QUANTITATIVE RESEARCH

A Deep Dive into Portfolio Optimisation

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Abstract

Background: Selecting an optimal portfolio allocation across a universe of assets is a central problem in investment management. While mean-variance optimisation, first proposed by Markowitz (1952), provides a theoretically principled framework, its out-of-sample performance has proven disappointing in practice. This study empirically compares four portfolio strategies to assess whether model sophistication translates into superior investment performance.

Methodology: This study implements and compares four portfolio strategies: equal weight, Markowitz mean-variance, Black-Litterman and Risk Parity applied to an 18-asset UK multi-asset portfolio comprising 15 FTSE 100 equities, a UK government bond ETF, a gold ETC and a broad commodities ETF, over the period January 2015 to December 2025. Strategies are evaluated using a monthly rebalancing framework and assessed across annualised return, volatility, Sharpe ratio, maximum drawdown and portfolio turnover.

Results: We find results broadly consistent with DeMiguel et al. (2009), with the naive benchmark proving difficult to displace on a risk-adjusted basis over the full sample period. The optimisation strategies exhibit meaningfully different risk profiles, however, suggesting that Sharpe ratio alone does not fully capture the practical trade-offs between approaches.

1 Introduction

1.1 The Portfolio Allocation Problem

The portfolio allocation problem is the fundamental question in investment management. It asks: How should capital be distributed across available investment opportunities to best achieve an investor's objectives?

The problem was intractable until 1952, when Harry Markowitz published *Portfolio Selection* in the *Journal of Finance* and turned it into a clean optimisation problem, and the "efficient frontier" gave investors a principled answer. Markowitz's mean variance model minimises portfolio variance for a given level of expected return. It was revolutionary in the consideration of variance as the risk, standing as a hallmark of modern portfolio theory. In practice however, the model has proven difficult to implement reliably.

Michaud (1989) challenged this view directly, arguing that small estimation errors in expected returns produce wildly unstable portfolio recommendations.

In 2009, DeMiguel et al. delivered a provocative result: he found that even sophisticated strategies incorporating shrinkage estimators and Bayesian methods failed to consistently outperform 1/N out-of-sample, suggesting that estimation error dominates theoretical optimality for typical portfolio problems. This tension between the theoretical elegance of optimisation and its empirical fragility motivates the present study.

1.2 Background and Research Objectives

This study addresses three main questions:

i. Do optimisation models outperform a naive 1/N benchmark out-of-sample, applied to a multi-asset UK portfolio over a ten-year period?

ii. Which model offers the best risk-adjusted performance, as measured by the Sharpe ratio?

iii. How does model performance vary across distinct market regimes: the Brexit referendum (2016), the COVID-19 crash (2020), and the 2022 rate-hiking cycle?

1.3 Summary of Findings

1.4 Structure of the Report

The remainder of this report is organised as follows:
Section 2 reviews the theoretical foundations of each model and derives their key mathematical results. Section 3 describes the data, implementation choices, and backtesting framework. Section 4 presents

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Key Points

- The equal-weight portfolio is robust because it does not rely on estimated parameters and is therefore unaffected by estimation error.
- The Efficient Frontier is the set of portfolios that deliver the minimum possible variance for each level of expected return.
- Empirical evidence suggests that many optimisation models fail to consistently outperform the equal-weight portfolio out of sample.

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the empirical results, including full-period performance metrics and a breakdown by market regime. Section 5 discusses the findings and their practical implications. Section 6 concludes.

2 Literature and Theoretical Background

This section details the design of your study, the data used, the models applied, and the analytical framework. The goal is to provide enough detail for another researcher to replicate your work.

2.1 Notation

Describe the data you used, where you got it from (e.g., Bloomberg, Refinitiv, web scraping), and any steps you took to clean, process, or transform it. Mention the time period and frequency of the data.

2.2 The Equal Weight Puzzle

The equal-weight (1/N) portfolio assigns an identical weight of 1/N to each asset in the portfolio, holding N assets, regardless of any asset-specific characteristics such as return, volatility, or correlation. DeMiguel et al. [1] tested 14 optimisation models across 7 datasets. None consistently outperformed the naive equal-weight portfolio:

$$w_i = \frac{1}{N} \quad \text{for} \quad 1 \le i \le N$$

But why? When expected returns are estimated from historical data in order to determine optimal portfolio weights, the estimates improve as more data is collected, but they do so slowly.
Formally, the estimation error decays as

$$\hat{\mu} - \mu = O\left(\frac{1}{\sqrt{T}}\right)$$

where T is the number of observations.

Because the error shrinks at rate $\frac{1}{\sqrt{T}}$, halving the error requires four times as much data, meaning that with monthly data and realistic return distributions, approximately 500 years of observations are required for statistically reliable estimates, especially since these errors are amplified through the term $\Sigma^{-1}\hat{\mu}$.

The equal-weight portfolio sidesteps this problem entirely: by assigning a fixed weight to each asset, it requires no parameter estimation and is therefore immune to estimation error by construction.
The cost of this simplicity, however, is that the portfolio ignores all information about assets' risk and return characteristics, makes no attempt to diversify risk efficiently and cannot adapt to changing market conditions.

2.3 Mean-Variance Optimisation

Markowitz [2] formulated *Portfolio Selection* as a constrained optimisation problem: minimise portfolio variance for a given level of expected return.

Formally:

$$\min_{\mathbf{w}} \hat{\sigma} = \mathbf{w}^\top \hat{\Sigma} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^\top \mu = \mu_p \quad ; \quad \mathbf{w}^\top \mathbf{1} = 1$$

with $\mathbf{w}, \mu \in \mathbb{R}^N$ and $\hat{\Sigma} \in \mathbb{R}^{N \times N}$ as the vector of portfolio weights, the vector of expected returns with μ_p as the scalar target portfolio return and the sample covariance matrix of asset returns estimated from historical data respectively.

The first constraint, also known as the return constraint, fixes the portfolio to a specific point at the target level μ_p, pinning the solution to a specific point on the "Efficient Frontier". By varying μ_p across all feasible values, the complete set of minimum-variance portfolios - the Efficient Frontier - is traced out. The second constraint, i.e. the budget constraint, ensures the weights represent a valid allocation of wealth, hence that the sum of all weights is always equal to 1.

It is important to note that no domain-restriction condition is imposed on the budget constraint, each w_i may take any real value, in particular, negative ones. Practically speaking, a negative weight corresponds to a short position in the associated asset.
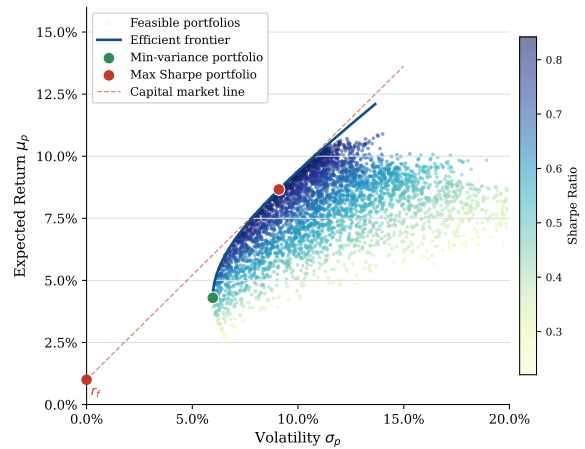

Figure 1. Efficient Frontier estimated using sample covariance

Because the objective function is quadratic and the constraints are linear equalities, we cannot solve the problem by direct substitution (two constraints, n unknowns). We therefore use the method of Lagrange multipliers. The Lagrangian formulation is:

$$\mathcal{L}(\mathbf{w}, \lambda, \gamma) = \mathbf{w}^\top \Sigma \mathbf{w} - \lambda\left(\mathbf{w}^\top \mu - \mu_p\right) - \gamma\left(\mathbf{1}^\top \mathbf{w} - 1\right)$$

where λ and γ are the Lagrange multipliers associated with the return and budget constraints, respectively. Intuitively, λ captures the marginal cost of requiring a higher return (i.e. how much additional variance must be accepted per unit increase in μ_p), and γ enforces full investment.

Taking the first-order condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = 2\Sigma\mathbf{w} - \lambda\mu - \gamma\mathbf{1} = \mathbf{0}$$

Solving gives:

$$\mathbf{w}_p = \frac{\lambda}{2}\,\Sigma^{-1}\mu + \frac{\gamma}{2}\,\Sigma^{-1}\mathbf{1}$$

Imposing the budget and return constraints leads to the following two-fund theorem: all efficient portfolios can be expressed as a linear combination of any two distinct efficient portfolios.

$$\mathbf{w}_A = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top\Sigma^{-1}\mathbf{1}} \qquad \mathbf{w}_B = \frac{\Sigma^{-1}\mu}{\mathbf{1}^\top\Sigma^{-1}\mu}$$

Note that $\mathbf{w}_A$ is the *global minimum variance* (GMV) portfolio, the leftmost point on the efficient frontier, obtained by dropping the return constraint and minimising variance subject only to full investment:

$$\mathbf{w}_{\text{GMV}} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top\Sigma^{-1}\mathbf{1}}$$

The *maximum Sharpe ratio* portfolio is:

$$\mathbf{w}_{\text{MSR}} = \frac{\Sigma^{-1}(\mu - r_f\mathbf{1})}{\mathbf{1}^\top\Sigma^{-1}(\mu - r_f\mathbf{1})}$$

Despite its theoretical elegance, Markowitz optimisation suffers from a well-documented problem. As mentioned above, the solution depends critically on μ and Σ, both estimated from historical data.

Small errors in μ can cause large shifts in optimal weights. Michaud [3] characterised the optimiser as an "error maximiser": it systematically overweights assets with overestimated returns and underestimated variances.

2.4 Black-Litterman and its Bayesian approach

If you are testing a strategy, describe the backtesting framework. Mention key parameters like rebalancing frequency, transaction cost assumptions, and performance metrics used.

2.5 Dalio's Risk Parity

3 Methodology

3.1 Data

3.2 Model Implementation

3.3 Backtesting Framework

Backtester method

3.4 Performance Metrics

All metrics we're using (Return, Sharpe, Sortino, Volatility, CAGR, Max DD, VaR, CVaR)

4 Results

This section presents the key findings from your analyses and interprets their significance. Use figures and tables to present your results clearly.

Present your main results. This could be in the form of tables summarizing statistical outputs, or charts showing trends and relationships.

Table 1. Descriptive caption for your table.

Category	Metric 1	Metric 2	Metric 3
Group A	0.00	0.00	0.00
Group B	0.00	0.00	0.00
Group C	0.00	0.00	0.00

Note: Explain any specific details about the data in the table.

4.1 Main Result: Performance Comparison

Discuss what your results mean. How do they relate to your initial research question? Are they consistent with existing literature? What are the implications of your findings?

4.2 Risk-Return Trade-off

4.3 Regime Dependence

4.4 Portfolio Characteristics

5 Discussion

Summarize the main conclusions of your research. Reiterate the key insights and their importance.

5.1 Why did 1/N perform well?

5.2 The risk parity puzzle

5.3 Practical Implications

5.4 Limitations

6 Conclusion

7 Declarations

Briefly describe the contribution of each author to the research and writing of the report. For example: "A.B. designed the research. C.D. collected the data. E.F. performed the analysis. All authors contributed to writing the report."

Future work could include:

· Exploring alternative methodologies or datasets.
· Addressing limitations of the current study.
· Expanding the research to a different market or asset class.

References

1. DeMiguel V, Garlappi L, Uppal R. Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? The Review of Financial Studies 2009 May;22(5):1915–1953. https://doi.org/10.1093/rfs/hhm075.
2. Markowitz H. Portfolio Selection. The Journal of Finance 1952;7(1):77–91. https://www.jstor.org/stable/2975974, publisher: [American Finance Association, Wiley].
3. Michaud RO. The Markowitz Optimization Enigma: Is 'Optimized' Optimal? Financial Analysts Journal 1989 Jan;45(1):31–42. https://doi.org/10.2469/faj.v45.n1.31, publisher: Routledge _eprint: https://doi.org/10.2469/faj.v45.n1.31.