$Gain Ratio_{Int.} = 0.311278$

$Gain_{PK} = 0.811278$

$Info\_split = -0.25 \log 0.25 - 0.25 \log 0.25 - 0.5 \log 0.5 = 1.5$

$Gain Ratio_{PK} = 0.540852$

$Gain_{CS} = 0.811278$

$Info\_split = -0.25 \log 0.25 - 0.25 \log 0.25 - 0.5 \log 0.5 = 1.5$

$Gain\ Ratio_{CS} = 0.54082$

Continuous Variables:

9.5 ✓	8.2 ✓	9.1 ✓	6.8 ✓	8.5 ✓	7.9 ✓	8.8 ✓
9.5		9.1				8.8

Ascending Order:

6.8	7.9	8.2	8.5	8.8	9.1	9.5
(1)	(1)	(1)	(1)	(2)	(1)	(2)

Range →	6.8		7.9		8.2		8.5		8.8		9.1		9.5	
JO ↓	≤	>	≤	>	≤	>	≤	>	≤	>	≤	>	≤	>
Yes	0	7	0	7	1	6	2	5	4	3	5	2	7	0
No	1	2	2	1	2	1	2	1	2	1	3	0	3	0
Entropy	0	0.764204	0	0.54353	0.71227	0.59163	1	0.65002	0.91829	0.81127	0.95443	0	0.88129	0
Gain	0.1935064		0.4444										0	

(iii) Using Gini Index:

$$GI_{JO} = 1 - (0.7)^2 - (0.3)^2 = 0.42$$

$$GI(<8, \{\geq 8, \geq 9\}) = (0.2)(1 - 0^2 - 1^2) + (0.8)\left(1 - \frac{0.7}{0.8}^2 - \frac{0.1}{0.8}^2\right)$$

$$= (0.8)($$

$$= 0.175$$