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A Deep Dive into Portfolio Optimisation

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Abstract

Background: Selecting an optimal portfolio allocation across a universe of assets is a central problem in investment management. A wide range of allocation strategies have been proposed, from naive diversification to theoretically grounded optimisation frameworks, yet their relative out-of-sample performance remains contested, particularly across varying market regimes. This study empirically compares four portfolio strategies to assess whether model sophistication translates into superior investment performance.

Methodology: This study implements and compares four portfolio strategies: equal weight, Markowitz mean-variance, Black-Litterman and Risk Parity applied to an 18-asset UK multi-asset portfolio comprising 15 FTSE 100 equities, a UK government bond ETF, a gold ETC and a broad commodities ETF, over the period January 2015 to December 2025. Strategies are evaluated using a monthly rebalancing framework and assessed across annualised return, volatility, Sharpe ratio, maximum drawdown and portfolio turnover.

Results: We find results broadly consistent with DeMiguel et al. (2009), with the naive benchmark proving difficult to displace on a risk-adjusted basis over the full sample period. The optimisation strategies exhibit meaningfully different risk profiles, however, suggesting that Sharpe ratio alone does not fully capture the practical trade-offs between approaches.

1 Introduction

1.1 The Portfolio Allocation Problem

The portfolio allocation problem is the fundamental question in investment management. It asks: How should capital be distributed across available investment opportunities to best achieve an investor's objectives?

The problem was intractable until 1952, when Harry Markowitz published *Portfolio Selection* in the *Journal of Finance* and turned it into a clean optimisation problem, and the "efficient frontier" gave investors a principled answer. Markowitz's mean variance model minimises portfolio variance for a given level of expected return. It was revolutionary in the consideration of variance as the risk, standing as a hallmark of modern portfolio theory. In practice however, the model has proven difficult to implement reliably.

Michaud (1989) challenged this view directly, arguing that small estimation errors in expected returns produce wildly unstable portfolio recommendations.

In 2009, DeMiguel et al. delivered a provocative result: he found that even sophisticated strategies incorporating shrinkage estimators and Bayesian methods failed to consistently outperform 1/N out-of-sample, suggesting that estimation error dominates theoretical optimality for typical portfolio problems. This tension between the theoretical elegance of optimisation and its empirical fragility motivates the present study.

1.2 Research Objectives

This study addresses three main questions:

i. Do optimisation models outperform a naive 1/N benchmark out-of-sample, applied to a multi-asset UK portfolio over a ten-year period?
ii. Which model offers the best risk-adjusted performance, as measured by the Sharpe ratio?
iii. How does model performance vary across distinct market regimes: the Brexit referendum (2016), the COVID-19 crash (2020), and the 2022 rate-hiking cycle?

1.3 Structure of the Report

The remainder of this report is organised as follows:
Section 2 reviews the theoretical foundations of each model and derives their key mathematical results. Section 3 describes the data, implementation choices, and backtesting framework. Section 4 presents

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Key Points

- The equal-weight portfolio is robust because it does not rely on estimated parameters and is therefore unaffected by estimation error.
- The Efficient Frontier is the set of portfolios that deliver the minimum possible variance for each level of expected return.
- Empirical evidence suggests that many optimisation models fail to consistently outperform the equal-weight portfolio out of sample.

the empirical results, including full-period performance metrics and a breakdown by market regime. Section 5 discusses the findings and their practical implications. Section 6 concludes.

2 Literature and Theoretical Background

2.1 Notation

Some common notation that is used across this paper is the following:

Symbol	Description
N	Number of assets in the portfolio
$\mathbf{w} \in \mathbb{R}^N$	Vector of portfolio weights
$\boldsymbol{\mu} \in \mathbb{R}^N$	Vector of expected returns
$\boldsymbol{\Sigma} \in \mathbb{R}^{N \times N}$	Covariance matrix of asset returns
$\mathbf{1} \in \mathbb{R}^N$	Vector of ones
$r_f \in \mathbb{R}$	Risk-free rate

2.2 The equal weight portfolio

The equal-weight (1/N) portfolio assigns an identical weight of 1/N to each asset in the portfolio, holding N assets, regardless of any asset-specific characteristics such as return, volatility, or correlation. DeMiguel et al. (2009) tested 14 optimisation models across 7 datasets. None consistently outperformed the naive equal-weight portfolio:

$$w_i = \frac{1}{N} \quad \text{for} \quad 1 \leq i \leq N \tag{2.1}$$

But why? When expected returns are estimated from historical data in order to determine optimal portfolio weights, the estimates improve as more data is collected, but they do so slowly.
Formally, the estimation error decays as

$$\hat{\mu} - \mu = O\left(\frac{1}{\sqrt{T}}\right) \tag{2.2}$$

where T is the number of observations.

Because the error shrinks at rate $\frac{1}{\sqrt{T}}$, halving the error requires four times as much data, meaning that with monthly data and realistic return distributions, approximately 500 years of observations are required for statistically reliable estimates, especially since these errors are amplified through the term $\Sigma^{-1}\hat{\mu}$.

The equal-weight portfolio sidesteps this problem entirely: by assigning a fixed weight to each asset, it requires no parameter estimation and is therefore immune to estimation error by construction.
The cost of this simplicity, however, is that the portfolio ignores all information about assets' risk and return characteristics, makes no attempt to diversify risk efficiently and cannot adapt to changing market conditions.

2.3 Markowitz's mean-variance optimisation

Markowitz (1952) formulated *Portfolio Selection* in 1952, approaching the asset allocation problem as a constrained optimisation problem: minimise portfolio variance for a given level of expected return, the foundation of the mean-variance model. His work stands as a hallmark of modern portfolio theory.

Formally, the problem states:

$$\min_{\mathbf{w}} \hat{\sigma} = \mathbf{w}^\top \hat{\boldsymbol{\Sigma}}\, \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^\top \boldsymbol{\mu} = \mu_p \quad ; \quad \mathbf{w}^\top \mathbf{1} = 1 \tag{2.3}$$

with μ_p as the scalar target portfolio return.

The first constraint, also known as the return constraint, fixes the portfolio to a specific point at the target level μ_p, pinning the solution to a specific point on the "Efficient Frontier". By varying μ_p across all feasible values, the complete set of minimum-variance portfolios - the Efficient Frontier - is traced out. The second constraint, i.e. the budget constraint, ensures the weights represent a valid allocation of wealth, hence that the sum of all weights is always equal to 1.

It is important to note that no domain-restriction condition is imposed on the budget constraint, each w_i may take any real value, in particular, negative ones. Practically speaking, a negative weight corresponds to a short position in the associated asset.

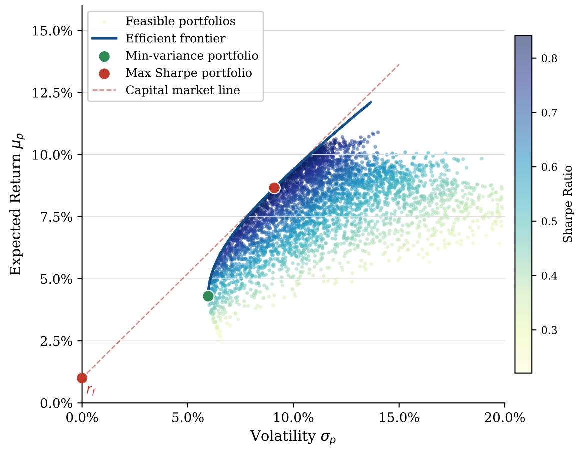

Figure 1. Efficient Frontier estimated using sample covariance

Because the objective function is quadratic and the constraints are linear equalities, we cannot solve the problem by direct substitution (two constraints, n unknowns). We therefore use the method of Lagrange multipliers. The Lagrangian formulation is:

$$\mathcal{L}(\mathbf{w}, \lambda, \gamma) = \mathbf{w}^\top \boldsymbol{\Sigma} \mathbf{w} - \lambda(\mathbf{w}^\top \boldsymbol{\mu} - \mu_p) - \gamma(\mathbf{1}^\top \mathbf{w} - 1) \tag{2.4}$$

where λ and γ are the Lagrange multipliers associated with the return and budget constraints, respectively. Intuitively, λ captures the marginal cost of requiring a higher return (i.e. how much additional variance must be accepted per unit increase in μ_p), and γ enforces full investment.

Taking the first-order condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = 2\boldsymbol{\Sigma}\mathbf{w} - \lambda\boldsymbol{\mu} - \gamma\mathbf{1} = \mathbf{0} \tag{2.5}$$

Key Points

- Black-Litterman uses market-cap weights as a neutral starting point for expected returns.
- Subjective investor views are blended with the equilibrium prior through a Bayesian update.
- The optimisation step in Black-Litterman is the same as MVO with μ_{BL} replacing the sample mean.

Solving gives:

$$\mathbf{w}_p = \frac{\lambda}{2}\boldsymbol{\Sigma}^{-1}\mu + \frac{\gamma}{2}\boldsymbol{\Sigma}^{-1}\mathbf{1} \tag{2.6}$$

Imposing the budget and return constraints leads to the following two-fund theorem: all efficient portfolios can be expressed as a linear combination of any two distinct efficient portfolios.

$$\mathbf{w}_A = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^\top\boldsymbol{\Sigma}^{-1}\mathbf{1}} \qquad \mathbf{w}_B = \frac{\boldsymbol{\Sigma}^{-1}\mu}{\mathbf{1}^\top\boldsymbol{\Sigma}^{-1}\mu} \tag{2.7}$$

Note that $\mathbf{w}_A$ is the *global minimum variance* (GMV) portfolio, the leftmost point on the efficient frontier, obtained by dropping the return constraint and minimising variance subject only to full investment:

$$\mathbf{w}_{\text{GMV}} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^\top\boldsymbol{\Sigma}^{-1}\mathbf{1}} \tag{2.8}$$

And the *maximum Sharpe ratio* portfolio is:

$$\mathbf{w}_{\text{MSR}} = \frac{\boldsymbol{\Sigma}^{-1}(\mu - r_f\mathbf{1})}{\mathbf{1}^\top\boldsymbol{\Sigma}^{-1}(\mu - r_f\mathbf{1})} \tag{2.9}$$

Despite its theoretical elegance, Markowitz optimisation suffers from a well-documented problem. As mentioned above, the solution depends critically on μ and $\boldsymbol{\Sigma}$, both estimated from historical data.

Small errors in μ can cause large shifts in optimal weights. Michaud (1989) characterised the optimiser as an "error maximiser": it systematically overweights assets with overestimated returns and underestimated variances.

2.4 Black-Litterman and its Bayesian approach

The Black-Litterman (BL) model was developed by Fischer Black and Robert Litterman at Goldman Sachs in the early 1990s to address the instability of Mean-Variance Optimisation (Black and Litterman; 1991, 1992).

The BL model addresses two shortcomings of classic mean-variance optimisation:

i. What expected returns should be used?
Rather than relying on historical data, BL uses returns implied by current market-cap weights as the neutral starting point.

ii. How should the views be incorporated?
BL offers a formal Bayesian framework for blending subjective investor views with this equilibrium prior, weighting each source inversely by its uncertainty.

Step 1. Equilibrium Returns

The model takes as its starting point the equilibrium expected returns implied by the market portfolio, i.e. the portfolio where each asset is held in proportion to its market capitalisation. Assuming the market portfolio is mean-variance efficient, the equilibrium excess returns are:

$$\Pi = \delta\Sigma w_m \tag{2.10}$$

where w_m is the market-cap weight vector, Σ is the covariance matrix of asset returns and δ is the risk-aversion coefficient

$$\delta = \frac{r_m - r_f}{\sigma_m^2} \tag{2.11}$$

with r_m the return of the market portfolio and σ^2 its variance.

Step 2. Express Views

A view can be absolute or relative, for example "asset A will return 5%" or "asset A will outperform asset B by 2%". Each view has two components: the expected return itself and a confidence level attached to it.

Formally, k views are expressed through the following system:

$$P\mu = Q + \varepsilon \qquad \varepsilon \sim \mathcal{N}(\mathbf{0}, \Omega) \tag{2.12}$$

where:

Symbol	Description
$P \in \mathbb{R}^{k\times n}$	*Pick matrix*: encodes which assets views involve
$\mu \in \mathbb{R}^k$	Vector of expected returns
$Q \in \mathbb{R}^k$	Expected view returns
$\Omega \in \mathbb{R}^{k\times k}$	View uncertainty matrix (diagonal)
$\varepsilon \sim \mathcal{N}(\mathbf{0}, \Omega)$	View error: deviation of true returns from stated views

Assets are ranked by their return over the previous 12 months, excluding the most recent month to eliminate short-term reversal effects. The highest-return tercile forms the winner portfolio, while the lowest-return decile constitutes the loser portfolio.

The view is expressed as:

$$p_k^\top \mu = q_k \tag{2.13}$$

p_k assigns equal positive weights to winner assets and equal negative weights to loser assets, forming a long-short momentum view. The expected return spread q_k is estimated over a rolling window at each rebalancing date as asset rankings change.

Step 3. Bayesian Update

The prior on expected returns is:

$$\mu \sim \mathcal{N}(\boldsymbol{\Pi}, \tau\boldsymbol{\Sigma}) \tag{2.14}$$

This encodes the belief that expected returns are centred on the equilibrium vector $\boldsymbol{\Pi}$, with uncertainty scaled by τ. Combining this prior with the view system via Bayes' theorem yields the posterior distribution over expected returns:

$$\mu \mid Q \sim \mathcal{N}(\mu_{BL}, \boldsymbol{\Sigma}_{BL}) \tag{2.15}$$

where the posterior mean is:

$$\mu_{BL} = \left[(\tau\Sigma)^{-1} + P^\top\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\boldsymbol{\Pi} + P^\top\Omega^{-1}Q\right] \tag{2.16}$$

The posterior mean μ_{BL} is a precision-weighted average of the equilibrium prior $\boldsymbol{\Pi}$ and the investor's views Q, with each source weighted inversely by its uncertainty. When views are diffuse (large $\boldsymbol{\Omega}$), μ_{BL} remains close to $\boldsymbol{\Pi}$. If the views are held with high confidence

Key Points

- Risk Parity allocates capital such that every asset contributes equally to total portfolio volatility, not equally to capital.
- Unlike MVO and Black-Litterman, Risk Parity requires no estimate of expected returns: allocations depend only on the covariance structure.
- The equal risk contribution condition $w_i(\boldsymbol{\Sigma}\mathbf{w})_i = c$ for all i has no closed-form solution and in general and is solved numerically.

(small $\boldsymbol{\Omega}$), μ_{BL} tilts toward $\mathbf{Q}$.

This blending property is central to the appeal of the BL framework: Rather than overriding the equilibrium prior, views are incorporated gradually and in proportion to their confidence. This acts as a natural regulariser, because the prior pulls weights toward the market portfolio, the model is far less prone to the concentrated, unstable allocations that plague classical MVO.

Step 4. Portfolio Optimisation
The optimal portfolio is obtained by applying the same Sharpe-maximising objective derived in Section 2.3 (equation 2.9), substituting μ_{BL} in place of the sample mean μ.

2.5 Risk Parity

The modern Risk Parity framework was popularised by Ray Dalio at Bridgewater Associates and underpins the All Weather strategy introduced in the 1990s. The approach was subsequently formalised by Maillard et al. (2010), who derived the mathematical conditions for equal risk contribution portfolios.

The central motivation departs entirely from the MVO and BL frameworks: rather than optimising expected return for a given level of risk, Risk Parity ignores return estimates altogether and instead focuses on how risk is distributed across the portfolio.

Risk Parity addresses a structural limitation shared by both MVO and the equal-weight portfolio:

i. In MVO and BL, allocations depend on estimated expected returns, which are noisy and prone to estimation error.
ii. In equal-weight portfolios, capital is diversified equally but risk is not. Assets with high volatility dominate total portfolio risk despite receiving the same capital allocation.

Risk Decomposition
Portfolio volatility $\sigma_p = \sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\mathbf{w}}$ is a homogeneous function of degree 1 in $\mathbf{w}$. By Euler's homogeneous function theorem, it admits the following decomposition:

$$\sigma_p = \sum_{i=1}^{N} w_i \frac{\partial \sigma_p}{\partial w_i} \tag{2.17}$$

The *marginal risk contribution* (MRC) of asset i is:

$$\mathrm{MRC}_i = \frac{\partial \sigma_p}{\partial w_i} = \frac{(\boldsymbol{\Sigma}\mathbf{w})_i}{\sigma_p} \tag{2.18}$$

and the total *risk contribution* (RC) of asset i is:

$$\mathrm{RC}_i = w_i \cdot \mathrm{MRC}_i = \frac{w_i\,(\boldsymbol{\Sigma}\mathbf{w})_i}{\sigma_p} \tag{2.19}$$

with the useful property that $\sum_{i=1}^{N} \mathrm{RC}_i = \sigma_p$.

The Risk Parity Condition
The portfolio is said to be *risk-balanced* when every asset contributes equally to total portfolio volatility:

$$\mathrm{RC}_i = \frac{\sigma_p}{N} \quad \forall\, i \tag{2.20}$$

Since σ_p appears on both sides, this reduces to the equivalent condition:

$$w_i\,(\boldsymbol{\Sigma}\mathbf{w})_i = c \quad \forall\, i \tag{2.21}$$

for some constant $c > 0$, subject to $\mathbf{1}^\top \mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$. This system has no closed-form solution in general and is solved numerically; the implementation details are described in Section 3.

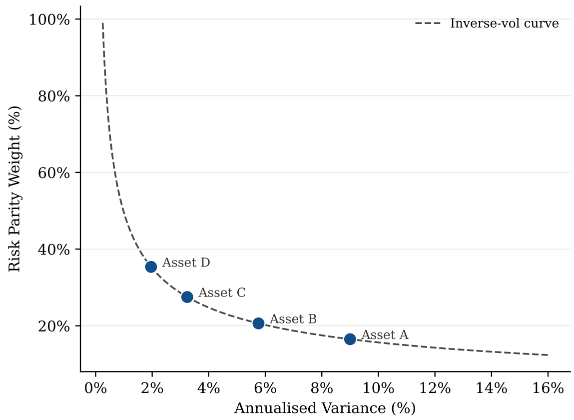

Figure 2. Risk parity weights as a function of asset variance

Special Case: Uncorrelated Assets
When $\boldsymbol{\Sigma}$ is diagonal, the risk parity condition reduces to an analytic solution. Equal risk contribution then requires each asset's weight to be proportional to the inverse of its volatility:

$$w_i^{\mathrm{RP}} = \frac{1/\sigma_i}{\sum_{j=1}^{N} 1/\sigma_j} \tag{2.22}$$

This inverse-volatility weighting is the closed-form limit of risk parity, and also serves as the natural initialisation point for the numerical solver in the general case.

Despite its appeal, Risk Parity has a well-known structural consequence: it mechanically overweights low-volatility assets. In a portfolio containing both equities and government bonds, bonds receive substantially larger capital allocations precisely because their volatility is lower, which raises the question of whether the resulting portfolio is truly diversified in an economic sense or merely in a statistical one.

3 Methodology

3.1 Data

The dataset comprises daily adjusted closing prices for 18 assets over an 11-year period from 1 January 2015 to 31 December 2025, sourced programmatically from *Yahoo Finance*. The universe is constructed to reflect a representative UK-centric multi-asset portfolio: 15 large-cap FTSE 100 equities spanning eight sectors, supplemented by three exchange-

Key Points

- The universe comprises 18 assets across diversified sectors, including equities, bonds, gold and commodities.
- Monthly rebalancing on the final trading day of each calendar month is applied consistently across all four strategies.
- The walk-forward design of the backtester enforces a strict information barrier, ensuring no look-ahead bias.

traded products providing exposure to UK government bonds, gold, and broad commodities. The full asset universe is listed in Table 1.

Asset Class	Name	Ticker
Banking	HSBC	HSBA.L
	Lloyds Banking Group	LLOY.L
	Barclays	BARC.L
Oil & Gas	Shell	SHEL.L
	BP	BP.L
Consumer Goods	Unilever	ULVR.L
	Tesco	TSCO.L
	Diageo	DGE.L
Pharmaceuticals	AstraZeneca	AZN.L
	GSK	GSK.L
Mining	Rio Tinto	RIO.L
	Glencore	GLEN.L
Utilities	National Grid	NG.L
Telecoms	Vodafone	VOD.L
Industrials	Rolls-Royce	RR.L
UK Govt Bonds	iShares Core UK Gilts ETF	IGLT.L
Precious Metals	Invesco Physical Gold ETC	SGLD.L
Commodities	WisdomTree Broad Commodities	WCOG.L

Table 1. Asset universe (18 assets)

Adjusted closing prices are used in preference to raw prices. Yahoo Finance's adjusted close corrects for corporate actions including stock splits, rights issues, and dividend distributions, ensuring that computed returns reflect the true total return experienced by an investor. Missing observations arising from non-synchronous trading calendars are handled by forward-filling the most recent available price prior to return calculation.

The risk-free rate is proxied by the 3-month UK Treasury Bill rate, sourced from the Federal Reserve Economic Data database and converted to a daily rate *FRED Database*. The rate is applied time-variably across the sample, capturing the material shift from near-zero rates during 2015–2021 to above 5% during the 2023–2024 tightening cycle.

3.2 Model Implementation

All models are implemented from first principles in Python using `NumPy` and `Pandas`, following an object-oriented design in which each strategy inherits from a common abstract base class. This ensures full transparency and reproducibility of all optimisation steps. The complete source code is publicly available at the project repository (Surrey Capital Research; 2026).

All models share a rolling estimation framework: at each monthly rebalancing date, covariance and return inputs are re-estimated using the preceding 252 trading days of data. Table 2 summarises the key implementation parameters for each strategy.

Equal Weight assigns $w_i = 1/N$ at each rebalancing date and requires no parameter estimation.

In the Mean-Variance model, the maximum Sharpe ratio portfolio is solved via an active-set method: assets with negative target weights are

Model	Window	Constraints	Solver
Equal Weight	None	$w_i = 1/N$	Closed-form
MVO	252 days	Long-only	Active-set
Black-Litterman	252 days	Long-only	Active-set
Risk Parity	252 days	Long-only	Newton-Raphson

Table 2. Implementation parameters by strategy

iteratively pruned from the investment set and the system is re-solved until all remaining weights satisfy the long-only constraint $w_i \geq 0$. A small ridge term $\lambda = 10^{-8}$ is added to the diagonal of $\hat{\Sigma}$ at each step to guarantee numerical positive definiteness.

Risk Parity solves the equal risk contribution system (equation 2.21) via Newton-Raphson iteration, initialised with inverse-volatility weights. Convergence is typically achieved within 10 iterations.

The Black-Litterman model requires two calibration parameters: the prior uncertainty scalar $\tau = 0.05$ and the risk-aversion coefficient $\delta = 3.0$. The investor views are constructed by the momentum effect documented by Jegadeesh and Titman (1993).

Assets are ranked by their return over the prior 12 months excluding the most recent month, and a long-short view is formed between the top and bottom terciles at each rebalancing date. The posterior expected returns μ_{BL} are then passed into the same active-set MVO solver used for the standard MVO strategy.

3.3 Backtesting Framework

The four strategies are evaluated using a walk-forward backtesting engine to ensure zero look-ahead bias. The engine enforces a strict information barrier by passing only prices up to the rebalancing date; the covariance dependent strategies (MVO, BL, RP) then estimate over a 252 trading day sub-window. This distinction is already implicit in Table 2 (Section 3.2).

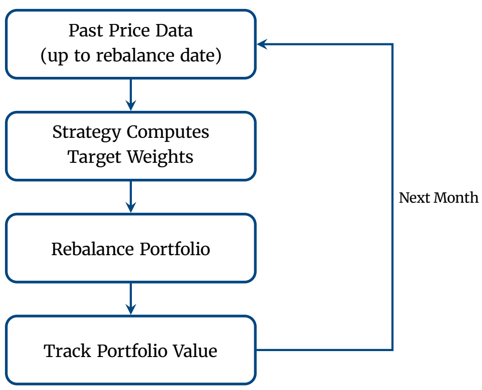

Figure 3. Walk-forward backtesting loop

A starting capital of £100,000 is deployed on the first available trading day. The portfolio is thereafter rebalanced on the final trading day of each calendar month, with all positions revalued daily at the adjusted closing price.

The simulation assumes perfect asset divisibility (i.e. the existence of fractional shares) and immediate execution at the closing price on each rebalancing date. Transaction costs are not modelled; portfolio turnover is instead reported as a standalone metric, allowing the reader to assess the practical trading burden of each strategy independently. Bid-ask spreads and market impact are not accounted for; given that all 18 assets are large-cap and exchange-traded, it is reasonable to assume sufficient liquidity to absorb the portfolio's order flow without material execution slippage.

Each simulation run produces a `BacktestResult` object containing the daily equity curve, full position history and a trade log. Performance metrics are computed from the equity curve at the conclusion of each run and are defined in Section 3.4.

3.4 Performance Metrics

Nine metrics are used to evaluate each strategy, grouped into three categories: return, risk, and risk-adjusted. Let $r_t = V_t/V_{t-1} - 1$ denote the daily portfolio return, V_0 and V_T the initial and terminal portfolio values, T the total number of trading days and r_f the mean annualised risk-free rate over the sample. Table 3 defines each metric formally.

Return metrics capture the magnitude of wealth creation. CAGR annualises the compounded total return to allow comparison across strategies regardless of sub-period variation.

Risk metrics characterise the severity and shape of losses: maximum drawdown measures the worst peak-to-trough decline an investor would have experienced; VaR and CVaR characterise tail losses from the empirical return distribution without parametric assumptions.

Risk-adjusted metrics normalise excess return by some measure of risk: the Sharpe ratio uses total volatility, while the Sortino ratio penalises only downside deviations, making it more appropriate when return distributions are asymmetric.

Metric	Formula	Category
Total Return	$V_T/V_0 - 1$	Return
CAGR	$(V_T/V_0)^{252/T} - 1$	Return
Volatility	$\hat{\sigma}(r_t) \cdot \sqrt{252}$	Risk
Max Drawdown	$\min_t \left(\frac{V_t}{\max_{s \le t} V_s} - 1 \right)$	Risk
95% VaR	Percentile$(r_t,\ 5\%)$	Risk
95% CVaR	$\mathbb{E}[\, r_t \mid r_t \le \text{VaR}\,]$	Risk
Sharpe Ratio	$(\text{CAGR} - r_f)\ /\ \hat{\sigma}_p$	Risk-Adjusted
Sortino Ratio	$(\text{CAGR} - r_f)\ /\ \hat{\sigma}_{\text{down}}$	Risk-Adjusted
Avg. Monthly Turnover	$\frac{1}{M} \sum_{m=1}^{M} \sum_{i=1}^{N} \lvert w_{i,m} - w_{i,m-1} \rvert$	Portfolio

Table 3. Performance metrics used to evaluate each strategy.

VaR and CVaR are computed from the empirical distribution of daily returns without a parametric assumption. $\hat{\sigma}_{\text{down}}$ denotes the annualised standard deviation of negative daily returns only, so the Sortino ratio penalises downside volatility exclusively. Average monthly turnover measures the mean total absolute weight change across all assets at each rebalancing date, providing a strategy-level indicator of trading activity.

4 Results

The results section evaluates four portfolio optimisation strategies, Equal Weight, Risk Parity, Mean-Variance Optimisation (MVO), and Black-Litterman, across the 2015 to 2025 period, using a starting portfolio value of £100,000. The analysis is organised across four subsections, covering full-period performance, risk-return trade-offs, regime dependence, and portfolio characteristics.

4.1 Full-Period Performance

Tables 1 and 2 present the return, risk, and downside risk metrics for all four strategies over the full sample period from 2015 to 2025, with Figure 3 showing the corresponding cumulative wealth curves and Figure 4 the drawdown profiles.

Table 4. Return and Risk Metrics (2015–2025)

Metric	Equal Weight	Risk Parity	MVO	Black-Litterman
Total Return	91.57%	27.34%	23.56%	22.17%
CAGR	6.08%	2.22%	1.94%	1.83%
Volatility	13.94%	9.39%	10.31%	12.03%
Sharpe Ratio	0.44	0.24	0.19	0.15
Max Drawdown	-29.68%	-22.60%	-22.60%	-27.35%

Table 5. Downside Risk Metrics (2015–2025)

Metric	Equal Weight	Risk Parity	MVO	Black-Litterman
Sortino Ratio	0.56	0.29	0.23	0.18
95% VaR	-1.28%	-0.86%	-0.95%	-1.05%
95% CVaR	-2.09%	-1.46%	-1.63%	-1.84%

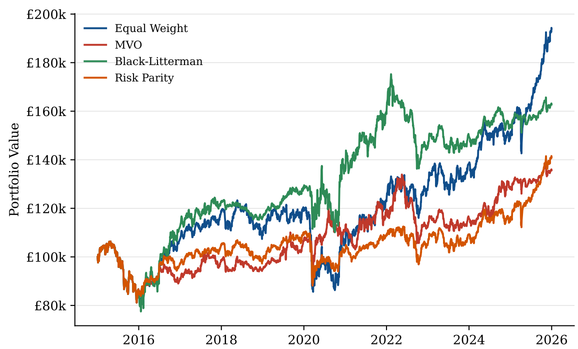

Figure 4. Cumulative wealth curves for all four strategies (2015–2025), indexed to £100,000 initial capital.

Which strategy won overall, and by how much?

On a raw return basis, Equal Weight was the clear winner, delivering a total return of 91.57% (CAGR: 6.08%) over the period. This result is more than three times the return of the next best strategy, Risk Parity (27.34%, CAGR: 2.22%). MVO and Black-Litterman lagged further still at 23.56% and 22.17% respectively. The cumulative return chart reinforces this: Equal Weight's portfolio value surpassed £190k by late 2025, while the others clustered between £135k–£163k, with Risk Parity the weakest performer in absolute terms.

Does the ranking change on a risk-adjusted basis?

The ranking is preserved but compressed. Equal Weight retains the top position with a Sharpe ratio of 0.44 and Sortino of 0.56, reflecting that its higher volatility (13.94%) was more than compensated by superior returns. Risk Parity comes second (Sharpe: 0.24, Sortino: 0.29), followed by MVO (0.19 / 0.23) and Black-Litterman (0.15 / 0.18).

Notably, the optimised strategies (MVO and Black-Litterman), which

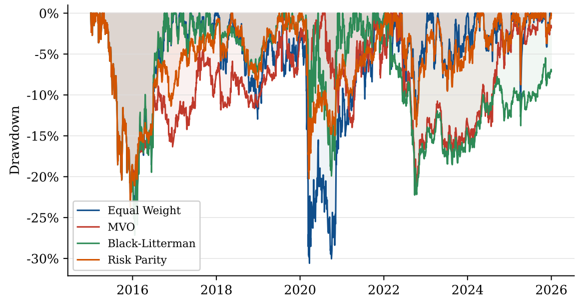

Figure 5. Drawdown profiles for all four strategies (2015–2025).

are theoretically intended to maximise risk-adjusted returns, actually underperformed the naive Equal Weight benchmark across all metrics. This is a well-documented phenomenon was mentioned above and is called the **"1/N puzzle"**. In practice, estimation error in expected returns and covariances often causes optimised portfolios to underperform simpler allocations out-of-sample.

What does the drawdown chart reveal that the cumulative returns do not?
The cumulative returns chart presents a broadly upward narrative, but the drawdown chart tells a more nuanced story about the cost of participating in that growth.

Equal Weight's hidden fragility. Despite delivering the highest returns, Equal Weight also experienced the largest maximum drawdown at −29.68%, most visibly during the COVID-19 market crash in early 2020. At that point, a £100,000 portfolio would have temporarily lost nearly £30,000 of value — a risk that is easily missed in the smooth upward trajectory of the cumulative chart.

Equal Weight's hidden fragility: Despite delivering the highest returns, Equal Weight also experienced the largest maximum drawdown at −29.68%, most visibly during the COVID-19 market crash in early 2020. At that point, a £100k portfolio would have temporarily lost nearly £30k of value.

Risk Parity's resilience under stress: Risk Parity and MVO share the best max drawdown (−22.50%), and the drawdown chart shows Risk Parity recovered relatively quickly from most dips. Its comparatively low volatility (9.39%) results in drawdowns that are both milder and shorter in duration, which is valuable for investors with lower loss tolerance or shorter time horizons.

Black-Litterman's 2022–2024 struggle: The drawdown chart highlights a sustained decline of over −22% between 2022 and 2024. This risk episode is partially masked in the cumulative return plot due to the strong performance recorded in 2020–2022.

4.2 Risk-Return Trade-off

The previous section showed that Equal Weight delivered the strongest absolute returns over the period. This section examines whether that outperformance holds up when risk is taken into account, and whether any strategy maintains a consistent edge throughout the sample or simply benefits from favourable conditions at certain points in time. Two figures inform this discussion: a risk-return scatter showing where each strategy sits in volatility-return space over the full period, and a rolling 12-month Sharpe ratio tracking how risk-adjusted performance evolved through time.

Positioning in Risk-Return Space
Equal Weight occupies the upper-right quadrant, offering the highest CAGR (~6.0%) but also the highest volatility (~14.0%). Risk Parity takes the opposite stance with the lowest volatility (~10%) and ~3.4% CAGR. Black-Litterman sits in the middle at ~4% CAGR, though at volatility

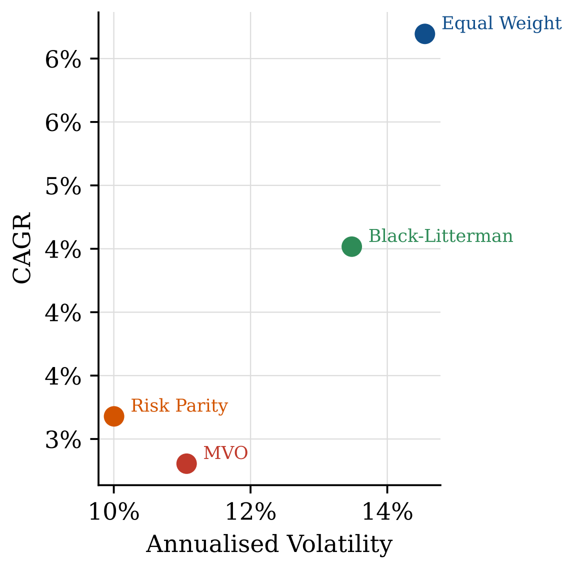

Figure 6. Risk-return scatter (one dot per strategy, volatility on x-axis, CAGR on y-axis).

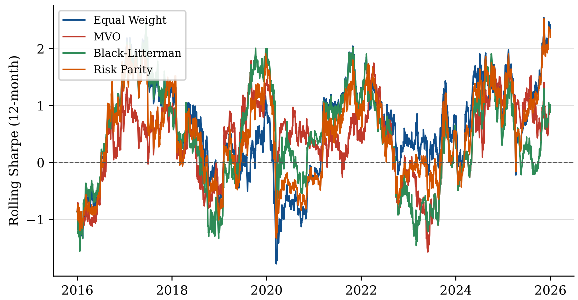

Figure 7. Rolling 12-month Sharpe ratio (all four strategies, full sample period).

nearly as high as Equal Weight, offering little reward for that additional risk. MVO ends up being the most difficult to justify: higher volatility than Risk Parity yet the weakest return of all four (~2.7%), placing it in a vulnerable position in risk-return space.

Consistency of Risk-Adjusted Performance Over Time
The rolling 12-month Sharpe ratio reveals that no strategy maintains a persistent performance advantage across the full sample. All four strategies move largely in tandem, suggesting that the choice of strategy offers limited protection during systemic shocks and that the macroeconomic regime is a stronger determinant of short-term risk-adjusted performance than the allocation model itself.

4.3 Regime Dependence

Full-period metrics can mask very different behaviour across market conditions. This section stress-tests the four strategies across three distinct episodes: the Brexit shock (Jun–Dec 2016), the COVID-19 crash (Feb–Sep 2020), and the 2022 rate-hiking cycle (Jan–Dec 2022), each representing a different type of market disruption. Table 3 and Figure 7 summarise cumulative returns across these periods.

Brexit (Jun–Dec 2016)
The Brexit referendum triggered a sharp sterling depreciation and a rapid rotation into exporters and internationally-exposed equities.

Table 6. Strategy Performance During Key Market Events

Event	EW	MVO	BL	RP
Brexit	15.95%	0.84%	20.28%	6.16%
COVID	-25.84%	3.59%	-13.44%	-13.69%
Rate Hikes	1.50%	-6.02%	-10.90%	-4.34%

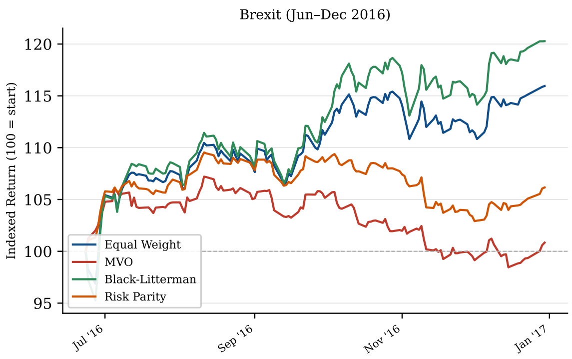

Figure 8. Regime: Brexit (Jun–Dec 2016).

Black-Litterman was the standout performer, gaining 20.25% over the period. Its view-adjusted allocations appear to have captured the post-referendum equity rebound effectively. Equal Weight followed at 15.95%, benefiting from its broad exposure across assets without concentration risk pulling it in the wrong direction. Risk Parity lagged considerably at 6.16%, its defensive, risk-equalising stance limiting upside participation during a strong directional move. MVO was essentially flat at 0.84%, suggesting its concentrated positioning was poorly aligned with the assets that benefited most from the shock.

COVID-19 (Feb–Sep 2020)

The COVID crash was sudden and indiscriminate, hitting all strategies hard in March 2020. What made a real difference is how quick and complete their recovery was. MVO recovered fastest and was the only strategy to finish the period in positive territory (+3.59%), suggesting its concentrated positioning happened to align with assets. Black-Litterman and Risk Parity ended the period in almost identical territory (−13.44% and −13.69% respectively), but went under different paths. Black-Litterman clawed its way back to breakeven by June 2020 before a second selloff dragged it back down, while Risk Parity simply never recovered, staying underwater for the entire period without any meaningful rebound. Equal Weight was the worst performer by far, ending down −25.84%. Its broad equity exposure amplified the drawdown, and the chart shows it had not recovered anywhere near its starting level by October 2020.

Rate Hikes (Jan–Dec 2022)

2022 was uniquely damaging: equities and bonds sold off together, breaking a well-established negative correlation between these assets. Equal Weight was the most resilient, ending the year roughly flat at +1.5%. Its lack of explicit bond exposure weighting meant it was less exposed to the bond sell-off than the more sophisticated models. Risk Parity (−4.34%) held up better than its structural weakness would suggest. While the equity-bond correlation breakdown directly undermines its core diversification logic, losses were contained to a level that outperformed both MVO and Black-Litterman. This suggests that the risk spreading across other assets have provided a cushion. MVO (−6.02%) fared worse than Risk Parity despite having no structural sensitivity to bond markets. Black-Litterman was the worst performer at −10.9%, its view-adjusted allocations, effective during Brexit, proving ill-suited to the speed and severity of the 2022 repricing.

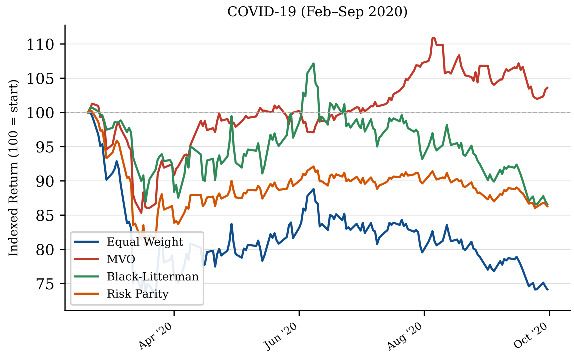

Figure 9. Regime: COVID-19 (Feb–Mar 2020).

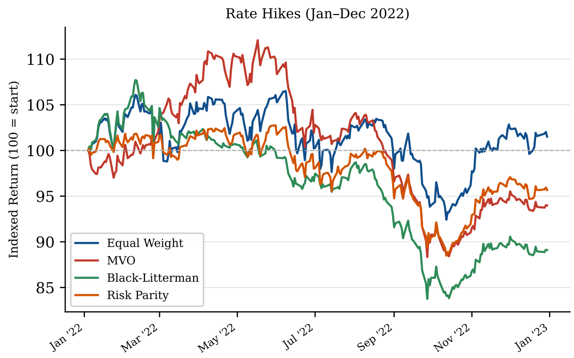

Figure 10. Regime: Rate Hikes (Jan–Dec 2022).

4.4 Portfolio Characteristics

This section looks under the hood at how each strategy allocates capital and how much it trades to get there, two practical considerations that matter significantly for real-world implementation.

Allocation Stability

The heatmaps reveal stark differences in how each strategy distributes and maintains its allocations over time.

MVO is visibly the most unstable. Allocations shift dramatically from period to period, with heavy concentration rotating between a handful of assets. Most notably IGLT.L and SGLD.L dominating at different points, while the majority of the portfolio sits near zero for extended stretches. This "all or nothing" behaviour is characteristic of mean-variance optimisation, which tends to produce corner solutions that look very different from one rebalancing period to the next. In practice, this makes the portfolio difficult to manage and exposes investors to sharp, unintended shifts in risk profile between rebalancing dates.

Black-Litterman appears to be smoother, with weights more evenly distributed across assets and less violent rotation between periods. IGLT.L consistently attracts a high allocation from 2018 onwards, which is worth noting as a point of concentration risk since a single asset commanding a persistently large share of the portfolio undermines the diversification the model is designed to provide.

Risk Parity is the most stable of the three, with allocations broadly uniform across assets for most of the period. This is consistent with its design objective of equalising risk contribution rather than capital. The periodic spikes in IGLT.L weight reflect its lower historical volatility, which mechanically results in a higher risk-budget allocation within the framework. For an investor, this predictability is attractive and reduces operational uncertainty.

Figure 11. Allocation heatmaps for MVO, Black-Litterman, and Risk Parity (assets on y-axis, time on x-axis) — Equal Weight omitted as trivially flat.

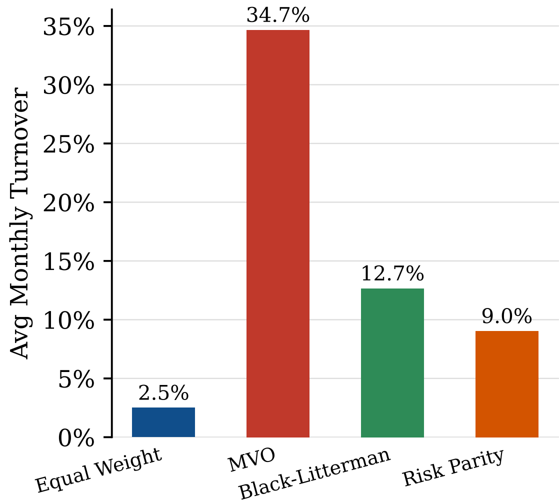

Figure 12. Average monthly turnover for all four strategies.

Table 7. Descriptive caption for your table.

Category	Metric 1	Metric 2	Metric 3
Group A	0.00	0.00	0.00
Group B	0.00	0.00	0.00
Group C	0.00	0.00	0.00

Note: Explain any specific details about the data in the table.

Turnover and the Cost of Complexity

High turnover is not a sign of a well-functioning strategy, it is a cost. The turnover chart shows MVO trading at an average monthly rate of 34.7%, by far the highest of the four: more than twice Black-Litterman (12.7%), nearly four times Risk Parity (9.0%), and almost fourteen times Equal Weight (2.5%).

Every rebalancing generates transaction costs such as brokerage fees and bid-ask spreads, which erode net returns. Given that MVO already delivered the weakest risk-adjusted performance on a gross basis, the addition of implementation costs would make its net performance even less competitive. MVO requires the greatest effort and cost to implement, yet provides the lowest return in comparison.

Equal Weight, by contrast, requires minimal intervention, yet outperforms all three more complex strategies over the full period. This is perhaps the most pointed finding of the section: **complexity does not necessarily pay.**

5 Discussion

Summarize the main conclusions of your research. Reiterate the key insights and their importance.

5.1 Why did 1/N perform well?

5.2 The risk parity puzzle

5.3 Practical Implications

5.4 Limitations

6 Conclusion

7 Declarations

Briefly describe the contribution of each author to the research and writing of the report. For example: "A.B. designed the research. C.D. collected the data. E.F. performed the analysis. All authors contributed to writing the report."

Future work could include:

- Exploring alternative methodologies or datasets.
- Addressing limitations of the current study.
- Expanding the research to a different market or asset class.

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