PH354

Assignment -6

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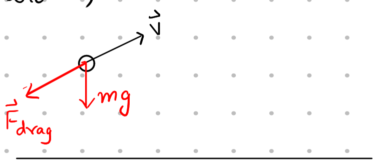
Q5). part -a)



Newtons 2^{nd} Law $\Rightarrow$ $\vec{F} = m\vec{a}$

$$\vec{F} = -mg\hat{y} - F_{drag}\left(\frac{\dot{x}\hat{x} + \dot{y}\hat{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}\right)$$

$$F_{drg} = \frac{1}{2}\pi R^2 \rho C(\dot{x}^2+\dot{y}^2)$$

EOM are

$$F_x = ma_x \Rightarrow \frac{-1}{2m}\pi R^2\rho C\dot{x}\sqrt{\dot{x}^2+\dot{y}^2} = \ddot{x}$$

$$F_y = ma_y \Rightarrow -g - \frac{1}{2m}\pi R^2\rho C\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} = \ddot{y}$$

Part -b)

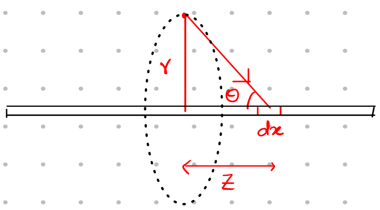
Lets change these equations to 4- first order ODE,

$x \equiv y_0$, $\dot{x} \equiv y_1$, $y \equiv y_2$, $\dot{y} \equiv y_3$

$$\boxed{\begin{aligned} \frac{dy_0}{dt} &= y_1 \\ \frac{dy_1}{dt} &= \frac{-1}{2m}\pi R^2\rho C y_1\sqrt{y_1^2+y_3^2} \\ \frac{dy_2}{dt} &= y_3 \\ \frac{dy_3}{dt} &= -g - \frac{1}{2m}\pi R^2\rho C y_3\sqrt{y_1^2+y_3^2} \end{aligned}}$$

We will be solving this system of First order ODEs using RK4.

Q6)



$r = \sqrt{x^2+y^2}$

We will be summing up all the contribution from these point elements dx

$$dF = \frac{GdMm}{r^2+z^2} = \frac{GMm}{L(x^2+y^2+z^2)}dz$$

But only the component of force perpendicular to the rod contributes,

$$F_{central} = \int dF\sin\theta = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{GMm}{L(x^2+y^2+z^2)} \cdot \frac{\sqrt{x^2+y^2}}{(x^2+y^2+z^2)^{1/2}}dz$$

$$\boxed{F_{central} = \frac{GMm}{L}\sqrt{x^2+y^2}\int_{-L/2}^{L/2}\frac{dz}{(x^2+y^2+z^2)^{3/2}}}$$

Integrating,

take $z = \sqrt{x^2+y^2}\tan\theta$

$dz = \sqrt{x^2+y^2}\sec^2\theta\, d\theta$

$$\frac{L}{2} \longrightarrow \tan^{-1}\left(\overbrace{\frac{L}{2\sqrt{x^2+y^2}}}^{x}\right)$$

$$-\frac{L}{2} \longrightarrow \tan^{-1}\left(\frac{-L}{2\sqrt{x^2+y^2}}\right)$$

$$\int \frac{\sqrt{x^2+y^2}\sec^2\theta\, d\theta}{(x^2+y^2)^{3/2}\sec^3\theta} = \frac{1}{(x^2+y^2)} \cdot \int \cos\theta\, d\theta = \left.\frac{\sin\theta}{x^2+y^2}\right|_{\tan^{-1}(-x)}^{\tan^{-1}(x)}$$

$$\Rightarrow \quad F_{central} = \frac{GMm}{L}\sqrt{x^2+y^2}\left(\frac{1}{(x^2+y^2)}\cdot\frac{L/2}{(x^2+y^2+\frac{L^2}{4})^{1/2}} - \frac{1}{(x^2+y^2)}\cdot\frac{-L/2}{(x^2+y^2+\frac{L^2}{4})^{1/2}}\right)$$

$$\boxed{F_{central} = \frac{GMm}{\sqrt{(x^2+y^2)(x^2+y^2+\frac{L^2}{4})}}}$$

The equation of motion can be written using Newton's 2nd Law,

$$\vec{F}_{central} = m\vec{a}$$

$$\frac{-GMm}{\sqrt{(x^2+y^2)(x^2+y^2+\frac{L^2}{4})}}\frac{(x\hat{x}+y\hat{y})}{\sqrt{x^2+y^2}} = m(\ddot{x}\hat{x}+\ddot{y}\hat{y})$$

Note: We have determined by symmetry the particle doesnt have any motion along z-axis

$$\Rightarrow \quad \ddot{x} = \frac{-GM}{(x^2+y^2)}\cdot\frac{x}{\sqrt{x^2+y^2+\frac{L^2}{4}}}$$

$$\ddot{y} = \frac{-GM}{(x^2+y^2)}\cdot\frac{y}{\sqrt{x^2+y^2+\frac{L^2}{4}}}$$

Part -b) Converting the two 2nd Order ODE to 4- first order ODE,

$x = y_0$, $\dot{x} = y_1$, $y = y_2$, $\dot{y} = y_3$

$$\boxed{\begin{aligned}
\frac{dy_0}{dt} &= y_1 \\
\frac{dy_1}{dt} &= \frac{-GM}{(y_0^2+y_2^2)}\cdot\frac{y_0}{(y_0^2+y_2^2+\frac{L^2}{4})^{1/2}} \\
\frac{dy_2}{dt} &= y_3 \\
\frac{dy_3}{dt} &= \frac{-GM}{(y_0^2+y_2^2)}\cdot\frac{y_2}{(y_0^2+y_2^2+\frac{L^2}{4})^{1/2}}
\end{aligned}}$$

Q10) Using Newton Law of gravitation,

$$\frac{d^2 r_1}{dt^2} = Gm_2 \frac{\bar{r}_2 - \bar{r}_1}{|r_2 - r_1|^3} + Gm_3 \frac{\bar{r}_3 - \bar{r}_1}{|r_3 - r_1|^3}$$

$$\frac{d^2 r_2}{dt^2} = Gm_1 \frac{\bar{r}_1 - \bar{r}_2}{|r_1 - r_2|^3} + Gm_3 \frac{\bar{r}_3 - \bar{r}_2}{|r_3 - r_2|^3}$$

$$\frac{d^2 r_3}{dt^2} = Gm_1 \frac{\bar{r}_1 - \bar{r}_3}{|r_1 - r_3|^3} + Gm_2 \frac{\bar{r}_2 - \bar{r}_1}{|r_2 - r_1|^3}$$

In component form,

$$\boxed{\begin{aligned}
\frac{d\dot{x}_1}{dt} &= Gm_2 \frac{(x_2 - x_1)}{|r_2 - r_1|^3} + Gm_3 \frac{(x_3 - x_1)}{|r_3 - r_1|^3} & \dot{x}_1 &= \frac{dx_1}{dt} \\
\frac{d\dot{y}_1}{dt} &= Gm_2 \frac{(y_2 - y_1)}{|r_2 - r_1|^3} + Gm_3 \frac{(y_3 - y_1)}{|r_3 - r_1|^3} & \dot{y}_1 &= \frac{dy_1}{dt} \\
\frac{d\dot{x}_2}{dt} &= Gm_1 \left(\frac{x_1 - x_2}{|r_1 - r_2|^3} \right) + Gm_3 \left(\frac{x_3 - x_2}{|r_3 - r_2|^3} \right) & \dot{x}_2 &= \frac{dx_2}{dt} \\
\frac{d\dot{y}_2}{dt} &= Gm_1 \frac{(y_1 - y_2)}{|r_1 - r_2|^3} + Gm_3 \frac{(y_3 - y_2)}{|r_3 - r_2|^3} & \dot{y}_2 &= \frac{dy_2}{dt} \\
\frac{d\dot{x}_3}{dt} &= Gm_1 \frac{(x_1 - x_3)}{|r_1 - r_3|^3} + Gm_2 \frac{(x_2 - x_3)}{|r_2 - r_3|^3} & \dot{x}_3 &= \frac{dx_3}{dt} \\
\frac{d\dot{y}_3}{dt} &= Gm_1 \frac{(y_1 - y_3)}{|r_1 - r_3|^3} + Gm_2 \frac{(y_2 - y_3)}{|r_2 - r_3|^3} & \dot{y}_3 &= \frac{dy_3}{dt}
\end{aligned}}$$