



QUANTITATIVE RESEARCH

A Deep Dive into Portfolio Optimisation

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Abstract

Background: Selecting an optimal portfolio allocation across a universe of assets is a central problem in investment management. A wide range of allocation strategies have been proposed, from naive diversification to theoretically grounded optimisation frameworks, yet their relative out-of-sample performance remains contested, particularly across varying market regimes. This study empirically compares four portfolio strategies to assess whether model sophistication translates into superior investment performance.

Methodology: This study implements and compares four portfolio strategies: equal weight, Markowitz mean-variance, Black-Litterman and Risk Parity applied to an 18-asset UK multi-asset portfolio comprising 15 FTSE 100 equities, a UK government bond ETF, a gold ETC and a broad commodities ETF, over the period January 2015 to December 2025. Strategies are evaluated using a monthly rebalancing framework and assessed across annualised return, volatility, Sharpe ratio, maximum drawdown and portfolio turnover.

Results: We find results broadly consistent with DeMiguel et al. (2009), with the naive benchmark proving difficult to displace on a risk-adjusted basis over the full sample period. The optimisation strategies exhibit meaningfully different risk profiles, however, suggesting that Sharpe ratio alone does not fully capture the practical trade-offs between approaches.

1 Introduction

1.1 The Portfolio Allocation Problem

The portfolio allocation problem is the fundamental question in investment management. It asks: How should capital be distributed across available investment opportunities to best achieve an investor's objectives?

The problem was intractable until 1952, when Harry Markowitz published *Portfolio Selection* in the *Journal of Finance* and turned it into a clean optimisation problem, and the “efficient frontier” gave investors a principled answer. Markowitz’s mean variance model minimises portfolio variance for a given level of expected return. It was revolutionary in the consideration of variance as the risk, standing as a hallmark of modern portfolio theory. In practice however, the model has proven difficult to implement reliably.

Michaud (1989) challenged this view directly, arguing that small estimation errors in expected returns produce wildly unstable portfolio recommendations.

In 2009, DeMiguel et al. delivered a provocative result: he found that even sophisticated strategies incorporating shrinkage estimators and Bayesian methods failed to consistently outperform 1/N out-of-sample, suggesting that estimation error dominates theoretical optimality for

typical portfolio problems. This tension between the theoretical elegance of optimisation and its empirical fragility motivates the present study.

1.2 Research Objectives

This study addresses three main questions:

- i. Do optimisation models outperform a naive 1/N benchmark out-of-sample, applied to a multi-asset UK portfolio over a ten-year period?
- ii. Which model offers the best risk-adjusted performance, as measured by the Sharpe ratio?
- iii. How does model performance vary across distinct market regimes: the Brexit referendum (2016), the COVID-19 crash (2020), and the 2022 rate-hiking cycle?

1.3 Structure of the Report

The remainder of this report is organised as follows: Section 2 reviews the theoretical foundations of each model and derives their key mathematical results. Section 3 describes the data, implementation choices, and backtesting framework. Section 4 presents

Key Points

- The equal-weight portfolio is robust because it does not rely on estimated parameters and is therefore unaffected by estimation error.
- The Efficient Frontier is the set of portfolios that deliver the minimum possible variance for each level of expected return.
- Empirical evidence suggests that many optimisation models fail to consistently outperform the equal-weight portfolio out of sample.

the empirical results, including full-period performance metrics and a breakdown by market regime. Section 5 discusses the findings and their practical implications. Section 6 concludes.

2 Literature and Theoretical Background

2.1 Notation

Some common notation that is used across this paper is the following:

Symbol	Description
N	Number of assets in the portfolio
$\mathbf{w} \in \mathbb{R}^N$	Vector of portfolio weights
$\mu \in \mathbb{R}^N$	Vector of expected returns
$\Sigma \in \mathbb{R}^{N \times N}$	Covariance matrix of asset returns
$\mathbf{1} \in \mathbb{R}^N$	Vector of ones
$r_f \in \mathbb{R}$	Risk-free rate

2.2 The equal weight portfolio

The equal-weight ($1/N$) portfolio assigns an identical weight of $1/N$ to each asset in the portfolio, holding N assets, regardless of any asset-specific characteristics such as return, volatility, or correlation. DeMiguel et al. (2009) tested 14 optimisation models across 7 datasets. None consistently outperformed the naive equal-weight portfolio:

$$w_i = \frac{1}{N} \quad \text{for } 1 \leq i \leq N \quad (2.1)$$

But why? When expected returns are estimated from historical data in order to determine optimal portfolio weights, the estimates improve as more data is collected, but they do so slowly.

Formally, the estimation error decays as

$$\hat{\mu} - \mu = O\left(\frac{1}{\sqrt{T}}\right) \quad (2.2)$$

where T is the number of observations.

Because the error shrinks at rate $\frac{1}{\sqrt{T}}$, halving the error requires four times as much data, meaning that with monthly data and realistic return distributions, approximately 500 years of observations are required for statistically reliable estimates, especially since these errors are amplified through the term $\Sigma^{-1}\hat{\mu}$.

The equal-weight portfolio sidesteps this problem entirely: by assigning a fixed weight to each asset, it requires no parameter estimation and is therefore immune to estimation error by construction.

The cost of this simplicity, however, is that the portfolio ignores all information about assets' risk and return characteristics, makes no attempt to diversify risk efficiently and cannot adapt to changing market conditions.

2.3 Markowitz's mean-variance optimisation

Markowitz (1952) formulated *Portfolio Selection* in 1952 as a constrained optimisation problem: minimise portfolio variance for a given

level of expected return. His work stands as a hallmark of modern portfolio theory.

Formally, the problem states:

$$\min_{\mathbf{w}} \hat{\sigma} = \mathbf{w}^\top \hat{\Sigma} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^\top \boldsymbol{\mu} = \mu_p \quad ; \quad \mathbf{w}^\top \mathbf{1} = 1 \quad (2.3)$$

with μ_p as the scalar target portfolio return.

The first constraint, also known as the return constraint, fixes the portfolio to a specific point at the target level μ_p , pinning the solution to a specific point on the "Efficient Frontier". By varying μ_p across all feasible values, the complete set of minimum-variance portfolios – the Efficient Frontier – is traced out. The second constraint, i.e. the budget constraint, ensures the weights represent a valid allocation of wealth, hence that the sum of all weights is always equal to 1.

It is important to note that no domain-restriction condition is imposed on the budget constraint, each w_i may take any real value, in particular, negative ones. Practically speaking, a negative weight corresponds to a short position in the associated asset.

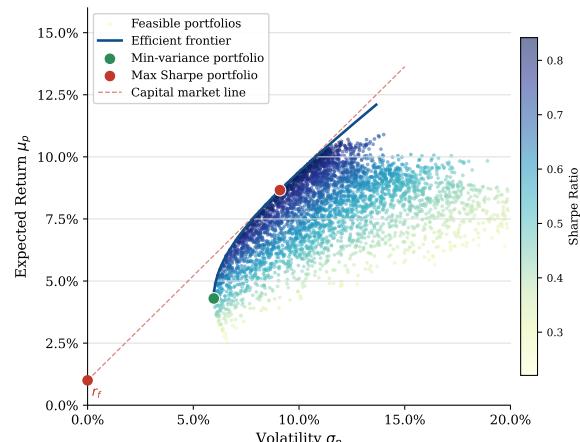


Figure 1. Efficient Frontier estimated using sample covariance

Because the objective function is quadratic and the constraints are linear equalities, we cannot solve the problem by direct substitution (two constraints, n unknowns). We therefore use the method of Lagrange multipliers. The Lagrangian formulation is:

$$\mathcal{L}(\mathbf{w}, \lambda, \gamma) = \mathbf{w}^\top \Sigma \mathbf{w} - \lambda (\mathbf{w}^\top \boldsymbol{\mu} - \mu_p) - \gamma (\mathbf{1}^\top \mathbf{w} - 1) \quad (2.4)$$

where λ and γ are the Lagrange multipliers associated with the return and budget constraints, respectively. Intuitively, λ captures the marginal cost of requiring a higher return (i.e. how much additional variance must be accepted per unit increase in μ_p), and γ enforces full investment.

Taking the first-order condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = 2\Sigma \mathbf{w} - \lambda \boldsymbol{\mu} - \gamma \mathbf{1} = \mathbf{0} \quad (2.5)$$

Key Points

- Black-Litterman uses market-cap weights as a neutral starting point for expected returns.
- Subjective investor views are blended with the equilibrium prior through a Bayesian update.
- The optimisation step in Black-Litterman is the same as MVO with μ_{BL} replacing the sample mean.

Solving gives:

$$\mathbf{w}_p = \frac{\lambda}{2} \Sigma^{-1} \boldsymbol{\mu} + \frac{\gamma}{2} \Sigma^{-1} \mathbf{1} \quad (2.6)$$

Imposing the budget and return constraints leads to the following two-fund theorem: all efficient portfolios can be expressed as a linear combination of any two distinct efficient portfolios.

$$\mathbf{w}_A = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}} \quad \mathbf{w}_B = \frac{\Sigma^{-1} \boldsymbol{\mu}}{\mathbf{1}^\top \Sigma^{-1} \boldsymbol{\mu}} \quad (2.7)$$

Note that \mathbf{w}_A is the *global minimum variance* (GMV) portfolio, the leftmost point on the efficient frontier, obtained by dropping the return constraint and minimising variance subject only to full investment:

$$\mathbf{w}_{GMV} = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}} \quad (2.8)$$

And the *maximum Sharpe ratio* portfolio is:

$$\mathbf{w}_{MSR} = \frac{\Sigma^{-1} (\boldsymbol{\mu} - r_f \mathbf{1})}{\mathbf{1}^\top \Sigma^{-1} (\boldsymbol{\mu} - r_f \mathbf{1})} \quad (2.9)$$

Despite its theoretical elegance, Markowitz optimisation suffers from a well-documented problem. As mentioned above, the solution depends critically on $\boldsymbol{\mu}$ and Σ , both estimated from historical data.

Small errors in $\boldsymbol{\mu}$ can cause large shifts in optimal weights. Michaud (1989) characterised the optimiser as an "error maximiser": it systematically overweights assets with overestimated returns and underestimates variances.

2.4 Black-Litterman and its Bayesian approach

The Black-Litterman (BL) model was developed by Fischer Black and Robert Litterman at Goldman Sachs in the early 1990s (Black and Litterman; 1991, 1992) to address the instability of Mean-Variance Optimisation.

The BL model addresses two shortcomings of classic mean-variance optimisation:

- What expected returns should be used?

Rather than relying on historical data, BL uses returns implied by current market-cap weights as the neutral starting point.

- How should the views be incorporated?

BL offers a formal Bayesian framework for blending subjective investor views with this equilibrium prior, weighting each source inversely by its uncertainty.

Step 1. Equilibrium Returns

The model takes as its starting point the equilibrium expected returns implied by the market portfolio, i.e. the portfolio where each asset is held in proportion to its market capitalisation. Assuming the market portfolio is mean-variance efficient, the equilibrium excess returns are:

$$\boldsymbol{\Pi} = \delta \Sigma \mathbf{w}_m \quad (2.10)$$

where \mathbf{w}_m is the market-cap weight vector, Σ is the covariance matrix

of asset returns and δ is the risk-aversion coefficient

$$\delta = \frac{r_m - r_f}{\sigma_m^2} \quad (2.11)$$

with r_m the return of the market portfolio and σ^2 its variance.

Step 2. Express Views

A view can be absolute or relative, for example "asset A will return 5%" or "asset A will outperform asset B by 2%". Each view has two components: the expected return itself and a confidence level attached to it.

Formally, k views are expressed through the following system:

$$P \boldsymbol{\mu} = Q + \boldsymbol{\varepsilon} \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \Omega) \quad (2.12)$$

where

- $P \in \mathbb{R}^{k \times n}$: the *pick matrix* (which assets are involved)
- $\boldsymbol{\mu} \in \mathbb{R}^k$: the expected return
- $Q \in \mathbb{R}^k$: the expected view returns
- $\Omega \in \mathbb{R}^{k \times k}$: view uncertainty (diagonal)
- $\boldsymbol{\varepsilon}$ (error-term): the deviation of true returns from the stated views, assumed to be normally distributed with mean zero

Assets are ranked by their return over the previous 12 months, excluding the most recent month to eliminate short-term reversals effects. The highest-return tercile forms the winner portfolio, while the lowest-return decile constitutes the loser portfolio.

The view is expressed as:

$$\mathbf{p}_k^\top \boldsymbol{\mu} = q_k \quad (2.13)$$

p_k assigns equal positive weights to winner assets and equal negative weights to loser assets, forming a long-short momentum view. The expected return spread q_k is estimated over a rolling window at each rebalancing date as asset rankings change.

Step 3. Bayesian Update

The prior on expected returns is:

$$\boldsymbol{\mu} \sim \mathcal{N}(\boldsymbol{\Pi}, \tau \Sigma) \quad (2.14)$$

This encodes the belief that expected returns are centred on the equilibrium vector $\boldsymbol{\Pi}$, with uncertainty scaled by τ . Combining this prior with the view system via Bayes' theorem yields the posterior distribution over expected returns:

$$\boldsymbol{\mu} | Q \sim \mathcal{N}(\boldsymbol{\mu}_{BL}, \Sigma_{BL}) \quad (2.15)$$

where the posterior mean is:

$$\boldsymbol{\mu}_{BL} = [(\tau \Sigma)^{-1} + P^\top \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \boldsymbol{\Pi} + P^\top \Omega^{-1} Q] \quad (2.16)$$

The posterior mean $\boldsymbol{\mu}_{BL}$ is a precision-weighted average of the equilibrium prior $\boldsymbol{\Pi}$ and the investor's views Q , with each source weighted inversely by its uncertainty. When views are diffuse (large Ω), $\boldsymbol{\mu}_{BL}$ remains close to $\boldsymbol{\Pi}$. If the views are held with high confidence (small Ω), $\boldsymbol{\mu}_{BL}$ tilts toward Q .

Key Points

- xx
- xx
- xx

This blending property is central to the appeal of the BL framework: Rather than overriding the equilibrium prior, views are incorporated gradually and in proportion to their confidence. This acts as a natural regulariser, because the prior pulls weights toward the market portfolio, the model is far less prone to the concentrated, unstable allocations that plague classical MVO.

Step 4. Portfolio Optimisation

The optimal portfolio is obtained by applying the same Sharpe-maximising objective derived in Section 2.3 (equation 2.9), substituting μ_{BL} in place of the sample mean μ .

2.5 Risk Parity

The modern Risk Parity framework was popularised by Ray Dalio at Bridgewater Associates and underpins the All Weather strategy introduced in the 1990s. The approach was subsequently formalised by Maillard et al. (2010), who derived the mathematical conditions for equal risk contribution portfolios.

The central motivation departs entirely from the MVO and BL frameworks: rather than optimising expected return for a given level of risk, Risk Parity ignores return estimates altogether and instead focuses on how risk is distributed across the portfolio.

Risk Parity addresses a structural limitation shared by both MVO and the equal-weight portfolio:

- i. In MVO and BL, allocations depend on estimated expected returns, which are noisy and prone to estimation error.
- ii. In equal-weight portfolios, capital is diversified equally but risk is not. Assets with high volatility dominate total portfolio risk despite receiving the same capital allocation.

Risk Decomposition

Portfolio volatility $\sigma_p = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}$ is a homogeneous function of degree 1 in \mathbf{w} . By Euler's homogeneous function theorem, it admits the following decomposition:

$$\sigma_p = \sum_{i=1}^N w_i \frac{\partial \sigma_p}{\partial w_i} \quad (2.17)$$

The *marginal risk contribution* (MRC) of asset i is:

$$MRC_i = \frac{\partial \sigma_p}{\partial w_i} = \frac{(\Sigma \mathbf{w})_i}{\sigma_p} \quad (2.18)$$

and the *total risk contribution* (RC) of asset i is:

$$RC_i = w_i \cdot MRC_i = \frac{w_i (\Sigma \mathbf{w})_i}{\sigma_p} \quad (2.19)$$

with the useful property that $\sum_{i=1}^N RC_i = \sigma_p$.

The Risk Parity Condition

The portfolio is said to be *risk-balanced* when every asset contributes equally to total portfolio volatility:

$$RC_i = \frac{\sigma_p}{N} \quad \forall i \quad (2.20)$$

Since σ_p appears on both sides, this reduces to the equivalent condition:

$$w_i (\Sigma \mathbf{w})_i = c \quad \forall i \quad (2.21)$$

for some constant $c > 0$, subject to $\mathbf{1}^\top \mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$. This system has no closed-form solution in general and is solved numerically; the implementation details are described in Section 3.

Special Case: Uncorrelated Assets

When Σ is diagonal, the risk parity condition reduces to an analytic solution. Equal risk contribution then requires each asset's weight to be proportional to the inverse of its volatility:

$$w_i^{RP} = \frac{1/\sigma_i}{\sum_{j=1}^N 1/\sigma_j} \quad (2.22)$$

This inverse-volatility weighting is the closed-form limit of risk parity, and also serves as the natural initialisation point for the numerical solver in the general case.

Despite its appeal, Risk Parity has a well-known structural consequence: it mechanically overweights low-volatility assets. In a portfolio containing both equities and government bonds, bonds receive substantially larger capital allocations precisely because their volatility is lower, which raises the question of whether the resulting portfolio is truly diversified in an economic sense or merely in a statistical one.

3 Methodology

3.1 Data

The dataset contains daily adjusted closing prices for an 18-asset universe over a 10-year period, spanning from January 1st 2015, to December 31st 2025. The universe is made to reflect a representative UK-centric multi-asset portfolio. It consists of 15 large-cap equities across various sectors, which include Financials, Energy, and healthcare, as well as three diversifying exchange-traded products: UK Government Gilts (IGLT.L), Gold (SGLD.L), and an extensive Commodities basket (WCOG.L). All of this pricing data is programmatically sourced via Yahoo Finance.

Standard closing prices were not chosen, instead adjusted closing prices were explicitly used because they account for corporate actions such as stock splits, rights issues, and dividend distributions. This is a crucial prerequisite for calculating precise, time-additive daily log returns. Moreover, the use of raw prices would distort the return metrics during dividend ex-dates, so adjusted prices are used since they accurately reflect the true total return an investor would capture over a decade-long horizon encompassing multiple macroeconomic regimes.

3.2 Model Implementation

While Section 2 establishes the theoretical framework of portfolio optimisation, this section details the practical parameters imposed to translate these theories into feasible algorithmic strategies. Rather than relying on generic, pre-packaged optimisation libraries, the architecture utilises a custom Object-Oriented Programming paradigm built on NumPy

and Pandas. An abstract base class (`BaseStrategy`) is defined to ensure modularity across models, utilising a rolling 252-trading-day estimation window to dynamically generate covariance (Σ) and expected return (μ) inputs.

To mitigate the “error maximisation” fragility highlighted in the literature review, our solvers mathematically enforce several structural constraints. In the case of Mean–Variance Optimisation (MVO), we derive the maximum Sharpe ratio portfolio via an iterative subspace method. Numerical stability and positive definiteness within highly correlated asset matrices are guaranteed by applying a Tikhonov regularisation penalty (a diagonal loading of 1×10^{-8}) to the covariance matrix. Concurrently, the algorithm strictly adheres to a long-only constraint ($w_i \geq 0$) by systematically pruning assets with negative target weights from the active matrix, re-solving iteratively until a strictly non-negative global optimum is achieved.

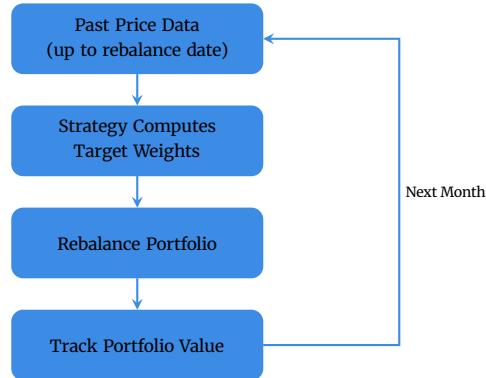
Moving to the Risk Parity framework, the equal risk contribution portfolio requires a distinct numerical approach. Here, a custom Newton–Raphson iteration serves as the root-finding algorithm. By utilising an analytically derived Jacobian matrix initialised with an inverse-volatility guess, this method ensures both rapid and precise convergence.

The Black–Litterman model likewise requires a systematic methodology to generate its subjective views (Q). We construct a cross-sectional 252-day price momentum signal, directly motivated by the momentum effect documented by Jegadeesh and Titman (1993). Because their research demonstrates that stocks with high returns over the prior 3 to 12 months continue to generate abnormal outperformance of approximately 1% per month, this empirically grounded signal provides a robust foundation for our directional views. These historical return signals are subsequently blended with the implied equilibrium returns via Bayesian updating to yield the final posterior expected returns.

3.3 Backtesting Framework

The strategies are evaluated using a walk-forward backtesting engine to ensure zero look-ahead bias. At any given rebalancing date t , the optimiser relies strictly on the historical data available in the preceding 252-day window. A starting capital base of £100,000 is initialised in the portfolio and deployed immediately on the first available trading day. Thereafter, the portfolio undergoes rebalancing on the final trading day of each calendar month.

Critically, the simulation relies on several foundational assumptions. We assume perfect asset divisibility, allowing the allocation of fractional shares to match exact target weights. In addition, we assume immediate execution at the end-of-day adjusted close price on the rebalancing date. To simulate real-world implementation frictions and measure the impact of portfolio turnover, a flat transaction cost of 5 basis points (0.05%) is deducted per unit of traded value during each monthly rebalance. Any capital not allocated to the active asset matrix is held as an uninvested cash balance yielding 0%. Furthermore, the simulation does not explicitly model non-linear market impact or bid–ask spread expansion during liquidity crises. It is assumed that the 18 selected large-cap assets possess sufficient market depth to absorb the hypothetical order flow without inducing severe execution slippage.



3.4 Performance Metrics

All metrics we're using (Return, Sharpe, Sortino, Volatility, CAGR, Max DD, VaR, CVaR)

4 Results

This section presents the key findings from your analyses and interprets their significance. Use figures and tables to present your results clearly.

Present your main results. This could be in the form of tables summarizing statistical outputs, or charts showing trends and relationships.

4.1 Main Result: Performance Comparison

Discuss what your results mean. How do they relate to your initial research question? Are they consistent with existing literature? What are the implications of your findings?

4.2 Risk–Return Trade-off

4.3 Regime Dependence

4.4 Portfolio Characteristics

5 Discussion

Summarize the main conclusions of your research. Reiterate the key insights and their importance.

5.1 Why did 1/N perform well?

5.2 The risk parity puzzle

5.3 Practical Implications

5.4 Limitations

6 Conclusion

7 Declarations

Briefly describe the contribution of each author to the research and writing of the report. For example: "A.B. designed the research. C.D. collected the data. E.F. performed the analysis. All authors contributed to writing the report."

Future work could include:

- Exploring alternative methodologies or datasets.
- Addressing limitations of the current study.
- Expanding the research to a different market or asset class.

Table 1. Descriptive caption for your table.

Category	Metric 1	Metric 2	Metric 3
Group A	0.00	0.00	0.00
Group B	0.00	0.00	0.00
Group C	0.00	0.00	0.00

Note: Explain any specific details about the data in the table.

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