First order scheme:

$$F(u_i, u_j, n_{ij}) = [V_n]^+ u_i + [V_n]^- u_j$$

Second order scheme:

$$F(u_{ij}, u_{ji}, n_{ij}) = [V_n]^+ u_{ij} + [V_n]^- u_{ji}$$

Neighborhood:
<NOME>

$$\tilde{u}_i(x, y) = ax + by + c$$

minimar E: for three neighbors + self:

$$E(a, b, c) = [\tilde{u}_i(x_{j_0}, y_{j_0}) - u_{j_0}^2]^2 + [\tilde{u}_i(x_{j_1}, y_{j_1}) - u_{j_1}^2]^2$$
$$+ [\tilde{u}_i(x_{j_2}, y_{j_2}) - u_{j_2}^2]^2 + [\tilde{u}_i(x_{j_3}, y_{j_3}) - u_{j_3}^2]^2$$

Objective:

$$min\, E(a, b, c) \Rightarrow \nabla E = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} \partial_a E = 0 \\ \partial_b E = 0 \\ \partial_c E = 0 \end{cases}$$

$$A =$$
$$\begin{bmatrix} (x_{j_0}^2 + x_{j_1}^2 + x_{j_2}^2 + x_{j_3}^2) & (x_{j_0}y_{j_0} + x_{j_1}y_{j_1} + x_{j_2}y_{j_2} + x_{j_3}y_{j_3}) & (x_{j_0} + x_{j_1} + x_{j_2} + x_{j_3}) \\ (x_{j_0}y_{j_0} + x_{j_1}y_{j_1} + x_{j_2}y_{j_2} + x_{j_3}y_{j_3}) & (y_{j_0}^2 + y_{j_1}^2 + y_{j_2}^2 + y_{j_3}^2) & (y_{j_0} + y_{j_1} + y_{j_2} + y_{j_3}) \\ (x_{j_0} + x_{j_1} + x_{j_2} + x_{j_3}) & (y_{j_0} + y_{j_1} + y_{j_2} + y_{j_3}) & 4 \end{bmatrix}$$