Introduction to Discrete mathematical structure

1) It developos your mathematical thinking
2) It improves your problem solving ability.
3) Discrete mathematics is important in subjects like Network security, compiler design. operating system DBMS e.t.c.
4) many problems can be solved by using discrete mathematics.

I. Some problems on finding the shortest path from your home to your friend home.

II. Draw a graph

III. How many different combination of passworols are possible eight alpha numeric characters.

IV Encript a message & deliver it to your friend any you ~~didn~~ don't want any body to read that message excepot your friend.

Statement :-

A proportion (or) statement is a declarative sentense that is either true (or) false but not both.

Ex :- 1) $8+3=11$

2) pairs is in England

3) $4-x=8$

The sentence 1 & 2 are statements the first is true and the second is False. The third statement is a declarative sentence but not a statement since it is true (or) False depends on the value of x.

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Connectives:

The words & symbols used to form compound proposition are called connectives

(or)

Any word or expression used to ~~from compound~~ connect two or more statement is called as connectives.

1) conjunction ($\wedge$)
2) Disjunction ($\vee$)
3) conditional ($\rightarrow$)
4) Bi-conditional ($\rightleftarrows$)
5) Negation ($\sim$)

1. Conjunction:

if P & Q are two students then conjunction two conjunction P and Q is the compound statement. The compound statement $P \wedge Q$ is True when both P & Q are true otherwise it is false.

Truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Disjunction:

disjunction ~~conjunction~~ if P and Q are two statements then P and Q is the compound statement. The compound statement. $P \vee Q$ is true of P (or) Q is ~~True~~ True

if false when both P & Q are false.

Truth table

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

3. Conditional (Implication):- The operation $P \rightarrow Q$ as a truth value F when p has the truth T and Q has the truth value F. otherwise its truth value T.

Truth table

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

4. Bi-conditional ($\rightleftarrows$):

The truth value of the statement $P \rightleftarrows Q$ is T whenever both P, Q have identified Truth values.

Truth table

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	T
F	F	T

5. Negation (~):

if p is any proposition the negation of p denoted by ~p and read as not p is a proposition which is false when p is True and True when p is False.

Truth Table

P	$\sim P$
T	F
F	T

=> construct a Truth table for $(P \vee Q) \vee \sim p$

P	Q	$P \vee Q$	$\sim p$	$(P \vee Q) \vee \sim p$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

→ $Q \wedge (P \rightarrow Q)$

P	Q	$P \rightarrow Q$	$Q \wedge (P \rightarrow Q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	F

=> construct a Truth Table for the $((P \vee Q) \wedge \sim p) \rightarrow Q$

P	Q	$P \vee Q$	$\sim p$	$((p \vee Q) \wedge \sim p)$	$((P \vee Q) \wedge \sim p) \rightarrow Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Imp

Tautology:

A compound proposition that is always true for all possible truth values of its variables or in other words contain only T in the last coloumn of its truth table is called a Tautology.

*Show tho

P	
T	
T	
F	
F	

Hence

*since all table of tautology

*Show tho

P	
T	
T	
F	
F	

*Show th

P	
T	
T	
F	
F	

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* Show that $P \to (P \vee Q)$ is a tautology.

P	Q	$P \vee Q$	$P \to (P \vee Q)$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

Hence $P \to (P \vee Q)$ is a tautology

* since all the entries in the last coloumn of truth table of $P \to (P \vee Q)$ are True Hence $P \to (P \vee Q)$ is a tautology.

* Show that $(P \to Q) \rightleftarrows (\sim P) \vee Q$ is a tautology

P	Q	$P \to Q$	$\sim P$	$\sim P \vee Q$	$(P \to Q) \rightleftarrows (\sim P) \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	T	T	T	T

* Show that $P \vee \sim (P \wedge Q)$ is a tautology.

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$	$P \vee (\sim P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

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* show that $[(P \to Q) \wedge (Q \to R)] \to [P \to R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$	$(P \to R)$	$\chi(P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

* show that $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ is a Tautology.

P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \to Q$	$P \to R$	$(P \to Q) \to (P \to R)$	χ
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T	T
T	F	T	T	T	F	F	T	T
F	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T

contradiction:

A compound proposition i.e; always false for all possible Truth values of its variable or in other words contain only F in the last colourm of its Truth table is called a contradiction

show that $(\sim Q \wedge P) \wedge Q$ is a contradiction

P	Q	$\sim Q$	$\sim Q \wedge P$	$(\sim Q \wedge P) \wedge Q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

Equivalance:-

if any two compound statements obtained have the same truth values for all 2^n components They are said to be logically equivalent.

if M, N are two compound statements and if the truth values in the last coloumn of M are last coloumn of N then N is said to be logically equivalent to 'N'

Notation $\Leftrightarrow$ or $\equiv$

Eg:- prove that $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

p	q	r	$(p \wedge q)$	$(p \wedge q) \wedge r$	$(q \wedge r)$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	T	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

$\therefore (p \wedge q) \wedge r = p \wedge (q \wedge r)$

a) show that $(P \to Q) \to R) \to (P \to Q) \to (P \to R))$ is a universally valid Formula.

P	Q	R	$P \to Q$	$(P \to Q) \to R$	$P \to R$	$(P \to Q) \to (P \to R$	$(P \to Q) \to R) \to (P \to Q) \to (P \to R))$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	T	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	T	T	T

b) show that $((P \to Q) \to R) \to ((p \to Q)) \to (P \to R)$ is a Tautology

$$P \to (q \vee r) = (P \to Q) \vee (P \to r)$$

p	q	r	$q \vee r$	$p \to (q \vee r)$	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$
T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	T	T	T	T	T
T	T	F	T	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T

$P \to (q \vee r) = (P \to q) \vee (P \to r)$ is equivalence

connectives truth table :- $\uparrow$ NAND $\downarrow$ NOR $\oplus$ XOR

P	Q	P∨Q	P∧Q	P→Q	P↔Q	P⊕Q	P↑Q	P⊽Q	~P
T	T	T	T	T	T	F	F	F	F
T	F	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	F	F	T	T	T	T	T	T

contigency :-

A proportion that is neither a tautology nor a contradiction is called a contigency.

Eg :- $(p \vee q) \rightarrow p$

p	q	p∨q	(p∨q)→p
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

Since all the entries in the last coloumn of the truth table are True and False.

$(p \vee q) \rightarrow p$ is contigency.

Equivalence of formulas

1.) Identify laws @ $P \wedge T = P$ b) $P \vee F \equiv P$

2.) Domination laws @ $P \vee T = T$ b) $P \wedge F \equiv F$

3.) Idempotent law @ $P \vee P = P$ b) $P \wedge P \equiv P$

4.) Double Negation: $\sim(\sim P) \equiv P$

5) commulative laws a) $P \vee q = q \vee P$ b) $P \wedge P = q \wedge P$

6) Associative laws a) $(P \vee q) \vee r = P \vee (q \vee r)$ b) $(P \wedge q) \wedge r = P \wedge (q \wedge r)$

7) Distribution laws : a) $P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$

b) $P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

8) Demorgan's law : a) $\sim(P \wedge q) \equiv \sim P \vee \sim Q$

b) $\sim(P \vee q) \equiv \sim P \wedge \sim q$

9) absorption law ; a) $P \vee (P \wedge q) \equiv P$

b) $P \wedge (P \vee q) = P$

10) Negation laws ; a) $P \vee \sim P \equiv T$

b) $P \wedge \sim P \equiv F$

In this equivalence T denotes the compound proposition i.e always True and False denotes the compound proposition i.e always False.

=> use

p	q
T	T
T	T
T	F
T	F
F	T
F	T
F	F
F	F

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⇒ use the Truth table to prove the distribution

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$\therefore p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Replacement process:

Consider the formula $A: p \to (q \to r)$ the formula $q \to r$ is a part of the formula A. If we replace $q \to r$ by equivalent formula $\sim q \vee r$ in A, we can another formula $B: p \to (\sim q \vee r)$

Hence we can easily verify that the formulas A and B are equivalent to each other. This process of obtaining B from A is known as the replacement process.

Ex:- show the following formula is a Tautology without constructing Truth table

$p \wedge (p \to q)) \to q$

$\Rightarrow p \wedge (\sim p \vee q)) \to q$ $[\because p \to q \equiv \sim p \vee q]$

$\Rightarrow ((p \wedge \sim p) \vee (p \wedge q)) \to q$ $[\because p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)]$

$\Rightarrow ((F) \vee (p \wedge q)) \to q$ $[\because p \wedge \sim p \equiv F]$

$\Rightarrow (p \wedge q) \to q$ $[\because p \vee F \equiv p]$

$\Rightarrow \sim(P \wedge Q) \vee Q$ $[\because P \vee F \equiv P]$

$\Rightarrow \sim P \vee \sim Q \vee Q$ $[\because \sim P \vee T = T]$

$\Rightarrow \sim P \vee T$

$\Rightarrow T$

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Normal Forms

The Problem of finding weather a given statement is Tautology or contradiction or satisfiable in finite no. steps is called a decision problem. for, decision Problem construction of Truth table may not be practical always. we can alternative procedure known as the reduction to normal form.

These are four types of normal forms

1) DNF (Disjunctive Normal Form)
2) CNF (conjuctive Normal form)
3) PDNF (principle Disjunctive Normal Form)
4) PCNF (principle conjuctive Normal form)

1. DNF

A Formula which is equivalent to a given formula and which consists of sum of elementary product is called DNF

ex :- $P(\wedge Q) \vee (Q \wedge \sim R) \vee (\sim P \wedge \sim Q)$

=> The product of variables & their negation is called an elementary sum.

ex-①

$P \wedge (P \to Q)$

$\Rightarrow P \wedge (\sim P \vee Q)$ $\quad [\because P \to Q \equiv \sim P \vee Q]$

$\Rightarrow (P \wedge \sim P) \vee (P \wedge Q)$

procedure

① Remove all $\to$, $\rightleftarrows$ by an equivalent expression containing the connectives $\wedge, \vee, \sim$ only.

2) Eliminate $\sim$ before sum & product by using double negation or by using demorgan's law.

Apply the distribution law untill a sum of elementary product is obtained

ex:- $\sim(P \to (q \wedge r))$

$\Rightarrow \sim(\sim P \vee (q \wedge r))$ $\quad [\because P \to Q \equiv \sim P \vee Q]$

$\Rightarrow (P \wedge \sim(q \wedge r))$

$\Rightarrow (P \wedge \sim q) \vee (P \wedge \sim r)$

② $(P \to q) \to (\sim r \wedge q)$

$\sim(P \to q) \vee (\sim r \wedge q)$

$[\sim(\sim P \vee q)] \vee [\sim r \wedge q)$

$(P \wedge \sim q) \vee (\sim r \wedge q)$

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CNF (conjuctive normal Form):-

A Formula which is equivalent to a given formula and which consist of a product of elementary sums is called a conjuctiv n. formula of a given formula

Ex:- $(P \vee q) \wedge (Q \vee R) \wedge (\sim P \vee Q)$

* CNF is not unique
* if every elementary sum in CNF is tautology then the given formula is also totology

Ex:- $P \wedge (P \rightarrow Q)$

$\Rightarrow P \wedge (\sim P \vee Q)$. $[P \rightarrow Q \equiv \sim P \vee Q]$

$\Rightarrow (P \vee P) \wedge (\sim P \vee Q)$

Ex:- $\sim (P \rightarrow (Q \wedge R)$

$= \sim (\sim P \vee (Q \wedge R))$

$= P \wedge \sim (Q \wedge R)$

$= P \wedge (\sim Q \vee \sim R)$

$= (P \vee P) \wedge (\sim Q \vee \sim R)$

$\sim (P \rightarrow (Q \wedge R)$

$= \sim (\sim P \vee (Q \wedge R)$

$= \sim (\sim P \wedge (Q \wedge R)$

$= \sim (\sim P \wedge \sim (Q \wedge R)$

$= P \wedge (\sim Q \vee \sim R)$

$= (P \wedge \sim Q) \vee (P \wedge \sim R)$

$P \rightarrow (Q \wedge R) \wedge (\sim P \rightarrow (\sim Q \wedge \sim R))$

Sol:-

given that;

$P \rightarrow (Q \wedge R) \wedge (\sim P \rightarrow (\sim Q \wedge \sim R))$

$= \sim P \vee Q) \wedge (\sim P \vee R) \wedge (P \vee \sim Q) \wedge (P \vee \sim R)$

PDNF (F

=> principal
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This i
Canonical

let p a
$P \wedge q$, $\sim P \wedge$
P and Q
are 2^n
Truth tab

F

select the
for the s
variables

* $P \rightarrow q <=$

Ex: $\sim (P$

P	q
T	T
T	F
F	T
F	F

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PDNF (principle Disjunctive Normal form):-

=> principle disjuctive Normal form of a given can be defined as ar equivalent formula consisting of disjuction of "min-terms" only.

This is also called the sum of products cononical Canonical form.

let p and Q are two statements variable then $P \wedge q$, $\sim P \wedge q$, $P \wedge \sim q$, $\sim P \wedge \sim q$ are called minterms P and Q. The number of minterms in n-variables are 2^n

Truth table:-

For every truth value t of the given formula, select the minterms which is also has the value 'T' for the same combination of truth values of variables

* $P \rightarrow q \Leftrightarrow (P \wedge q) \vee (\sim P \wedge q) \vee (\sim P \wedge \sim q))$

Ex: $\sim (P \wedge q)$

P	q	$P \wedge q$	$\sim P \wedge q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

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PCNF (principle conjective Normal Form) :-

PCNF of a given formula can be defined as an equivalent formula consists of conjective of max terms only. This is also called the product of sum of canonical form.

Two variables P and q There are 2^2 max terms

$p \vee q$, $\sim p \vee q$, $p \vee \sim q$, $\sim p \vee \sim p$

EX:- $p \wedge q$

$p \wedge q \Rightarrow (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

EX:- $(\sim p \rightarrow r) \wedge (q \rightarrow p)$

p	q	r	$\sim p$	$\sim p \rightarrow r$	$q \rightarrow p$	$(\sim p \rightarrow r) \wedge (q \rightarrow p)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

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$\Leftrightarrow (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee \sim r)$

Theory :-

consider a set of propositions $P_1, P_2, P_3, \ldots P_n$ and proposition q. Then a compound proposition of the form $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots P_n) \rightarrow q$ is called an argument here $P_1, P_2, P_3 \ldots P_n$ are called the premises of the argument and q is called a conclusion of the argument.

Ex :- It is a practise to write the above argument.

$$\begin{array}{l} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \hline \therefore q \end{array}$$

an argument with premises P_1 and $P_2, P_3 \ldots$ and conclusion q is said to valid is whenever each of premises True and the conclusion also true otherwise the argument is invalid.

Ex :- Determine wether the following statement is valid of invalid

$$\begin{array}{l} H_1 : \sim q \\ H_2 : p \rightarrow q \\ \hline C : \sim p \end{array}$$

Truth table

p	q	$\sim q$	$p \rightarrow q$	$\sim p$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

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We see in the above table there is only one case in which both premises are true, namely the last case and in this case the conclusion is true and hence the argument is valid

Ex: $p \to (q \vee \sim r)$

$q \to (p \wedge r)$

$\therefore p \to r$

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$(p \to q \vee \sim r)$	$(q \to p \wedge r)$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	T
T	F	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

=> Tautologycal implication :-

A statement A is said to be Tautologycally imp a statement B if and only if $A \to B$ is a Tautology we shall denote it by $A \Rightarrow B$ which is read as 'A implies B'

Note :- is not connective

$A \Rightarrow B$ is a statement formula

Ex :- $(p \to q) \Rightarrow (\sim q \to \sim p)$

p	q	$\sim p$	$\sim q$	$(p \to q)$	$(\sim q \to \sim p)$	$(p \to q) \Rightarrow (\sim q \to \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

∴ All th
true then
show that
(A ⊕ B)
(A ⊕ B)

A	B	A
T	T	
T	F	
F	T	
F	F	

(~P/

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	
F	F	
F	F	

(~

PCNF

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∴ All the entries in the last coloumn are true then $(P \to q) \Rightarrow (\sim q \to \sim p)$ is a tautology.

Show that :-

$(A \oplus B) \vee A\ (A/B)$ is equivalent to $A \wedge B$

(or)

$(A \oplus B) \vee (A \downarrow B) \equiv A \uparrow B$

A	B	$A \oplus B$	$A \downarrow B$	$(A \oplus B) \vee (A \downarrow B)$	$A \uparrow B$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	T	T	T

$(\sim P \wedge R) \wedge (Q \leftrightarrow D)$ → PDNF, PCNF

P	Q	R	$\sim P$	$\sim P \wedge R$	$Q \leftrightarrow P$	$(\sim P \wedge R) \wedge (Q \leftrightarrow R)$
T	T	T	F	F	T	F
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

$(\sim P \wedge R) \wedge (Q \leftrightarrow P)$ $(\sim P \wedge \sim q \wedge r) \to$ PDNF

PCNF :-

$(P \vee q \vee r) \wedge (P \vee q \vee \sim r) \wedge (P \vee \sim q \vee r) \wedge (P \vee \sim q \vee \sim r) \wedge$
$(\sim P \vee q \vee r) \wedge (\sim P \vee q \vee \sim r) \wedge (\sim P \vee \sim q \vee \sim r)$.

Rules of Inference (or) I^{nd} method.

The following are two important rules for

a. Rule P:- Formulas may be introduced at any point in the derivation.

Rule T :- A formula S may be introduced in derivation if S^5 is a Tautologically implied by 1 or more of the preceding formulas in the derivation.

Ex:- Demonstrate that R is a valid from the premises

Ex:- $P \to Q$, $Q \to R$, P.

R

{1}	(1)	$P \to Q$	Rule P
{2}	(2)	P	Rule P
{1,2}	(3)	Q	Rule T $[\because P, P \to Q \Rightarrow Q]$
{4}	(4)	$Q \to R$	Rule P
{1,2,4}	(5)	R	Rule T $[Q, Q \to R \Rightarrow R]$

Method - I

P	Q	R	$P \to Q$	$Q \to R$	
T	T	T	T	T	→ valid statement
T	T	F	T	F	
T	F	T	F	T	
T	F	F	F	T	
F	T	T	T	T	
F	T	F	T	F	
F	F	T	T	T	
F	F	F	T	T	

Consistency of premises:

A set of premises $H_1, H_2 \ldots H_m$ are said to be consistant if there conjunction $H_1 \wedge H_2 \wedge H_3 \ldots \wedge H_m$ has the Truth values T in atleast one Possible situation.

ex: $P \vee Q \sim P$

show that the following premises are consistance

. $P \vee Q, \sim P$

P	Q	$P \vee Q$	$\sim P$	$(P \vee Q) \wedge \sim P$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

The above premises are consistance because $P \vee Q$ & $\sim P$ is true when p is False and Q is true.

Inconsistancy of premisses:

A set of premises $h_1, h_2, h_3 \ldots H_m$ are said to be inconsistancy if their conjunction h_1 and h_2 and $h_3 \ldots h_m$ as the truth values F in every Possible situation.

Ex :- show that the following permises are in consistncy

$\sim P \wedge Q, P$

P	Q	$\sim P$	$\sim P \wedge Q$	$(\sim P \wedge Q) \wedge P$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

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∴ The above premises are inconsistance because $(\sim P \wedge Q) \wedge P$ is false in all possible situations.

⇒ Implication formulas:-

J_1 : $P \wedge q \Rightarrow P$

J_2 : $P \wedge q \Rightarrow q$

J_3 : $P \Rightarrow P \vee q$

J_4 : $q \Rightarrow P \vee q$

J_5 : $\sim P \Rightarrow P \rightarrow q$

I_6 : $q \Rightarrow P \rightarrow q$

I_7 : $\sim(P \rightarrow q) \Rightarrow \sim P$

I_8 : $\sim(P \rightarrow q) \Rightarrow \sim q$

I_{10} : $\sim P,\ P \vee q \Rightarrow q$

I_{11} : $P,\ P \rightarrow q \Rightarrow q$

I_{12} : $\sim q,\ P \rightarrow q \Rightarrow \sim P$

I_{13} : $P \rightarrow q,\ q \rightarrow r \Rightarrow P \rightarrow r$

I_{14} : $P \vee q,\ P \Rightarrow q,\ q \rightarrow r \Rightarrow r$.

Indirect methods of proof
(or)
proof By contradiction
(or)
Inconsistancy of premises

Proof by contridiction is often reffered to has indirect Proof, in contract with the Direct proofs which have appeared earlier. A contradiction is in the form $R \wedge \sim R$ where R is any statement.

Ex :- $P \rightarrow Q,\ Q \rightarrow R,\ P \vee R \Rightarrow R$.

In order
logically f
we assume
solutions
P→Q,
{1}
{2}
{1,2}
{4}
{1,2,4}
{6}
{1,2,2,6
{1,2,4,
A
properit
is called
predicat
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Rami
girl. wh
is

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In order to show that a conclusion c follows logically from the premisis, H_1, H_2, H_3 - - - we assumed.

Solutions

$P \to Q$, $Q \to R$, $P \vee R \Rightarrow R$

{1}	(1)	$P \to Q$	Rule P
{2}	(2)	$Q \to R$	Rule P
{1, 2}	(3)	$P \to R$	Rule T (1) (2) and I 13 [$P \to Q, Q \to R \Rightarrow P \to R$]
{4}	(4)	$\sim R$	Rule T (∴ Additional premisses)
{1, 2, 4}	(5)	$\sim P$	Rule T (3) (4) & I 12 [∴ $\sim R, P \to R \Rightarrow \sim P$]
{6}	(6)	$P \vee R$	Rule P
{1, 2, 2, 6}	(7)	R	Rule T (5) (6) & I 10 [∴ $\sim P, P \vee R \Rightarrow R$]
{1, 2, 4, 6, 7}	(8)	$R \wedge \sim R$	Rule T (4) (7) & [$\sim R, R \Rightarrow \sim R \wedge R$]

A part of declarative sentence describing the properities of an object (or) relation among objects is called a predicate. The logic Base upon the analysis the predicate in any statement is called a predicate.

Rani and sitha the subject of statement is a girl. which refers to the property let the subject is

Statement function :

Statement function is defined as an expression that contains a predicate symbol and a variable.

$H(B) =$

B is a Bachelor

variables in predicate logic :- variables are the letters used as a base holder and has object name.

let us consider Predicate (s)

Here s is a student.

Ex :- Raju is a Bachelor and This Painting is red.

ex :- $B(R) \cap R(P)$

=> Raju is a Bachelor (or) This Painting is Red

$B(R) \vee R(P)$

=> if Raju is a Bachelor then this Painting is Red.

$B(R) \sim R(P)$

Ex :- let $P(x)$ denote the statement $x > 3$. What are the truth triangle of $P(4)$ and $P(2)$

$P(x) \rightarrow x > 3$

Sol :- $P(4)$ and $P(2)$

$P(4>3) \wedge P(2>3)$

$T \wedge F \Rightarrow F$

predicate logic, variables, Quantifiers, statements functions: (predicate calculus):-

eg :- $s(x)$: x is a student

$s(x)$ → statement function (or) Propositional logic of x

x → variable, subject, object

"is a student" → predicate

eg 1 :- $e(x)$: x is a even number

$e(x)$ → s.f

x → var, one place

"is a even number" → predicate

eg 2 :- x is taller than y

$T(x,y)$ → 2 place predicate

eg 3 :- x is an b/w y and z

$B(x,y,z)$ → m place predicate

eg :- Ramu is a good boy

$g(R)$ → one place

compound statements in predicate logic :-

S_1 :- John is a bachelor

S_2 :- This painting is red

S_1S :- $B(J)$: John is a bachelor

S_2S :- $R(P)$: This painting is red.

(1) John is a bachelor and this painting is red.

A :- $B(J) \wedge R(P)$

(2) John is a bachelor or this painting is red

$B(J) \vee R(P)$

(3) if John is a bachelor than this Painting is not red

$B(J) \rightarrow \sim R(P)$

M Place :-

1. Raju sits Between Ramu, Ravi

A :- B(Raju, Ramu, Ravi) – 3 (or) m place predicate

2. canada is to the North of united states

A :- N(C, U) – 2 places

3. Green and miller played bridge against John and smith

A :- P(G, M, J, S) – M Place.

Types of Quantifiers

1. universal Quantifiers : $\forall$
2. Existential Quantifiers : $\exists$

universal quantifiers

The quantifier "All" is called universal quantifier. It is denoted by $\forall$. The symbol : $\forall$. The following phrases have some meaning for all, for every, for every thing.

Existential Quantifier :

The Quantifier some is called existential Quantifier. It is denoted by $\exists$. The symbol $\exists$ represents the following phrases of same meaning :- There exists, "There is atleast" "There is an".

Eg :

write the following statement in symbolic form ?

S1. All men are good

S2. No men are good

S3. Some men are good

S4. Some men are not good.

S1. All men are good

A :- for all x; if x is a mean, then x is a good.

$$\forall x (m(x) \rightarrow G(x))$$

S2 : No men are good

A : for all x, if x is a men, then x is a not good.

$$\forall x (m(x) \rightarrow \sim G(x))$$

S3 : Some men are good.

A : There is atleast one x, if x is a men and x is a good

$$\exists x (m(x) \wedge G(x))$$

S4 : Some men are not good

A : There is atleast one x, if x is a men and then x is not good.

$$\exists x (m(x) \wedge \sim G(x))$$

——— o ———

S_1 : All monkey have tails

S_2 : No monkeys has tails

S_3 : Some monkeys have tails

S_4 : Some monkeys have no tail.

A :

S1 :- All monkeys have tails.

$m(x)$: x is a monkey

$G(x)$: x have a tail.

S1 :- All monkey have tails

A : for all x, if x is a monkey then x have a tail

$$\forall x\,(m(x) \rightarrow G(x))$$

S2 :- No monkeys has tails.

A :- for all x, if x is a monkey, then x have no tail

$$\forall x\,(m(x) \rightarrow \sim G(x))$$

S3 :- Some monkeys have tail.

A : there is atleast one x, x is a monkey and x has a tail

$$\exists x\,(m(x) \wedge G(x))$$

S4 :- Some monkeys have no tail.

A : There is atleast one x, x is a monkey and then x have no tail

$$\exists x\,(m(x) \wedge \sim G(x))$$

— o —

S1 :- Some people who trusted others are rewarded.

S2 :- Some one is teasing.

S3 :- No one is ambitious.

S4 :- if any one is good then John is good.

S5 :- It is not true that all roads leads to delhi.

Answer :-

$P(x)$: x is a people

$T(x)$: x is trusted

$R(x)$: x is Rewarded.

S1 : Some People who trusted other are rewarded

-> There is atleast one x, x is a People and x is

trusted and x is rewarded

$$\exists x (P(x) \wedge T(x) \wedge R(x))$$

S2 :- Some one is teasing

-> There is atleast one x, x is a people and x is teasing.

$$\exists x (P(x) \wedge te(x))$$

S3 : No one is ambitious

-> for all x, if x is a people, then x has no ambitious.

$$\forall x (P(x) \rightarrow \sim A(x))$$

S4 : if any one is good then John is good.

-> for all x if x is a person and x is good then John is good

$$\forall x \, [P(x) \wedge G(x)] \rightarrow G(J)$$

S5 :- it is not true that all roads leads to delhi

-> for all of x, if x is a road then x leads to delhi

$$\sim \forall x \, (R(x) \rightarrow L(x))$$

Bound variable / free variable / scope of Quantifiers :-

Bound variable :-

A occurance of variable quantifiers (either $\forall$ (or) $\exists$) is said to be a bound variable.

$\therefore$ All mens are Tails

$$\forall x (m(x) \rightarrow T(x))$$

B : x

Free variable :

A occurance of variable that is not bounded by a quantifier (either $\forall$ or $\exists$) is said to be a free variable.

Eg :- All means are tails

$\rightarrow (m(x) \rightarrow T(x))$

f : x

Scope of a Quantifier :

A scope of a quantifier is the formula immediately following the quantifier

Eg :- All mens have tails

$\forall x\, (m(x) \rightarrow T(x))$

S :- $\forall x : m(x) \rightarrow T(x)$

① $\forall x\, P(x,y) \rightarrow$ B : x

f : y

S : (P, y)

② $\forall x\, (P(x) \rightarrow Q(x)) \rightarrow$ B : x

f : –

S : $P(x) \rightarrow Q(x)$

③ $\forall x\, (P(x) \rightarrow (\exists y)\, R(x,y)) \rightarrow$ B : x, y

F : –

S : $P(x) \rightarrow (\exists y)\, R(x,y)$

$R(x,y)$

④ $\forall x\, (P(x) \wedge Q(x) \vee \forall y R(y)) \rightarrow$ B : x, y

F : –

S : $P(x) \wedge Q(x) \vee \forall y\, R(y)$

$R(y)$

⑤ $\exists x\, (P(x) \wedge Q(x)) \rightarrow$ B : x

F : –

S : $P(x) \wedge Q(x)$

⑥ $\exists x\, P(x) \wedge Q(x) \rightarrow$ B : x

F : –

S : $P(x)$

Unit - II

Sets, Relations and functions :-

1. sets, Representation of sets and operations on set.

set :- set is a collection of elements. A set can be represented by capital letters. the elements in set can be represented in small letters.

eg :- group of students.

member / element of a set :-

members are element of a set can be represented by "ε" (epsilon) -> reads as belongs to.

$\therefore$ eg : $A = \{a, b, c\}$

$\rightarrow a \in A, \rightarrow d \in A$

finite set :-

A set contains a fixed number of elements.

Infinite set :

A set does not contain a fixed number of elements.

singleton set :

A set contain only one element.

Representation / Notation of a set :-

-> Represented in two types.

1. Roster method eg :- $X\{1, 2, 3\}$
2. set Builder method :- contain same properties.

Eg :- $X = \{2, 4, 6, 8, 10, 12, 14\}$

$X = \{x / P(x)\}$

$X = \{x / x$ is a even number$\}$

Various terminalogies in set :-

1. Cardinality :- No. of elements in a set

$\therefore$ eg $x = \{1, 2, 3\}$

$|x| = 3$

$\therefore$ eg2 :- $x = \{1, \{2,3\}, \{4,5,6\}\}$

$|x| = 3$

2. subset (or) inclusion set :-

Representation of a subset is "$\subseteq$". Every elementy of A in element of B

$\therefore A \subseteq B$

eg1 :- $A = \{1,2,3\}$ $B = \{1,2,3,4,5\}$

$A \subseteq B$

eg2 :- $A = \{1,2,3\}$ $B = \{1,3,4,5\}$ X

$A \subseteq B \rightarrow$ not satisified

3. Equality of set :

All elements in A is equal to elements of B

Eg :- $A = \{1,2,3\}$ $B = \{1,2,3\}$

$A \subseteq B$

4. Proper subset :-

$A \subset B = A \subseteq B$ and $A \neq B$

5. universal set :-

→ Represented by "U" and "E"

eg :- $A = \{1,2\}$, $B = \{3,4\}$, $C = \{5,6\}$

$E = \{1, 2, 3, 4, 5, 6\}$

6. Empty set / Null set :-

does not contain any elements

$X = \{\phi\}$

7. Power set :

if A set contain n elements Then power contain 2^n elements.

eg:- $A = \{1, 2, 3\} = 2^n = 2^3 = 8$

$P(A) = \{\phi, 1, 2, 3, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$X = \{a, b, c, d\} = 2^n = 2^4 = 16$

$P(A) = \{\phi, a, b, c, d, \{a,b\}, \{a,c\} \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{c,d\}, \{b,c,d\}, \{a,c,d\} \{a,b,c,d\}$

operation of sets :-

2. union → 1. insertion → $A \cap B$ and $A \cap B$
→ $(A \cup B)$ (or) $A \vee B$

3. Disjoint → $A \cap B = \phi$

4. Difference → $A - B$

5. complement – A^c (or) $\bar{A}$ (or) A'

6. symetric → $A \Delta B = (A-B)$ or $(B-A)$

7. carolition Product → $A \times B$

1. Intersection :-

$A = \{1, 2, 3, 5\}$, $B = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{1, 2, 3, 5\}$

2. union :-

$A = \{1, 2, 3\}$, $B = \{1, 3, 4, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 6, 7\}$

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Disjoint :-

$A = \{1, 2, 3\}$, $B = \{3, 6, 7, 8\}$

Collection of ordered pairs :-

7. cartition Product :- $A \times B = \{(x, y) / (x \in A) \wedge (y \in B)\}$

eg :- $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5)\}$

$B \times A = \{(3,1)\ (3,2)\ (3,3)\ (4,1), (4,2), (4,3), (5,1), (5,2)\ (5,3)$

$\therefore A \times B \neq B \times A$.

Relation :

Relation is defined as a collection of ordered pair. An ordered pair (x, y), $(x, y) \in R$, R is a Relation.

Types of Relations (or) properties of Relation :

5 types of Relations

1. Reflective Relation
2. irreflective Relation
3. Symmentric Relation
4. Anti - Symmetric Relation
5. Transitive Relation.

Reflective Relation :-

A Relation R on a set A, it said to be a reflexive, if $(x, x) \in R \ \forall x \in A$

eg :- $A = \{1, 2, 3\}$

$R = \{(1,1) \ (1,2) \ (1,3) \ (2,1) \ (2,2) \ (2,3) \ (3,1) \ (3,2) \ (3,3)\}$ → $\{\}$ - Not reflexive

$\therefore (x, x) \ RA \Rightarrow x \in A$

eg2 :- $R = \{(1,1) \ (2,2)\}$ → Is not Reflexive

eg3 :- $R = \{(1,1) \ (2,2) \ (3,3) \ (1,2)\}$ → it is reflexive

eg4 :- $R = \{(x, y) / x - y$ is a integer$\}$ check which of the above is Reflexive Relation on a set of all Natural Numbers → ✓

eg5 :- $R = (x, y) / x - y$ is a odd number) ✗

2. IRReflexive Relation :-

A Relation R on a set A, is said to be irreflexive, if $(x \not R x)$ x is not Related to x, but Every $x \in A$ That is if $(x, x) \notin R$.

→ $\{\}$ → it is consioler as Pevreflexive.

eg :- $A = \{1, 2, 3\}$

$R = \{(1,2) \ (1,3) \ (1,3) \ (2,1) \ (2,2) \ (2,3) \ (3,1) \ (3,2) \ (3,3)\}$

$\therefore n^2 - n = 3^2 - 3 = 9 - 3 = 6$ → if we have all oroleved Pairs we can use it.

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1.5

Reflexive Relation on a set A

A Relation R, it is said to be reflexive if $(x,x) \in R$, $\forall x \in A$

eg :- $A = \{1,2,3\}$

$$R = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{matrix} \right\} \rightarrow \text{is reflexive}$$

eg :- $R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\}$ - is reflexive

Irreflexive Relation

A Relation R on set A, is said to be irreflexive if (x,x), $\forall x \in A$

eg :- $A = \{1,2,3\}$

~~$R = \{(1,2)\ (1,3$~~

$(x,y) \in R$ and $x = y$

eg :- $\{(2,1)\ (2,3)$

Transit

A Rela

said to

$(y,z) \in$

eg :-

R_1

ex :-

(b

(c

(c

(

A Relation R on set ... symetric, if (xRy) then y Relats x (yRx) for all (x,y) ∈ A.

A = {1, 2, 3, 4}

R1 = {(1,2) (1,3) (1,4) (2,1) (2,3) (2,4) (3,1) (3,2) (3,4) (4,1) (4,2) (4,3)}

Anti-symmetric

A Relation R on set A is anti-symetric if (x,y) ∈ R and x = y then (y,x) does not belongs to R.

(x,y) ∈ R and x = y (y,x) ∉ R. ∀ (x,y) ∈ R

eg:- {(2,1) (2,3) (1,1)}

Transitive Relation

A Relation R on set A, said to be transitive (x,y) ∈ R, (y,z) ∈ R the (x,z) ∈ R

eg:- A {1,2,3}

R1 = {(1,2) (2,3) (1,3)}

ex:- R = {(a,a) (b,c) (c,b) (d,d) (b,b) (c,c)}

(b,c) ∈ R, (c,b) ∈ R } (b,b)

(c,b) ∈ R, (b,c) ∈ R } (c,c) — present in set so it is Transit

(b,b) ∈ R, (b,c) ∈ R } (b,c)

...e Relation on a set A

...n R, it is said to be reflexive

...∈ R, ∀ x ∈ A

... = {1, 2, 3}

(1,1) (1,2) (1,3)
(2,1) (2,2) (2,3)
(3,1) (3,2) (3,3) } → is reflexive

{(1,1) (2,2) (3,3) (1,2)} – is reflexive

Relation

R on set A, is said to be irreflexive

x ∈ A

2, 3}

~~(1,1~~, 3

Representation of Relation

1. Relational matrix
2. Di graph

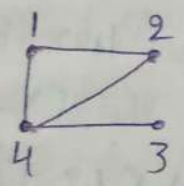
$A = \{1,2,3\}$ $B = \{0,1,2,4\}$ $(x,y) \in R$ if $x \le y$

$R = \{(1,0)\ (1,1)\ (1,2)\ (1,4)\ (2,0)\ (2,1)\ (2,2)\ (2,4)\ (3,0)\ (3,1)\ (3,2)\ (3,4)\}$

$R = \{(1,1)\ (1,2)\ (1,4)\ (2,2)\ (2,4)\ (3,4)\}$

	0	1	2	4
1	0	1	1	1
2	0	0	1	1
3	0	0	0	1

→ Relational matrix

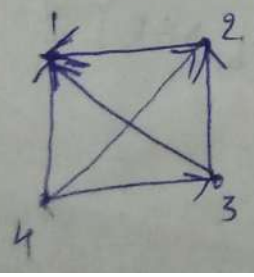


Eg 2 :- $A = \{1,2,3,4\}$ and $R = (x,y)\ x \ge y$.

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\}$

$R = \{(2,1)\ (3,1)\ (3,2)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\}$

	1	2	3	4
1	0	0	0	0
2	1	0	0	0
3	1	1	0	0
4	1	1	1	1



Equivalence Relation

A Relation R on set X is called eq. Relation if it is in reflexive, symmetric, anti-symmetric.

$X = \{1,2,3,4,5,6\}$

$R = \{(1,1)\ (1,3)\ (1,5)\ (3,5)\ (3,3)\ (5,5)\ (2,6)\ (2,2)\ (6,6)\ (6,2)\ (3,1)\ (5,1)\ (5,3)\ (4,4)\}$

Reflexive

$(x,x) \in R$ for $\forall x \in X$

$(1,1) \in R$ $\quad (4,4) \in R$

$(2,2) \in R$ $\quad (5,5) \in R$

$(3,3) \in R$ $\quad (6,6) \in R$

$\therefore$ Hence it is in reflexive

symmetric

if a relation R, $(x,y) \in R$ then (y,x) also $\in R$

$(1,3)\ (3,1) \in R$

$(1,5)\ (5,1) \in R$

$(3,5)\ (5,3) \in R$

$(2,6)\ (6,2) \in R$

it is in symmetric.

Transitive

if a relation $(x,y) \in R$, $(y,z) \in R$ then $(x,z) \in R$

$(1,1)\ (1,3) \rightarrow (1,3) \in R$

$(1,1)\ (1,5) \rightarrow (1,5) \in R$

$(2,6)\ (6,2) \rightarrow (2,2) \in R$

$(2,2)\ (2,6)\ (2,2) \rightarrow (2,6)\ (2,2) \in R$

$(3,3)\ (3,1) \rightarrow (3,1) \in R$

$(3,1)\ (3,3) \rightarrow (3,3) \in R$

$(4,4)\ (4,4) \rightarrow (4,4) \in R$

$(5,1)\ (1,5) \rightarrow (5,5) \in R$

$(5,5)\ (5,5) \rightarrow (5,5) \in R$

$\therefore$ R is transitive

compatibality Relation

* A Relation R on set A is said to be Reflexive, symmetric
* so, all equivalence Relation is represented by "$\approx$"

Ex: $A = \{1,2,3\}$ $R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (2,1)\ (3,2)\ (1,3)\ (3,1)\}$

Reflexive

$(x,x) \in R$ for any $x \in A$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

Hence it is Reflexive

symmetric

if $(x,y) \in R$ then $(y,x) \in R$ for any $(x,y) \in A$

$(1,2) \in R$ then $(2,1) \in R$

$(2,3) \in R$ & $(3,2) \in R$

$(1,3) \in R$ & $(3,1) \in R$

$\therefore$ Hence it is symetrice

so this is compatibility Relation.

Partial ordering Relation & Partial order set (POSET)

-> A Relation R on a set A is said to be partial ordering Relation if it is reflexive, anti-symetric, and transitive.

-> if the relation is partial order relation the the corresponding set called partial order set (poset)

-> poset is denoted by $(P, \leq)$ (P, R)

Ex: $P = \{1, 2, 3, 4, 6, 12\}$ $(x, y) \in R$ is x divides y

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,12)\ (2,2)\ (2,4)\ (2,6)\ (2,12)\ (3,3)\ (3,6)\ (3,12)\ (4,4)\ (4,12)\ (12,12)\}$

Reflexive

$(x, x) \in R$ for any $\forall x \in P$

$(1,1) \in R$ $(6,6) \in R$
$(2,2) \in R$ $(12,12) \in R$
$(3,3) \in R$
$(4,4) \in R$

anti-symmetric

if $(x, y) \in R$ & $x \neq y$ then $(y, x) \notin R$

$(1,2) \in R \rightarrow (2,1) \notin R$
$(1,3) \in R \rightarrow (3,1) \notin R$
$(1,4) \in R \rightarrow (4,1) \notin R$
$(1,6) \in R \rightarrow (6,1) \notin R$

$(2,4) \in R \rightarrow (4,2) \notin R$
$(2,6) \in R \rightarrow (6,2) \notin R$
$(2,12) \in R \rightarrow (12,2) \notin R$

$(3,6) \in R \rightarrow (6,3) \notin R$
$(3,12) \in R \rightarrow (12,3) \in R$

$(4,12) \in R \rightarrow (12,4) \notin R$

$\therefore$ it is in-anti-symetric form

Transitive

if The Relation is said to be Transitive

if $(x, y) \in R$, $(y, z) \in R$ then $(x, z) \in R$

$(1,2) \in R$ $(2,2) \in R \rightarrow (1,2) \in R$
$(1,3) \in R$ $(3,6) \in R$ $(3,12) \rightarrow (1,6)$ $(1,12) \in R$
$(1,4) \in R$ $(4,4)$ $(4,12) \rightarrow (1,4)$ $(1,12) \in R$
$(1,6)$ (

(1,1) (1,2) (1,3) (1,4) (1,6) → (1,2) (1,3) (1,4) (1,6)

~~(2,4) (4,4) (4,12)~~ → ~~(2,4), (2,12) ∈ R~~

(3,12) (2,12) → (3,12) ∈ R

(4,4) (4,12) → (4,12) ∈ R

(2,2) (2,4) (2,6) (2,12) → (2,4) (2,6) (2,12) ∈ R

(3,3) (3,6) (3,12) → (3,6) (3,12) ∈ R

(12,12) (12,12) → (12,12) ∈ R

it is Transitive.

∴ Hence it is partial ordering Relation.

Transitive closure of a Relation

$R^+ = R \cup R^2 \cup R^3 \cup \dots \cup R^n$

Ex :- $X = \{1,2,3,4\}$ $R = \{(1,2), (2,3), (3,4)\}$

Ex :- $A = \{0,1,2,3\}$ $R = \{(0,1), (1,2), (2,3)\}$

$R^+ = R \cup R^2 \cup R^3 \cup \dots \cup R^n$

$R = \{(1,2)(2,3), (3,4)\}$

Transitive relation is $(a,b)(b,c) \in R$ then $(a,c) \in R$

$R^2 = R.R = \{(1,3), (2,4)\}$

$R^3 = R^2.R = \{(1,4)\}$

$R^4 = R^3.R = \phi$

$R^+ = \{(1,2)\ (2,3)\ (3,4)\} \cup \{(1,3)\ (2,4)\} \cup \{(1,4\} \cup \phi$

$R^+ = \{(1,2), (2,3)\ (3,4)\ (1,3), (2,4), (1,4)\}$

operations on Relation

1. Intersection

→ It consists of order pairs common to both relations

$x(R \wedge S)y = xRy \wedge xSy.$

Ex:

$A = \{1,2,3\}$

$B = \{1,2,3,4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\}$

$S = \{(1,1)\ (1,2)(1,3)\ (1,4)\}$

$R \wedge S = \{(1,1)\}$

union

→ It consist of order pairs of Relation or Relation S. $x(R \vee S)y = xRy \vee xSy.$

$R \vee S = \{(1,1)(2,2)\ (3,3)\ (1,2)\ (1,3)\ (1,4)\}$

Difference

All the order pairs of R which are not present in S. $x(R-S)y = xRy \wedge x \not S y$

$R - S = \{(2,2)\ (3,3)\}$

Compliment

Set of order pairs which belongs to both sets but they are not present in R.

$x(\sim Ry) = x \not R y$

$\sim R = (A \times B) - R$

$= \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\} - \{(1,1)\ (2,2)\}$

$= \{ (1,2)\ (1,3)\ (1,4)\ (2,1)\ (2,3)\ (2,4)$
$(3,1)\ (3,2)\ (3,4)\ (4,1)\ (4,2)\ (4,3)\ (4,4) \}$

Hasse diagram

① show that the relation "greater than or equal" is a partially or relation on a set of Integers.

given that $Z = \{$

$R = \{ (2,6)\ (2,12)\ (2,24)\ (2,36)\ (3,6)\ (3,12)\ (3,24)$
$(6,12)\ (6,24)\ (6,36)\ (12,12)\ 6(12,36)\ (24,24)\ (3,$

① let $x = \{2,3,6,12,24,36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram.

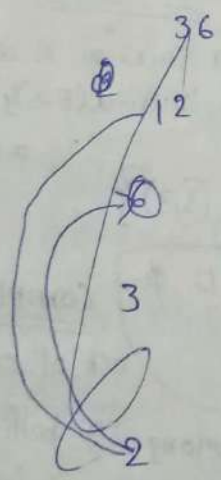
$2 \to 6, 12, 24, 36 = \{(2,6)(2,12)\ (2,24)\ (2,36)\}$

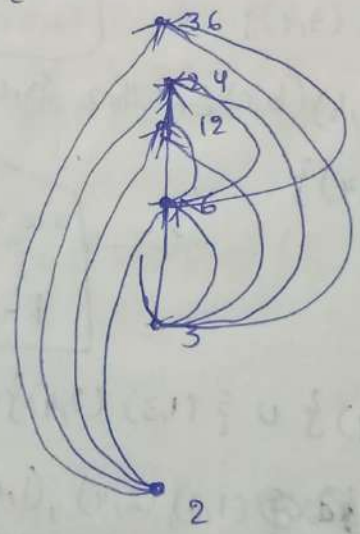
$3 \to 6, 12, 24, 36 = \{(3,6)\ (3,12)\ (3,24)\ (3,36)\}$

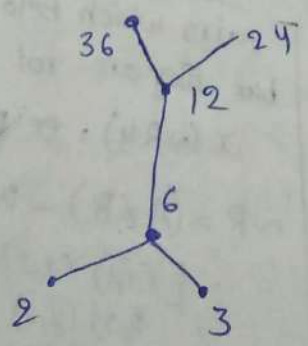
$6 \to 12, 24, 36 = \{(6,12)\ (6,24)\ (6,36)\}$

$12 \to 24, 36 = \{(12,24)\ (12,36)\}$

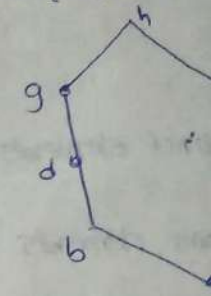
$36 \to 36 = \{(36,36)\}$







set $\{b, d, g\}$



upper Bound

$\therefore$ UB $\{b,$

$\therefore$ least up

glB :-
lower

b =>

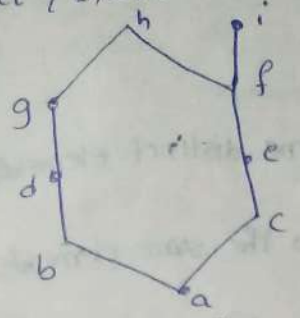
d =>

g =>

lB $\{b, d$

$\therefore$ glB =

set $\{b, d, g\}$



upper Bound of $\{b, d, g\} = \{b \to b, b \to d, b \to g, b \to h\}$

$= \{d \to d, d \to g, d \to h\}$

$= \{g \to g, g \to h\}$

$\therefore$ UB $\{b, d, g\} = \{g, h\}$

$\therefore$ least upper bound $= g$

GLB :-

lower Bounds of $\{b, d, g\}$

$b \Rightarrow b \to b, b \to a$

$d \Rightarrow d \to d \quad d \to b \quad d \to a$

$g \Rightarrow g \to g \quad g \to d \quad g \to b \quad g \to a$

LB $\{b, d, g\} = \{b, a\}$

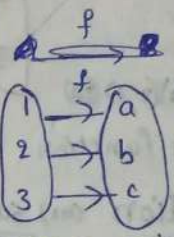
$\therefore$ GLB $= b$

Introduction to function

Let A and B be any 2 sets A function f from A to B denoted by $f: A \to B$, is a relation from A to B such that every element in set A i.e $a \in A$, there is Exactly one element in set B i.e $b \in B$ such that $(a, b) \in f$

$f: A \to B$

Here set A is called Domain of f, B is called range co-domain



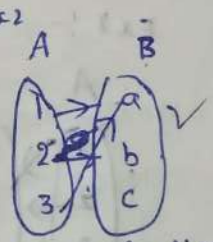
Ex1

1 is image a, 2 is image b

a is preimage 1, b is preimage of 2

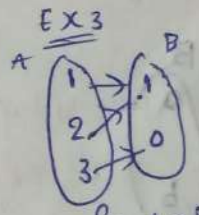
$f: A \to B$ it is a function

Ex2



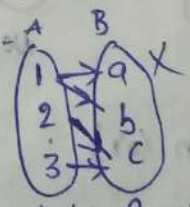
$f: A \to B$ is function

EX3



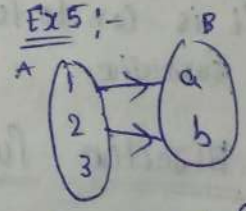
$f: A \to B$ is function

EX4 :-



$f: A \to B$ not a function

Ex5 :-



$f: A \to B$, not function

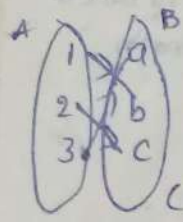
Types of functions

1) one to one / Injective function

-> A function $f: A \to B$ is called one to one function is distinct elements of set A are mapped to one distinct element of set B.

-> Two elements in set A can't be ~~mapped~~ mapped to the same element of set B.

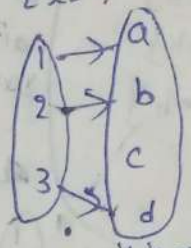
Ex:-



(1,b)(2,c)(3,a) ✓

it is one-one function

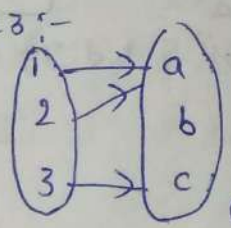
Ex2 :-



(1,a) (2,b) (3,d) ✓

it is one-one function

Ex3 :-



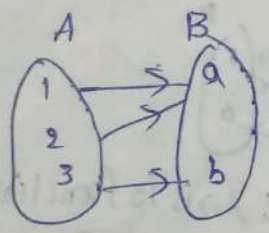
(1,a) (2,a) (3,c) ✗

it is Not one-one function

on to function (or) surjective function

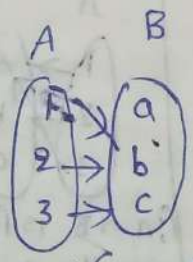
A function $f: A \to B$ is said to be on to function if every element of b will be an image of same element of set A

Ex:-



it is (on-to function)
surjective

Ex2 :-

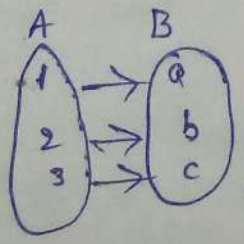


Not (on to function) ✗

Bijective function (or)

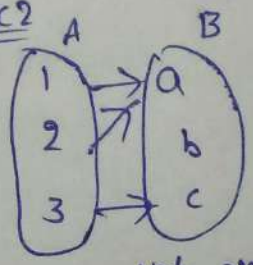
A function $f: A \to B$ is called Bijective function (or) one-to-one on to function if it is both one to one function and on-to function

Ex:-



it is one-one function
it is on-to function
Bijective

Ex2

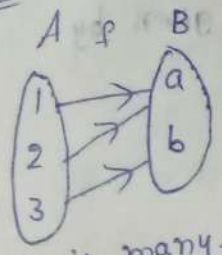


Not one-one function ✗
(on-to function) ✗
Bijective ✗

many-to-one function :-

A function $f: A \to B$ is called many-to-one function if many elements of set A is mapped to one single element of set B

Ex:-

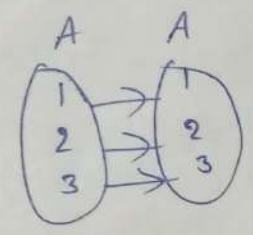


It is many-to function

Identity function

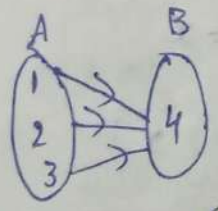
A function of $A \to A$ is called the identity function if the image of every element of set A is itself.

$A = \{1, 2, 3\}$



constant function :

A function $f: A \to B$ is called constant function if all the elements of set A have to same have the same image in set B.



composite function (or) composition of function :

~~let X = {1,2,3}, Y = {~~ more than on function are using then it is composit function, it is denoted by 'o' (circle)

Ex:1 Let $X = \{1,2,3\}$, $Y = \{p, q\}$ $Z = \{a, b\}$ Also let $f: X \to Y$ be $f = \{(1,p), (2,p), (3,q)\}$ and $g = \{(p,b)\ (q,b)\}$

$f(1) = f(2) = p$, $f(3) = q$

$g(p) = b$, $g(q) = b$

X, Y, Z

$f: X \to Y$

$g: Y \to Z$

$gof: X \to Z$

$gof(1) = g(f(1)) = g(P) = b$

$gof(2) = g(f(2)) = g(P) = b$

$gof(3) = g(f(3)) = g(P) = b$

Ex: $X = \{1,2,3\}$ and f, g, h be functions from X to X given by

$f = \{(1,2), (2,3), (3,1)\}$

$g = \{(1,3), (2,1), (3,3)\}$

$h = \{(1,1), (2,2), (3,1)\}$

find fog, gof, $fohog$.

$fog = \{(1,3), (2,2), (3,1)\}$

$gof = \{(1,1), (2,3), (3,2)\}$

$fohog = (foh) o g$

$= \{(1,2)\ (2,3), (3,2)\} o g$

$= \{(1,1)\ (2,3)\ (3,1)\}$

Ex 3 :-

let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$

find gof, fog, fof, gog, foh, hog

$gof(x) = g(f(x)) = g(x+2) = (x+2) - 2 = x$

$fog(x) = f(g(x)) = f(x-2) = (x-2)+2 = x$

$fof(x) = f(f(x)) = f(x+2) = (x+2)+2 = x+4$

$gog(x) = g(g(x)) = g(x-2) = (x-2)-2 = x-4$

$foh(x) = f(h(x)) = f(3x) = 3x+2$

$hog(x) = h(g(x)) = h(x-2) = 3(x-2) = 3x-6$

Ex4 :-

let $f(x) = 5x+1$, $g(x) = 3x-2$,

find $f(og)(x)$, $(gof)(x)$

$$fog(x) = f(g(x))$$
$$= f(3x-2)$$
$$= 5(3x-2)+1$$
$$= 15x-10+1$$
$$= 15x-9$$

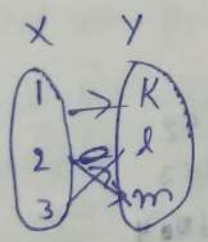
$$gof(x) = g(f(x))$$
$$= g(5x+1)$$
$$= 3(5x+1)-2$$
$$= 15x+3-2$$
$$= 15x+1$$

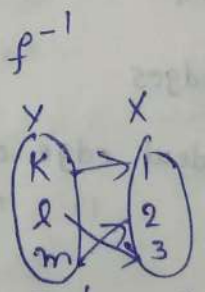
Inverse of a function

Ex :1 $X = \{1,2,3\}$ $Y = \{K, l, m\}$ $f = \{(1,K)\ (2,m)\ (3,l)\}$

$f : A \to B$ is a Bijective function

$f^{-1} : B \to A$





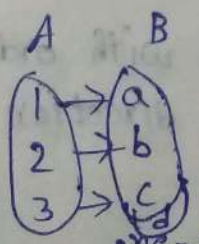
1-1 function ✓

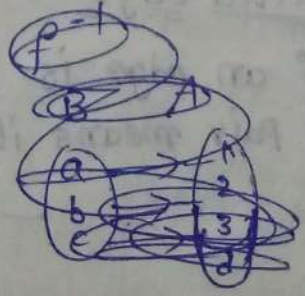
onto function ✓

Bijective function ✓

$f^{-1} = \{(K,1)\ (m,2)\ (l,3)\}$

Ex 2 :- $A = \{1,2,3\}$ $B = \{a,b,c,d\}$ $f : \{(1,a), (2,b), (3,c)\}$





one-one ✓

on-to ✗

~~Bijective~~ Bijective ✗

find inverse of $f(x) = x^3 - 2$

1. Replace $f(x)$ with y.
2. swap x & y (or) interchange x & y
3. solve y.
4. Replace y with $f^{-1}(x)$.

$f(x) = x^3 - 2$

$y = x^3 - 2$

$x = y^3 - 2$

$y^3 = x + 2$

$y = \sqrt[3]{x+2}$

$f^{-1}(x) = \sqrt[3]{x+2}$

Ex 4:-

$f(x) = \frac{5x-3}{2x+1}$

$y = \frac{5x-3}{2x+1}$

$x = \frac{5y-3}{2y+1}$

$x(2y+1) = (5y-3)$

$2xy + x = 5y - 3$

$2xy - 5y = -3 - x$

$y(2x-5 = -3-x)$

$y = \frac{-3-x}{2x-5}$

$f^{-1}(x) = \frac{-3-x}{2x-5}$

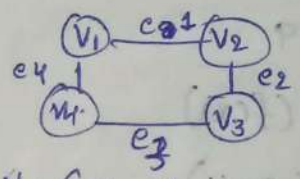
3. Graph Terminologies

Graph

$G = (V, E)$

$V \rightarrow$ set of Vertices (or) nodes

$E \rightarrow$ set of Edges



$V = \{V_1, V_2, V_3, V_4\}$

$E = \{e_1, e_2, e_3, e_4\}$

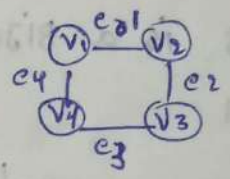
E_1 = an edge b/w v_1 and e_2

$E_1 = \{(V_1, V_2), (V_2, V_3)\ (V_3, V_4)\ (V_1, V_4)\}$

2) Adjacent nodes

...if there is a connection b/w two nodes called

ex: V_1, V_2
V_2, V_3
V_3, V_4
V_1, V_4



3) Incident edges

e_1 is incident edge on V_1 and e_2
e_2 is " " " V_2 and e_3
e_3 is V_3 and e_4
e_4 is V_1 and V_4

If An edge Joins two vertices (or) two nodes, we call Incident edges.

4) Directed edge

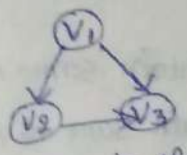
if an edge is associated with order Pair ordered Pair means it has some direction

$\longrightarrow$

5) Directed graph

if every edge in the graph is Directed edge. Then we can call D. graph

Ex:

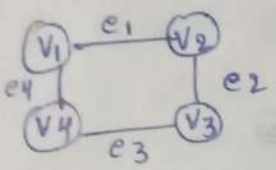


* every edge in the graph is Directed

(V_1, V_3) (V_2, V_3) (V_1, V_2)

6) un-directed edge

* if an edge is associated with unordered pair of vertices. it is called un-D-edge.
* unorder pair means it have no order no direction



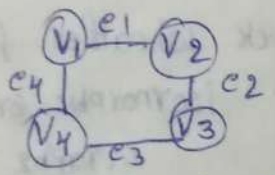
(V_1, V_2) (V_2, V_1)

(V_2, V_3) then also say that (V_3, V_2)

(V_4, V_3) " (V_3, V_4)

7) un-directed graph

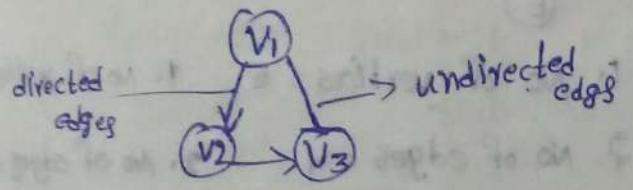
* if every edge in the graph is undirected edge
* it is called un-directed graph



every edge do not contain any direction

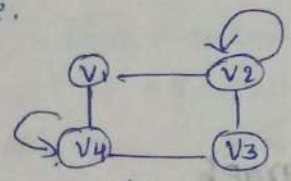
8) Mixed - graph

some edges are directed edges & some edges are undirected edges called Mixed graph



9) Loop

loop means where starting vertices and the Ending vertices are same.

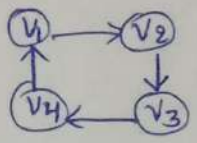


(V_1, V_1) starting ending

* loop have only one edge

10) cycle

* cycle means set of edges.
* where starting vertices and ending vertices are same.



starting vertices

ending vertices

$V_1 \to V_2 \to V_3 \to V_4 \to V_1$

$V_2 \to V_3 \to V_4 \to V_2 \to V_2$

another cycle

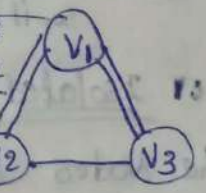
cycle mean set of edges

11) parallel edges

Parallel edge means more than one edge b/w a pair of vertices called Parallel

$(V_1 - V_2)$ pair of vertics

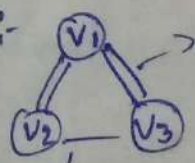
more than 1 edge



12) Multi-graph

if a graph contain some parallel edges. that graph is multi-graph

Ex :-



some edges parallel

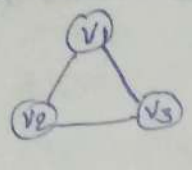
out of 5 edges 4 edges Prll

Some are normal

* all the edges need not to be Parallel edges

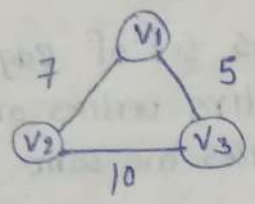
13) Simple graph

A graph with no parallel edges are called as simple graph



14. Weighted graph

Every edge contain some weight, cost Then it is called weighted graph

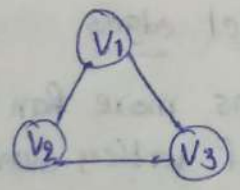


v_1 and v_2 is 7

15. Isolated Node

* Isolated node means The node is seperated from graph.
* Isolated Node desn't contain any adjacent nodes.

Ex:

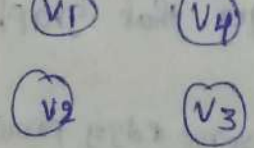


V_4 → Isolated Node

V_4 have not edge to v_1, v_2, v_3 so, it is Isolated Node.

16. Null graph or Isolated graph

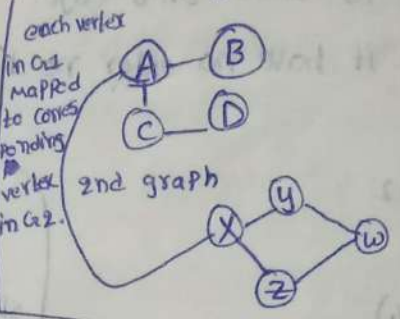
* if all the Nodes of graph are Isolated Nodes
* it is called Null graph (or) Isolated graph



Isomorphic graphs

2 graphs are said to be isomorphic if satisfy following conditions

1. Both graphs must contain same No.
2. Both graphs must contain same No. of
3. Both graphs mus contain same No. of Degree
4. one-to one corespondence between vertices of 2 graphs must be same (i.e for every vertex in graph there is an equivalent vertex in graph2)

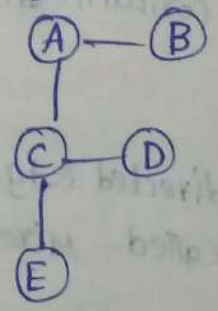


5) Edge preserving should be satisfied (i.e Each edge in graph1 is equal to an edge in graph2).

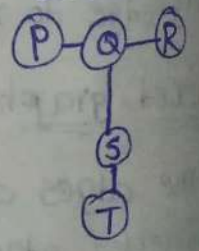
6) Adjancey matrix of both graphs must be same.

Ex:- Check wheather following graphs are isomorphic (or) not

Graph-1



Graph2



1. No of vertices = 5
2. No of edges = 4

1. No. of vertices
2. No of edges

3. Degree sequence -> No. of edges associated with that vertics

$(A, B, C, D, E) = (2, 1, 3, 1, 1)$

3. Degree sequence $(P, Q, R, S, T) = (1, 3, 1, 2, 1)$

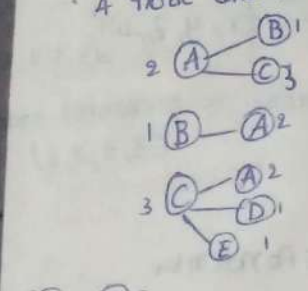
* All these degress are availabe in both. but in different order so implement Degree with Decreasing order (or) Ascending

Degree sequence in Decreasing order = (3, 2, 1, 1, 1)

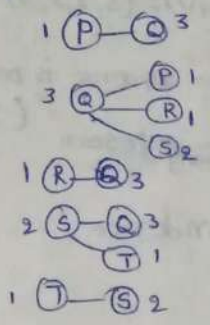
Degree in Decreasing order = (3, 2, 1, 1, 1)

4. one-to-one correspondence =

· A node and take Adjecent vertices that vertex and find degree



4.



C <=> Q

A <=> S

B <=> T

D <=> P

E <=> R

5) Edge preserving

A - B (=) S - T

A - C (=) S - Q

C - D (=) Q - P

C - E (=) Q - R

6) Adjancey matrix

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ S \\ T \\ Q \\ P \\ R \end{array} \begin{array}{c} \begin{array}{ccccc} S & T & Q & P & R \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

∴ These two graphs it satisfies all conditions so it is Isomorphic graphs.

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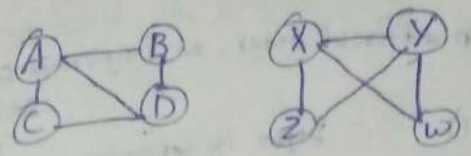
Ex :- check wheather following graphs are Isomorphic or not



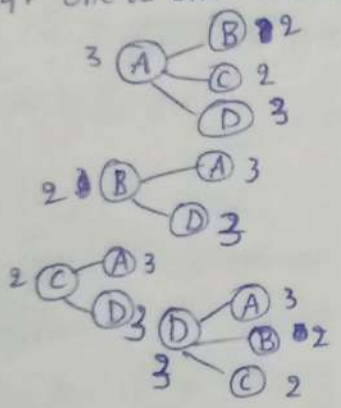
1. No. of vertices = 4 No. of vertices = 4
2. No. of edges = No. of edges = 5
3. Degree of sequence $(A,B,C,D) = (3,2,2,3)$ Degree of sequence = (x, y, z, w)

Degree sequence is decresing order $(3,3,2,2)$

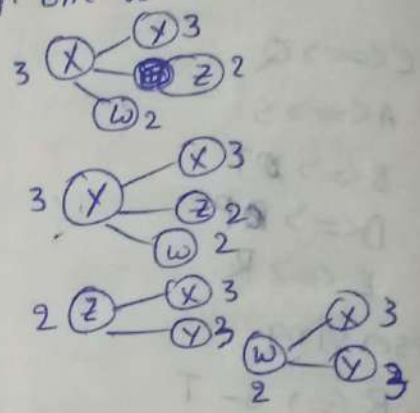
Degree sequence is decreasing (3,3

~~Degree of sequence using Deseen~~

4. one to one correspondence



4. one to one corespondence



A (=) X
B (=) Z
C (=) W
D (=) Y

~~5. Adjacent Matrix~~ 5.) Edge preserving:

A - B (=) X - Z
A - C (=) X - W
A - D (=) X - Y
B - D (=) Z - Y
C - D (=) W - Y

6. Adjacent Matrix

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 0 & 1 \\ C & 1 & 0 & 0 & 1 \\ D & 1 & 1 & 0 & 0 \end{array} \qquad \begin{array}{c|cccc} & X & Z & W & Y \\ \hline X & 0 & 1 & 1 & 1 \\ Z & 1 & 0 & 0 & 1 \\ W & 1 & 0 & 0 & 1 \\ Y & 1 & 1 & 1 & 0 \end{array}$$

∴ it is a Isomorphic graphs.

Matrix Representation of Graphs

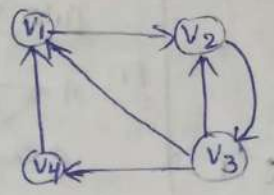
1. Adjancey matrix
2. Incidence matrix

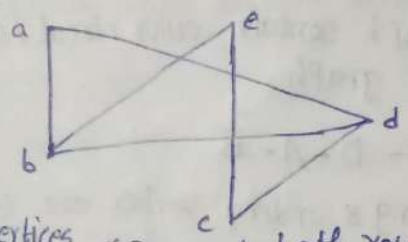
1. Adjancey matrix

* it means value may be either 1 or zero
* if there is an edge between 2 vertices then we have to fill the value as 1. whereas no edge b/w vertices the value is 0.

undirect graph

Directed graph





* vertices represent both row value & column value.

$$\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{bmatrix} a & b & c & d & e \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

→ ~~indirected graph based upon column to return value~~

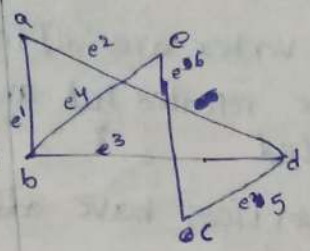
constructing adencancy matrix for Directed graph

$$\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

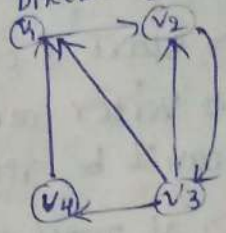
Incedence matrix

in Incedence matrix we construct matrix for undirected graph & Directed graph

undirected - graph



Directed - graph

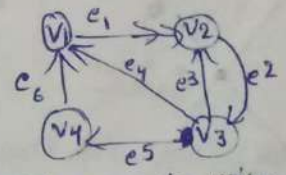


rows are vertices
coloumns are edges

$$\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

→ based upon color to return value

construct incedence matrix for Directed graph.



using
+1 → outgoing edge
-1 → incoming
0 → no edge

based on row to write values

$$\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ +1 & 0 & 0 & -1 & 0 & -1 \\ -1 & +1 & -1 & 0 & 0 & 0 \\ 0 & -1 & +1 & +1 & +1 & 0 \\ 0 & 0 & 0 & 0 & -1 & +1 \end{bmatrix}$$

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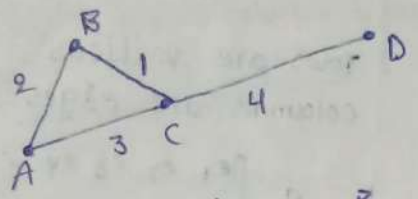
Euler path, Euler circuit & Euler Graph

Euler path (or) Eluler walk (or) Euler trail

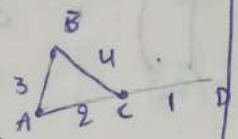
-> visit each edge of the graph exactly once
-> start & end vertex are different
-> vertex can be repeated but an edge can't be repeated.
-> At most 2 vertices have odd degree.

Ex 1 :-



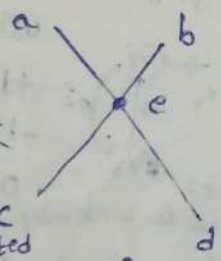
A -> 2
B -> 2
C -> 3
D -> 1
-> CD vertices have odd degree

C - B - A - C - D



2) D - C - A - B - C

Ex2 :-



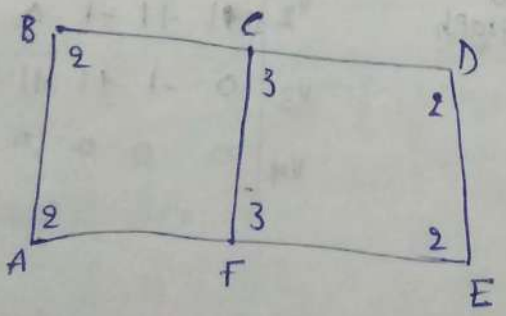
a - e - d - e so X
visted & be visted again

almost 2 degree odd only so it is not eluer X

a -> 1
b -> 1
c -> 1
d -> 1
e -> 4
4 odd degrees

∴ Degree means the no. of edges associated with that vertex

Ex



C -> D - E - F - C - B - A - F

So, This is eluer path
starting and ending vertex different

Elur circuit (or) Eular cycle

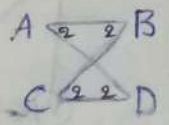
1) visit each edge of graph exactly once
2) vertex can be repeated but edge can't be repeated
3) start and end vertex are same
4) if each vertex degree is even.

Euler grajoh (or) Eulerian graph

A graph is called Euler graph if contain Euler circuit.

Euler graph (or) Euler circuite

Ex:

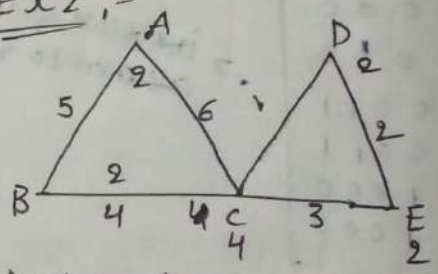


This graph contain euler circut, so is euler graph.

B - C - D - A - B

∴ Starting & ending vertex are same

Ex2 :-

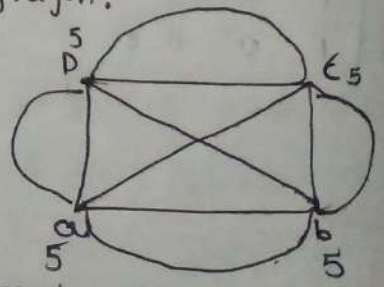


∴ all numbers are even

C - D - E - C - B - A - C

∴ start & ending vertex are same, it is Euler circuite, so it is called Eluer grajoh.

Ex3



Every vertex is odd. So it Not Euler circuit.
So it is not Euler graph or Eulerian

Hamiltonian path, Hamiltonian cycle, Hamiltonian graph

Hamiltonian Path

Hamiltonian path focus on vertices

-> Each vertex of the graph will be visited exactly once.

-> start and end vertex must be different

A - B - C - D - E

↳ visiting all vertices

starting (A) & ending (E) vertex diff

Hamiltonian circuit (or) Hamitonian cycle

-> Each vertex of graph will be visited exactly once except start vertex.

-> because start & end vertex must be same.

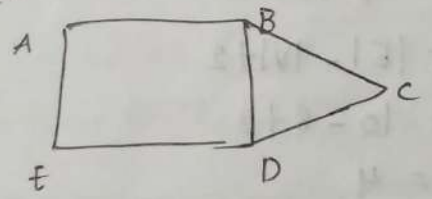
start A A end

start C C end

Hamiltonian Graph

-> Hamiltonian graph means if the graph contains Hamiltonian circuit (or) Hamiltonian cycle.

Ex1:

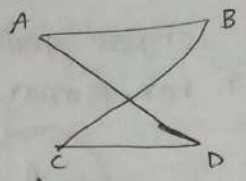


A B C D E A C D E A B C

This is called Hamitonian circuit (or) cycle.

So it is called Hamiltonian graph

Ex2:



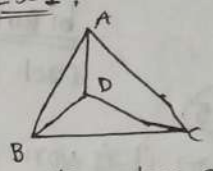
A B C D A

it is a Hamiltonian graph

Planar graph

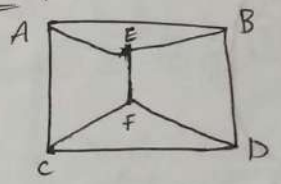
A graph is called planar graph if it can be drawn without any crossing edges.

Ex1:



The edges are not crossing each other so, it is planar graph

Ex2:-

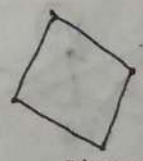


∴ The edges are not crossed, so it is Planar

Non-Planar Graph

A graph is called Non-Planar graph if it can be drawn with any crossing edges

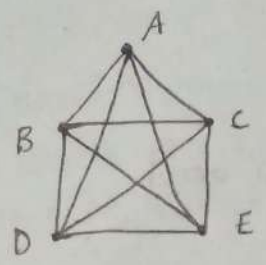
Non-Planer graph because it is crossed

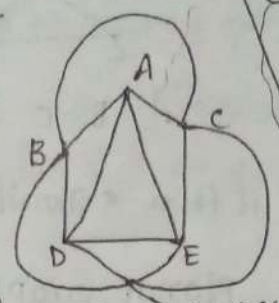


Planar graph

Planar graph

Ex1: show that a complete graph of 5 vertices in non-planar

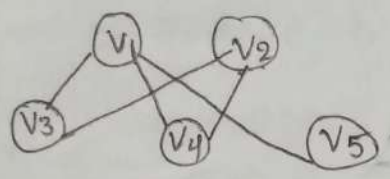




crossed

It is no-planar graph
∴ several edges are crossed here so, again it is non-planar

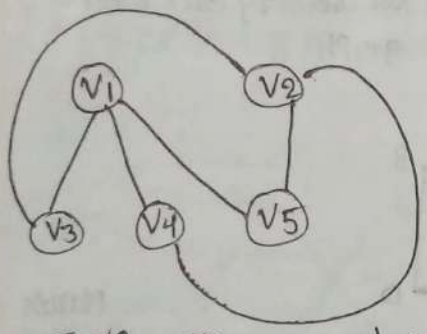
Ex2: check the graph K_2, K_3 is planar or not



bi partite graph

Each vertex of section 1 an edge to each vertex of

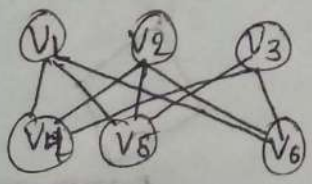
→ it is Non-planar graph

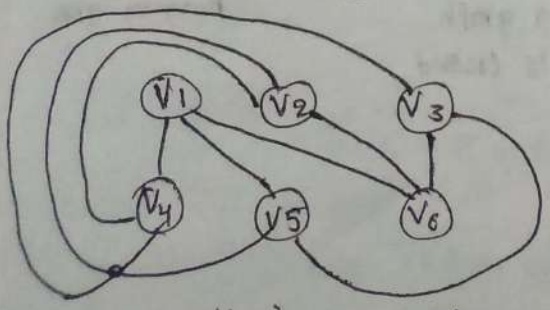


There are no edge crossing
∴ it is a planar graph

Ex3: Check the graph $K_{3,3}$ is planar or not

1st section, 2nd section





So, it is non-planar graph

Euler's formula (or) Euler's theorem

A connect planar graph G with |V| and |E| edges has exactly |E| - |V| + 2

* connected graph means from each vertex we should have an edge or path to a vertex in graph.
* planar graph means if we draw the graph without crossing any edge

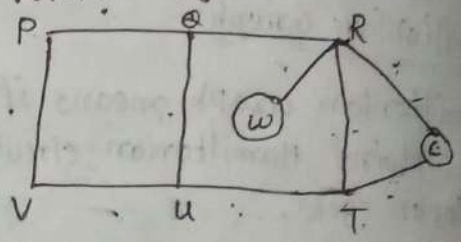
$$|R| = |E| - |V| + 2$$

(or)

$$|V| - |E| + |R| = 2$$

∴ regions cycle the

Ex1:- show that following graph is verified by Euler's theorem.



1) $|V| = 8$, $|E| = 10$

$$|R| = |E| - |V| + 2$$
$$= 10 - 8 + 2$$
$$= 4$$

no. of regions = 4

The Regions are

$R_1 \rightarrow P-Q-U-V-P$

$R_2 \rightarrow Q-R-W-R-T-U-Q$

$R_3 \rightarrow R-S-T-R$

outer region $R_4 \rightarrow P-Q-R-S-T-U-V-P$

we have 4 regions

Euler's formula

$|V| - |E| + |R| = 2$

$8 - 10 + 4 = 2$

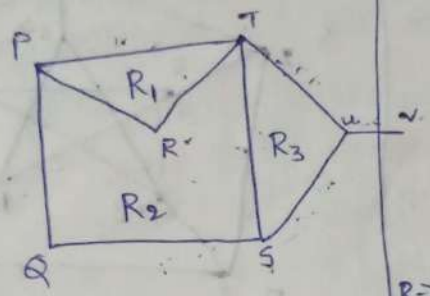
$-2 + 4 = 2$

$2 = 2$

left hand side = right hand side

∴ So Euler's formula verified

Ex 2 :- Find the degree of each Regions & verify that sum of these degree is equal to twice the no. of edges show that one graph is verified by Euler's formula.



Sol) $|R| = |E| - |V| + 2$

(or)

$|V| - |E| + |R| = 2$

$|V| = 7 \quad |E| = 9$

$|R| = 9 - 7 + 2$

$= 4$

$R_1 \rightarrow P - T - R - P$ (3) → degree of R_1 is 3 in question mentioned ~~types~~ degree of each region

$R_2 \rightarrow P - R - T - S - Q - P$ (5)

$R_3 \rightarrow T - u - S - T$ (3)

$R_4 \rightarrow P - T - u - V - u - S - Q - P$ (7)

Sum of all degree = 18

∴ twice the no. of edges = $9 \times 2 = 18$

Sum of all degre is equal to no. of edges $18 = 18$

Euler's formula

$|V| - |E| + |R| = 2$

$7 - 9 + 4 = 2$

$2 = 2$

LHS = RHS

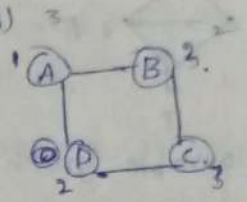
∴ So, it is verified by Eluer's formula

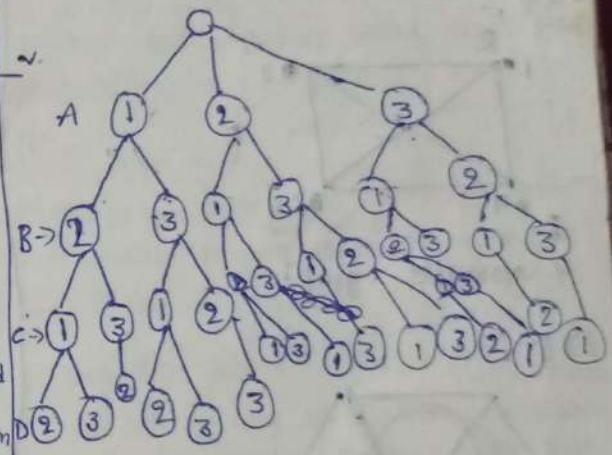
Graph - coloring using Back tracking

* Back tracking is an algorithm designed strategy technique.
* The Major Aim is we have to color all the vertices of a graph in such a way that no two Adjacent vertices should have the same color.

Ex1 :- m = 3

(1,2,3)

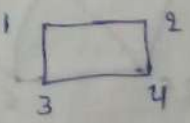




graph coloring

we have to color all the vertices of the graph in such a way but no two adjacent vertices have the same color.

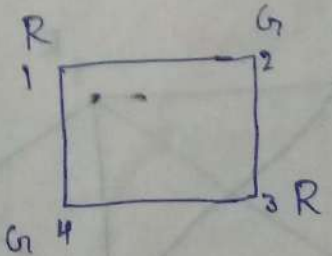
Ex 1 :-



R → Red

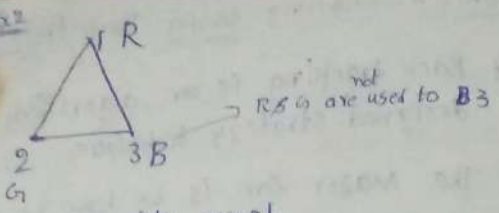
G → Green

B → Blue



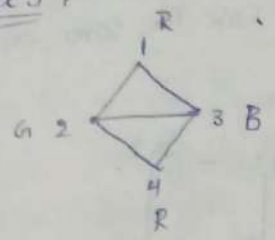
our Aim should be use minimum no. of colors.

∴ 2-colorable graph



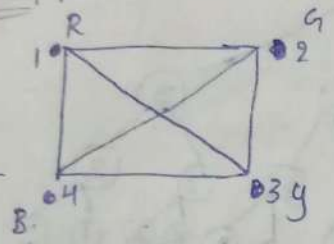
∴ 3- colourable graph

EX3 :-



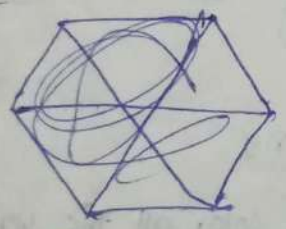
∴ 3- colourable graph

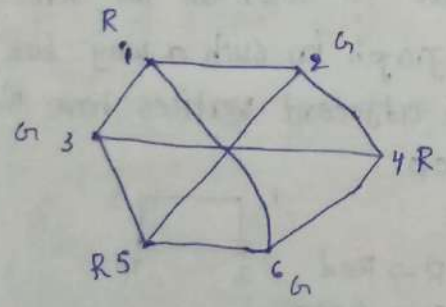
EX4 :-



4- colourable graph

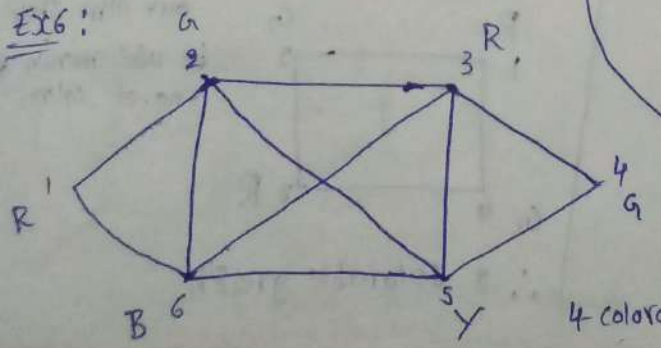
EX5 :-





2- colourable graph

EX6 :



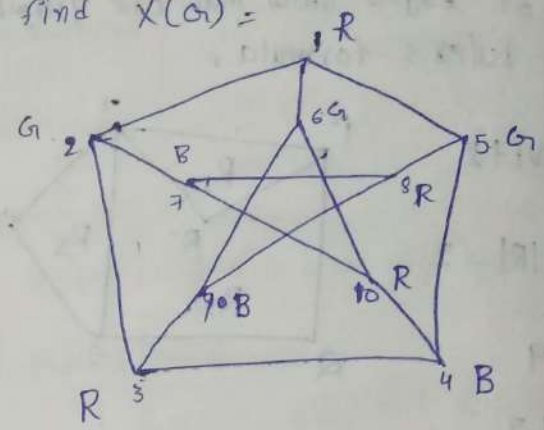
4- colorable graph

Chromatic number of a graph

* The minimum no. of colours requis. to colour all the vertices of graph take min colour 2, 3, 4, . . .
* No two Adjacent vertices have the same colour
* we can denote choromatic no. of a gr by using $\chi(G)$ → symbol

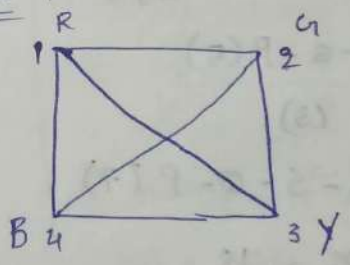
EX 1 :- Peterson graph (10 vertices 15 edges) find $\chi(G)$ =

R → Red
B → Blue
G → Green



choromatic no. of graph = 3

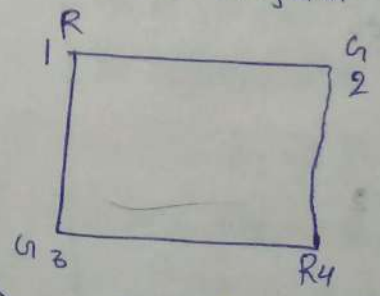
EX2 :-



$\chi(G)$ choromatic no. of graph = 4

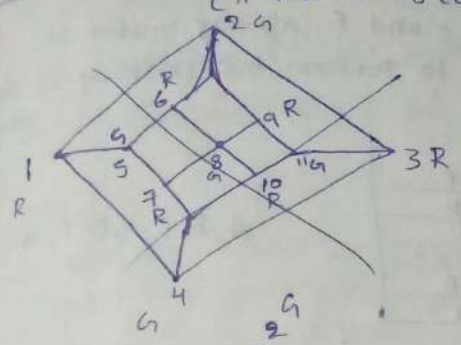
EX3 :

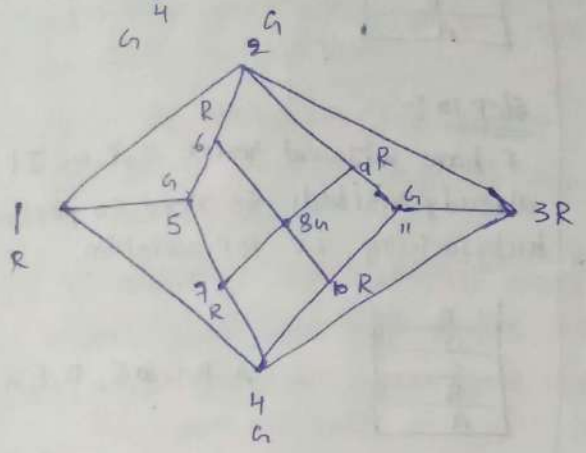
let us analyze this diagram



chromatic No. of graph $\chi(G)$ = 2

Ex2 :- Herschel Graph
(11 vertices 18 edges)

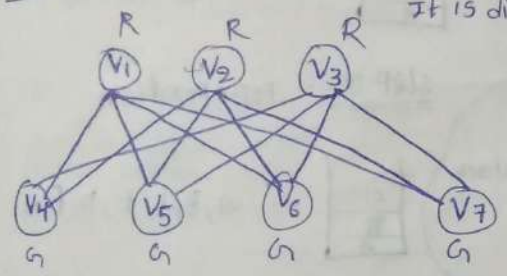




chromatic No of graph $X(G) = 2$

Ex 3 :- $K_{3,4}$ bipartite graph

It is divided into 2 sections



The section 1 vertices are all are edge to section 2 vertices

The chromatic No. of any bipartiate graph = 2 always

Graph Traversing

graph Traversing means visiting vertices of the graph exactly once.

we can traverse the graph in two ways.

① DFS (Deepth first search)

② BFS (Bredth first search)

DFS

visited all the vertices upto the deepest level and continue

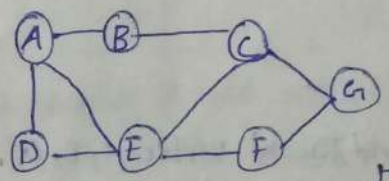
* start from starting vertex and ~~examine~~ visit all vertexs till the deepest level

* DFS uses a Data structure called Stack for implementation

Algorithm :-

1. take any vertex as starting vertex and insert it top of the stack make the status as visited.

2. why the stack is not empty
   a) select any unvisited edges & vertex of corresponding vertex and push it to the stack.
   b) if there is no unvisited Adjacent vertex then perform backtracking and then pop a element to stack.



A	B	C	D	E	F	G
1	1	1	1	1	1	1

Step 1 :-

stack Result

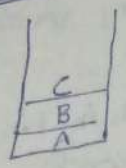
A

Step-2 :-

A, B

Result

Step 3

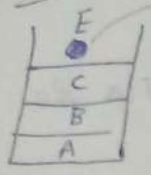


A, B, C

Step 4 :-

selecting for back tracking using

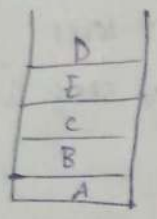
instead of G directly get output



Result

A, B, C, E

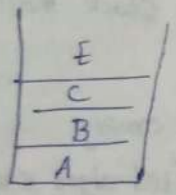
Step 5 :-



A, B, C, E, D

Step 6 :-

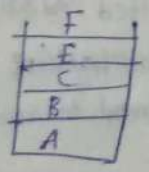
D have adjacent vertex D, E it is already visited so no node to visit so use backtraking i.e pop the element stack



A, B, C, E, D

Step 7 :- The Adjacent vertex of E is C and F

F is not visited so, it is pushed to stack

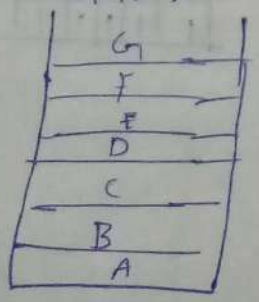


A, B, C, E, D, F

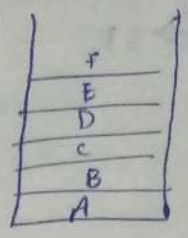
Step 8 :-

F have Adjacent vertex C, E, G. C & E are already visited

G is pushed to stack



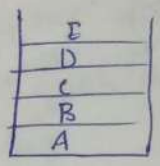
G have Adjacent vertex C, but C and F already visited need to perform backtracking



A, B, C, E, D, F

Step 10 :-

F have Adjacent vertex C, E, G. It already visited. So need to perform backtracking i.e POP operation.



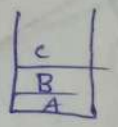
A, B, C, E, D, F, G

Step 11 :-

POP operation

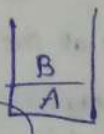
A, B, C, E, D, F

Step 12 :- POP operation



A, B, C, E, D, F

Step 13 :-



A, B, C, E, D, F

Step 14 :-

A, B, C, E, D, F

Step 15 :-

A, B, C, E, D, F, G

Now stack is empty

A, B, C

Now First ... after D

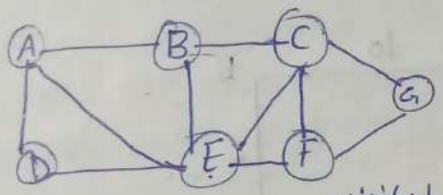
Breadth First Search (BFS)

* it means level by level searching
* to implement BFS we used a data structure called Que (i.e FIFO)

Algorithm

1. select any vertex as starting vertex insert that vertex into Queue. and make the status as visited
2. while the Queue is Not empty
   a) delete a vertex from the Queue and display it.
   b) insert all the unvisited Adjacent vertices of that vertex deleted vertex into the Queue and make the status as visited.

Ex:-



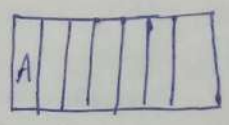
visited vertex matrix

A	B	C	D	E	F	G
1						

let us assume A is starting vertex.

Step 1 :-

Queue

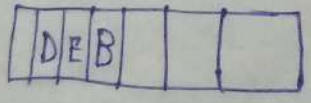


Result

empty

Step2 :-

if the Que is not empty then delete a vertex from queue and display it

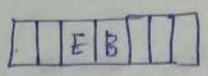


Result

A

Step 3 :-

while the queue is Not empty delete an element from Queue



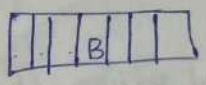
A, D

we have to insert all the unvisited Adjacent vertices of D

D Adjacent vertices are A and E it is already visited

So there is No need to perform enqueue op

Step 4



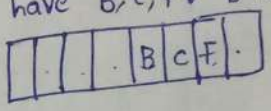
A, D, E

*first we have to delete a element from Queue

*so, E is Deleted

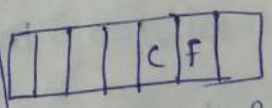
*so, next we have to insert all the unvisited Adjacent vertices E into Queue

*E have B, C, F. B is visited so C & F are insert



Step 5 :-

*Queue is Not empty delete an element from Queue.



A, D, E, B

*so, delete B from Queue.

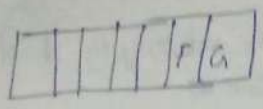
* insert all the unvisited Adjacent vertices of B into Queue

B have C and E, C & E are already visited

Step 6 :-

* delet an element from Queue
* so, next insert all unvisited Adjacent vertices of C
* C have F and G, F are alredy visited insert G and make it visited

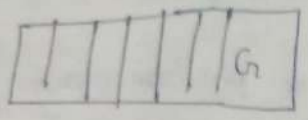
DEF followed by G.



A, D, E, B, C

Step 7

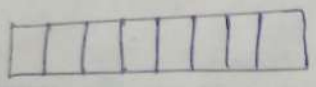
* delete an element from Queue
* Starting element is F, F is deleted
* Adjacent vertices of F are C, E and G
* But all the nodes are visited no unvisited Adjacent vertices.
* So no need to perform enqueue operation



A, D, E, B, C, F

Step 8 :-

* Delete G from Queue
* There are no unvisited vertices of G
* So the output is



A, D, E, B, C, F, G

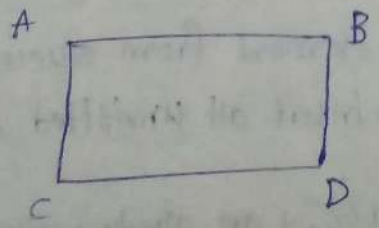
zero level is A D
first level is E B
Second level is C F
Third level is G

Spanning tree

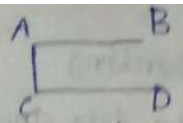
A spanning tree is a subgraph of the given graph

2) A spanning tree must satisfies 3 properties.

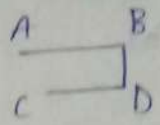
1) it should contain all vertices of graph
2) if graph contain n vertices then the spanning tree contain n-1 edges
3) spanning tree should not contain any cycle



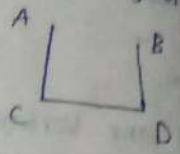
1)



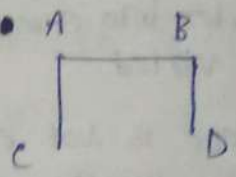
2)



3rd spanning tree



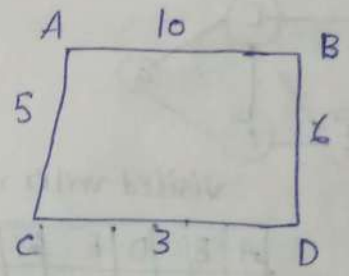
4th spanning tree



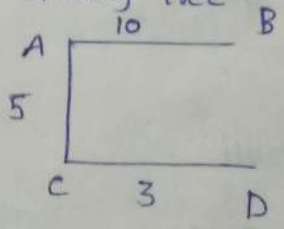
minimum cost spanning tree

A spanning tree in which cost is minimum. Here the cost can also be cost of minimum spanning tree is small when compared to remaining

~~spanning tree~~

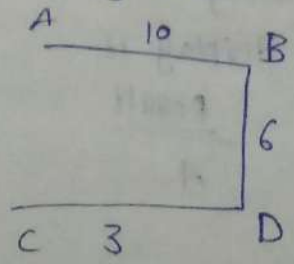


1st spanning tree



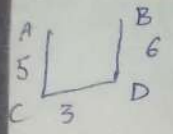
cost = 10+5+3
= 18

2nd spanning tree



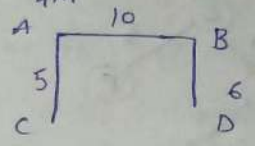
cost = 10+6+3

3rd



cost = 14

4th



cost = 21

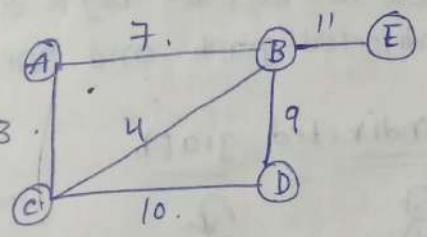
So, out of all the trees 3rd spaning tree is minimum spaning tree

Prim's Algorithm

1) start with any vertex of graph
2) find the edges associated with that vertex and add minimum cost edge to the spaning tree if it is not performing any cycle

* suppose By adding that edge if a cycle forms Then we have to discard that edge.

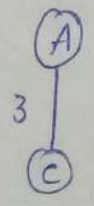
3) we have to repeat step 2 till all the vertices are covered



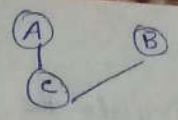
let us start with A vertex

A-B (7)
A-C (3)

out of these edges A-C has least cost edge. So we have to add this edge to spaning tree if it ~~perf~~ Not forming any ~~ge~~ cycle.



A-B (7)
C-B (4) — is least cost
C-D (10)

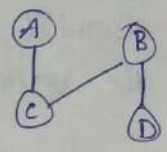


A-B (7)
C-D (10)
B-D (9)
B-E (11)

add what are edge associated with New edge.

A-B (7) is least cost. but cycle was formed so discard it

Next minimum cost B-D(9). add the edge it is Not formed cycle.

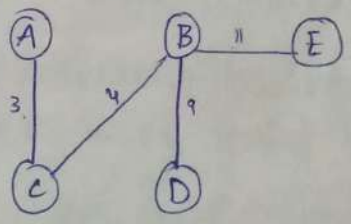


C-D (10)
B-E (11)

New edge is D. so add what are edges associat with D. ~~but there is~~ but there is No New edge. all are included.

C-D (10) is least. but cycle Formed so dis carded

The last edge B-E (11) is added. it is Not Form cycle.



cost = 27

* it should also contain all vertices of graph

* it ~~should~~ graph contain n ~~edg~~ vertices it should contain n-1 edges so 5 vertices and 4 edges

* it should ~~a~~ Not Contain any cycle

So It is correct minimum cost spaning tree with help of Prim's algorithm

Kruskal's algorithm

* it is mainly useful inorder to calculate the minimum cost spaning tree

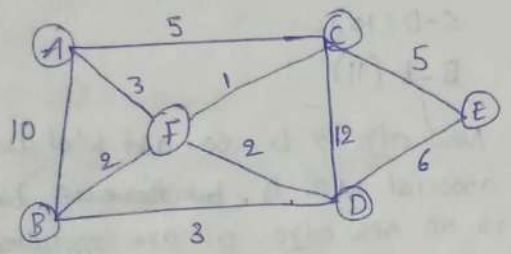
steps

1) we have to arranges all nodes in ascending order based upon their cost.

2) Select minimum cost edge from the edge list and add that edge to spaning tree if it is not forming cycle.

   * if it forms a cycle discard it

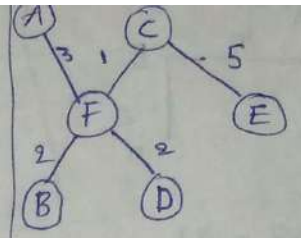
3) Repeat step 2 untill all the vertices are covered



6 vertices
5 edges

1. edge cost
   C-F 1 ✓
2. B-F 2 ✓
3. D-F 2 ✓
4. A-F 3 ✓
5. B-D 3 ✗
6. A-C 5 ✗
7. C-E 5 ✓
8. D-E 6 ✗
9. A-B 10 ✗
10. C-D 12 ✗

1st step over, arranging all edges in ascending order.



This is minimum cost spaning that is obtained by using Kruskal algorithm.

cost = 13

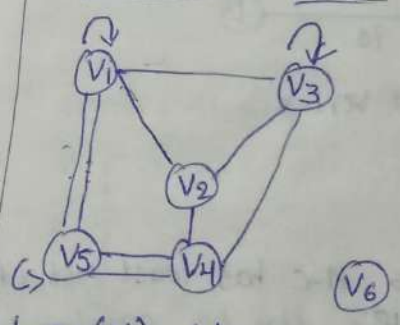
* our spaning tree also contain all (6) vertices
* spaning tree should contain n-1 edges are also 5.
* it should not contain any cycle.

In this way we construct a minimum cost of spaning tree with the help of Kruskal's algorithm.

Degree of Vertex

How to calculate Degree of Vertex in undirected graph and Directed graph

undirected graph



loop = 2

Isolated node :

degree $(V_1) = 4+2 = 6$

degree $(V_2) = 2$

degree $(V_3) = 3+2 = 5$

degree $(V_4) = 4$

degree $(V_5) = 4+2 = 6$

degree $(V_6) = 0$

In this way we calculate a degree of undirected graph.

Directed

degree =

Indgree =

outdegree

V1 :-

V2 :-

V3 :-

V4 :-

V5

In

degree

* A

are

* A

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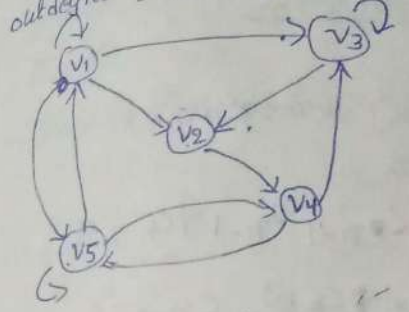
Directed graph

Degree = Indegree of vertex + outdegree of vertex

Indgree = No. of edges which are coming into a vertex

outdegree = coming out from the vertex



100 P = 1

V1 :- Indgree (V1) = 1+1 = 2
outdgree (V2) = 3+1 = 4 } 6

V2 :- Indgree (V2) = 2
outdegree (V2) = 1 } 3

V3 :- Indgree (V3) = 2+1 = 3
outdgree (V3) = 1+1 = 2 } 5

V4 :- Indgree (V4) = 2
outdegree (V4) = 2 } 4

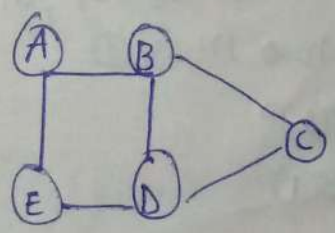
V5 :- Indegree (V5) = 2+1 = 3
outdegree (V5) = 2+1 = 3 } 6

In this way we have to calculate degree of vertex.

Hamiltonion cycle using Backtracking

* it means it is a path which all vertices are visited Exactly onces
* And after that we have to visit source vertex.

Ex

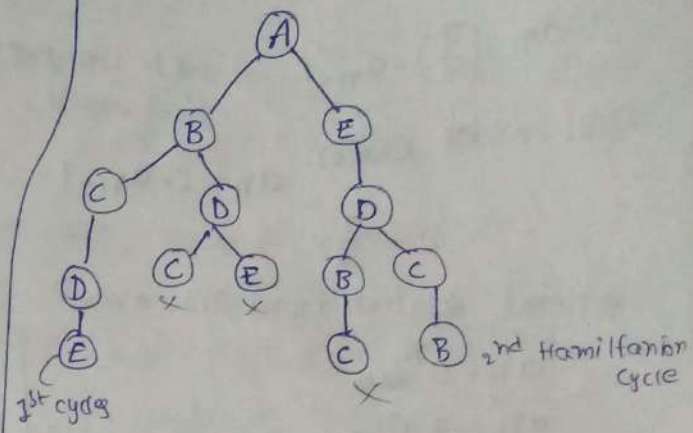


A B C D E A

A E D C B A

construct states in order to findout all the Hamiltonian cycle.

So, let we have a graph like this



5. Reccurance Relation

Definition

A Recurrance Relation for the sequence $a_0, a_1, a_2 \cdots a_n$ is an equation that relates a_n to its predecessors such as $a_0, a_1 \ldots a_{n-1}$

-> So, Recurrance Relation given a relation regards how a_n relates to its predecessors

Example of Recuran Relation is Fibanocci sequence

$F_n = F_{n-1} + F_{n-2}$ $n \geq 2$ $F_0 = 0$

$F_1 = 1$, $F_2 = F_1 + F_0$

$F_2 = 1 + 0$
$= 1$

$F_3 = F_2 + F_1$
$= 1 + 1$
$= 2$

Seq a = (0, 1, 1, 2, 3, 5)

Fibanoci means preview 2 elements added the 3rd element get

$F_4 = F_3 + F_2$
$= 2 + 1$
$= 3$

$F_5 = F_4 + F_3$
$= 3 + 2$
$= 5$

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First order

$6a_n - 7a_{n-1} = 0$ for ≥ 1 $a_3 = 343$

$6a_n = 7a_{n-1}$

$a_n = \left(\frac{7}{6}\right) \cdot a_{n-1}$ → to changing the first order

First order Relation $a_n = c \cdot a_{n-1}$

$c = \frac{7}{6}$

general solution from first order

$a_n = c^n a_0$

$a_n = \left(\frac{7}{6}\right)^n a_0$

Substitute $n = 3$

$a_3 = \left(\frac{7}{6}\right)^3 \cdot a_0$

$a_3 = \frac{343}{216} \times a_0$

$343 = \frac{343}{216} \times a_0$

$a_0 = 216$

$a_n = \left(\frac{7}{8}\right)^n \cdot 216$

2) $a_{n+1} = 4 \cdot a_n$ for $n \geq 0$ $a_0 = 3$

$a_{n+1} = 4 \cdot a_{n-1}$ $\forall$ $n-1 > 0$

$n > 1$

$a_n = 4a_{n-1}$

$a_n = c \cdot a_{n-1}$

$c = 4$

substitut

$\boxed{a_n = 4^n \cdot 3}$

Second order

Solve the following Recurrence Relation.

$a_n + a_{n-1} - 6a_{n-2} = 0 \ \forall \ n \geq 2$

$a_0 = -1$

$a_1 = 8$

Second order Homogenius Recurrence Relation is

$c_n \cdot a_n + c_{n-1} a_{n-1} + c_{n-2} a_{n-2}$

on comparing $c_n = 1$, $c_{n-1} = 1$, $c_{n-2} = -6$

Auxiliary eq

$c_n k^2 + c_{n-1} \cdot k + c_{n-2} = 0$

$1 \cdot k^2 + 1 \cdot k + (-6) = 0$

$k^2 + k - 6 = 0$

$k^2 + 3k - 2k - 6 = 0$

$k(k+3) - 2(k+3) = 0$

$(k+3)(k-2) = 0$

$k_1 = 3$ $k_2 = 2$

Thus roots are real and disjoint because the general sol form of 2nd order homog recur relation

case (i)

$a_n = A(k_1)^n + B(k_2)^n$

$= A(-3)^n + B(2)$ → ①

Initial conditions $a_0 = -1$, $a_1 = 8$

substitute $n = 0$ in eq ①

$a_0 = A(-3)^0 + B(2)^0$

$-1 = A(1) + B(1)$

$-1 = A + B$

$A + B = -1$

substitute $n=1$ in eq ①

$a_1 = A(-3)^1 + B(2)^1$

$8 = -3A + 2B$

$-3A + 2B = 8$

$A + B = -1 \quad \times 2$

$-3A + 2B = 8$

$2A + 2B = -2$

$-3A + 2B = 8$

$5A = -10$

$A = \frac{-10}{5}$

$A = -2,$

$A + B = -1$

$B = -1 - A$

$= -1 - (-2)$

$= -1 + 2$

$= 1$

$a_n = (-2)(-3)^n + 1(2)^n$

2) $a_{n+2} - 3a_{n+1} + 2a_n = 0 \ \forall \ n \geq 0$

$a_{n-2+2} - 3a_{n-2+1} + 2a_{n-2} = 0 \ \forall \ n-2 \geq 0$

$a_n - 3 \cdot a_{n-1} + 2a_{n-2} = 0 \ \forall \ n \geq 2$

general

$C_n \cdot a_n + C_{n-1} a_{n-1} + C_{n-2} a_{n-2} = 0$

$C_n = 1, \ C_{n-1} = -3, \ C_{n-2} = 2$

Auxilary

$C_n K^2 + C_{n-1} \cdot K + C_{n-2} = 0$

$1K^2 - 3K + 2K = 0$

$K^2 - 2K - K + 2K = 0$

$K(K-2) - 1(K-2) = 0$

$K = 1, 2$

$K_1 = 1 \quad K_2 = 2$

roots are real & different

case 1:

$a_n = A(K_1)^n + B(K_2)^n$

$= A(1)^n + B(2)^n$

Initial conditions $a_0 = 1, \ a = 6$

substitute $n=0$ in eq ①

$a_0 = A(1)^0 + B(2)^0$

$a_0 = A + B$

$A + B = 1$

substitute $n=1$ eq ①

$a_1 = A(1)^1 + B(2)^1$

$a_1 = A + 2B$

$A + 2B = 6$

$A + 2B = 6$

$A + B = 1$

$A + 2B = 6$

$-B = -5$

$\boxed{B = 5}$

$A = 1 - B$

$= 1 - 5$

$\boxed{A = -4}$

$a_n = (-4)(1)^n + 5(2)^n$

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Case - 2 ← second order

2) $a_n - 6 \cdot a_{n-1} + 9 \cdot a_{n-2} = 0 \ \forall \ n \geq 2$ given that

$a_0 = 5, \ a_1 = 12$

~~Auxilary eq~~ Second order Homog

$c_n \cdot a_n + c_{n-1} a_{n-1} + c_{n-2} a_{n-2}$

$c_n = 1, \ c_{n-1} = -6, \ c_{n-2} = 9$

Auxilary

$c_n \cdot k^2 + c_{n-1} \cdot k + c_{n-2} = 0$

$1 \cdot k^2 - 6k + 9 = 0$

$k^2 - 6k + 9 = 0$

$k^2 - 3k - 3k + 9 = 0$

$(k-3)(k-3) = 0$

$k = 3, 3$

The root are equal we follow case 2

case - 2

$a_n = (A + Bn) k^n$

$a_n = (A + Bn) 3^n$

Substitute $n = 0$ eq ①

$a_0 = (A + B \times 0) 3^0$

$5 = (A) 1$

$A = 5$

substitut $n = 1$ eq ①

$a_1 = (A + B \cdot 1) 3^1$

$12 = (A + B) 3$

$A + B = \frac{12}{3}$

$= 4$

$B = 4 - A$

$= 4 - 5$

$= -1$

$a_n = (5 - n) 3^n$

$a_n = 2(a_{n-1} - a_{n-2}) \ \forall \ n$

~~$a_n = 2a_n$~~ $a_n - 2a_{n-1} =$

second order form $= a_{n-2}$

$c_n \cdot a_n + c_{n-1} a_{n-1} + c_{n-2}$

$c_n = 1, \ c_{n-1} = 2, \ c_{n-2} = -2$

Auxilary equation

$c_n \cdot k^2 + c_{n-1} \cdot k + c_{n-2} = 0$

~~$1 \cdot k^2 - 2k - 2 = 0$~~

$k^2 - 2k + 2 = 0$ $a = 1, b = -2, c = 2$

$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$d = \frac{2 \pm \sqrt{4 - 8}}{2}$

$= \frac{2 \pm \sqrt{4 \cdot -2}}{2}$

$\frac{2 \pm 2 \times i}{2}$

$= 1 \pm i$

$k_1 = 1 + i, \ k_2 = 1 - i$

$= p + iq, \ p - iq$

$p = 1, \ q = 1$

case 3 :-

$a_n = r^n (A \cos n\theta + B \sin n\theta)$

$r = \sqrt{p^2 + q^2}$

$\theta = \tan^{-1}(q/p)$

$r = \sqrt{1^2 + 1^2}$

$r = \sqrt{2}$

$\theta = \tan^{-1}(1/1)$

$= 45^\circ$

$a_n = (\sqrt{2})^n (A\cos n45^\circ + B\sin 45^\circ)$

: sublitute $n=0$ in ①

$a_0 = (\sqrt{2})^0 (A\cos 0.45^\circ + B\sin 0.45^\circ)$

$1 = 1\,(A\cos 0 + B\sin 0)$

$1 = 1\,(A.1 + B.0)$

$1 = A$

$A = 1$

sublitute $n=1$ in eq ①

$a_1 = (\sqrt{2})^1 (A\cos 1.45^\circ + B\sin 1.45^\circ)$

$2 = (\sqrt{2})^2 \left(A\frac{1}{\sqrt{2}} + B\frac{1}{\sqrt{2}}\right)$

$2 = \sqrt{2}\left(\frac{A+B}{\sqrt{2}}\right)$

$A+B = 2$

$B = 2-A$

$B = 2-1$

$B = 1$

gen sol of problem

$a_n = (\sqrt{2})^n (1.\cos n45^\circ + 1.\sin 45^\circ)$

$2.a_{n-2} = 0$

$a_n = 6a_{n-1} - 12a_{n-2} + 8 - a_{n-3} \quad n \geq 3$

$a_0 = 1$

$a_1 = 4$

$a_2 = 28$

$a_{n-3}\, c_{n-3} = 0$

$+$

$a_n c_n + a_{n-1} c_{n-1} + a_{n-2} c_{n-2}$

$a_n - 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$

$c_n = 1 \quad c_{n-1} = -6 \quad c_{n-2} = 12 \quad c_{n-3} = -8$

Auxilary eq

$c_n k^3 + ck^2 + ck + c = 0$

$k^3 - 6k^2 + 12k - 8 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 12 & -8 \\ & 0 & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$k=2, \ (k-2)^2 = 0$

$k=2, \ k=2, 2$

The roots are equal so use case 2

Case 2 :-

$a_n = (A + Bn + Cn^2)k^n$

$a_n = (A + Bn + Cn^2)2^n \rightarrow ①$

subtitute $n=0$ in eq -①

$a_0 = (A + B.0 + C.0^2)2^n$

$1 = (A + 0 + 0)2^0$

$1 = (A + 0 + 0)1$

$1 = A$

$\boxed{A = 1}$

Subtit $n=1$ in eq ①

$a_1 = (A + B.1 + C1^2)2^1$

$\frac{4}{2} = (A + B + C)2$

$2 = A + B + C$

$a_2 = (A + B2 + C2^2)2^2$

$a_2 = (A + 2B + 4C)4$

$A + 2B + 4C = \frac{28}{4}$

$A + 2B + 4C = 7 \rightarrow$

$A + B + C = 2 \rightarrow *2$

$2A + 2B + 2C = 4$

$A + 2B + 4C = 7$

$-A \quad -2C = -3$

$-2C = -3 - A$

Sub $n=2$ in eq①

$-2C = -3-1$

$-2C = -4$

$C = \frac{4}{2}$

$\boxed{C = 2}$

$A + B + C = 2$

$B = 2 - A - C$

$= 2 - 1 - 2$

$\boxed{B = -1}$

$a_n = (A + Bn + Cn^2)2^n$

$a_n = (1 + (-1)n + 2n^2)2^n$

---

4. Algebraic System

we perform several operation on set is called $\langle S, +, * \rangle$

$\langle S \xrightarrow{\text{means}}$ non empty set.

They are two operation

1) unary $\longrightarrow$ performing operation of one operand $-a$ → unary meaning only one operand

2) binary $\longrightarrow$ performing operation on two operand $a+b$, $a-b$ → binary mi

* Algebraic system contains non empty set. And we perform various operations on non empty set.

like $+, *, -$ like several binary operations. mainly perform binary operations

general properties of Algebraic System

$\langle S, +, * \rangle$ → set name S

1. closure property

for any two elements $a, b \in S$ then $a+b \in S$ and $a*b \in S$

Ex:- $1, 2 \in S$

$1+2 \in S$

$3 \in S$

$1*2 \in S$

$2 \in S$

3 and 2 are present in S so closure $S = \{0, 1, 2, 3 \ldots \alpha\}$

2. Associative Property

we have to take 3 elements from the set $a, b, c \in S$ then

$(a+b)+c = a+(b+c)$

$(a*b)*c = a*(b*c)$ because algebraic system have $\langle S, +, * \rangle$ Two operations

if algebraic System satisfies these two properties then we can say that it is Associative property.

Ex:- $1, 2, 3 \in S$

$(1+2)+3 = 1+(2+3)$

$3+3 = 1+5$

$6 = 6$

$(1*2)*3 = 1*(2*3)$

$2*3 = 1*6$

$6 = 6$

Here two formulas satisfies so it is Associative property.

3. commutative property

we have to take any 2 elements $a, b \in S$ then

$a+b = b+a$

$a*b = b*a$

It satisfies these 2 it is commutative property

Ex: $1, 2 \in S$

$1+2 = 2+1$

$3 = 3$

$1*2 = 2*1$

$2 = 2$

it satisfies commutative property.

4. Identity element

for any 2 elements $a, e \in S$ then

Identity element

$a+e = e+a = a$

$a*e = e*a = a$

for addition operation the Identity element $= 0$

for multiplication operation the identity element $= 1$

Ex $a=1$

$1+0 = 0+1 = 1$

Ex $a=2$ → taking values

$2*1 = 1*2 = 2$ → and result are same it is Associative

5. Inverse Element

$a \in S$ There exists one inverse of element a

$a^{-1} \in S$ $\therefore a+a^{-1} = a^{-1}+a = e$ → The identity element $+ = 0$ so we take result 0 The calculated result also 0 so it satisfies

Ex $a = 5 \in S$

$5+(-5) = -5+5 = 0$

$0 = 0 = 0$

$a*a^{-1} = a^{-1}*a = e$

Ex :- $a = 3$

~~$3*(-3) = -3*3$~~

$a*a^{-1} = e$

$3*(a^{-1}) = 1$

$a^{-1} = 1/3$

Substitute a^{-1} value

$\cancel{3}*1/\cancel{3} = 1/\cancel{3}*\cancel{3} = 1$ → Ident element 1 for multi * It satisfies

$1 = 1 = 1$

6. Distributive property

we have to take any 3 elements $a, b, c \in S$

$a+(b*c) = (a+b)*(a+c)$

$a*(b+c) = (a*b)+(a*c)$

Ex : $a, b, c = 1, 2, 3 \in S$

$1+(2*3) = (1+2)*(1+3)$

$7 \neq 12$

It is Not distributive.

Ex: $1 * (2+3) = (1*2) + (1*3)$

$5 = 2+3$

$5 = 5$

It satisfies distributive

7. Cancellation Property

we have to take 3 elements from the set $a, b, c \in S$ & $a \neq 0$

~~a*b and a~~ $a*b = a*c$ then $b = c$

$b*a = c*a$ then $b = c$

8. Idempotent property

we have to only take 1 element from the set $a \in S$

$a+a = a$

$a*a = a$

It satisfies these it is Idempotent property.

Semi groups of Algebraic System

an algebraic system called $\langle S, *, + \rangle$ → Binary operations

↳ non empty set

An algebraic system $*, +$ is called semi group if it satisfies two property

1. closure Porperty
2. Associative Property

1. Closure Porperty

for any 2 Elements $a, b \in S$ then

$a+b \in S$ and

$a*b \in S$

Ex

$S = \{1, 2, 3 \ldots N\}$

$1, 2 \in S$

$1+2 \in S$ & $1*2 \in S$

$= 3$ $\quad = 2$

3 & 2 are present in set $<S$ so it satisfies

<u>Associative property</u>

for any 3 elements $a, b, c \in S$

$a+(b+c) = (a+b)+c$ &

$a*(b*c) = (a*b)*c$

<u>EX</u> 1, 2, 3

$1+(2+3) = (1+2)+3$

$1+5 = 3+3$

$6 = 6$

$1*(2*3) = (1*2)*3$

$1*6 = 2*3$

$6 = 6$

So, it both are same it satisfies associative property

-* Some Examples of semi group

E = set of even numbers

<u>EX</u>: $<E, +>$ ——————→ Quetion

$E = \{2, 4, 6, 8, 10 \ldots\}$

$2+(4+6) = (2+4)+6$

$2+10 = 6+6$

$12 = 12$

The 12 are present in set E and satisfies

<u>EX2</u> :-

$<W, *>$ is semigroup or not ?

W = set of whole numbers

$W = \{0, 1, 2, \ldots \infty\}$

we have to take 2 elements from set W

it satisfies closure or not

we have to take 3 elements from set

it satisfies Associativ or not

this is the process of semi groups of algebraic system.

Monoid in Algebraic system

$\langle S, +, * \rangle$

If it satisfies 3 properties

1. closure property
2. Associative property
3. Identity element

1. closure property

for any 2 elements $a, b \in S$ then

$a+b \in S$ &

$a*b \in S$

Ex :- $\langle W, +, * \rangle$

W = set of whole numbers

$W = \{0, 1, 2, 3, 4, 5 \ldots\}$

$1, 2 \in S$

$1+2 \in S \quad = 3$

$1*2 \in S \quad = 2$

two elements are present in set so it satisfies closure property.

2. Associative property

for any 3 elements $a, b, c \in S$ then

$a+(b+c) = (a+b)+c$

$a*(b*c) = (a*b)*c$

Ex :- $1, 2, 3 \in S$

$1+(2+3) = (1+2)+3$

$1+5 = 3+3$

$6 = 6$

$1*(2*3) = (1*2)*3$

$1*6 = 2*3$

$6 = 6$

So both are same so it satisfies Associative property.

3. Identity element

we have to take 2 elements from the set $a, e \in S$

$a+e = e+a = a$ → Identity element

$a*e = e*a = a$

$+ = 0$

$* = 1$

Ex :- $3 \in S$ = $a = 3$

$3+0 = 0+3 = 3$ ✓

$3*1 = 1*3 = 3$ ✓

It satisfies Indentity element.

* Monoid is always a semigroup

$\langle N, + \rangle$

N = set of Natural nos

$N = \{1, 2, 3, \ldots \infty\}$

It does not satisfies Identity element

$\langle N, + \rangle$ is not Monoid

because in Natural num No '0' are there

Abelian group

1) show that $G = \{1, -1, i, -i\}$ is an abelian group under multiplication → first we perform * only

~~$G = \{0, 1, 2, 3, 4, 5\}$ P.T G is Abelian group~~

$\langle S, *, + \rangle$ non empty set

→ Next no operation specified so taking + binary opn

1. closure property
2. Associative property
3. Identity property
4. Inverse
5. commutative

if These 5 Satisfies Then it is called Abelian group.

1) $\boxed{G = \{1, -1, i, -i\} \text{ first problem}}$

1. closures property

for any 2 elements $a, b \in G$ ors then

$a+b \in S$

$a*b \in S$

$G = \{1, -1, i, -i\}$

$a, b \in S \Rightarrow 1, -1 \in G$ → taking

$1 * -1 \in G$

$= -1$

It. Satisfies first property closure property

$\boxed{\text{2nd problem } G = \{0, 1, 2, 3, 4, 5\}}$

1. clossure property

$1, 2 \in G$

$1 + 2 \in G$

$= 3$

So 3 are present in group.

~~$1, 2 \in G \Rightarrow 1+2 \in G$~~

~~$= 3$~~

~~So, -1 are present in G and 3 are present in group~~

2. Associative property

for any 3 element $a, b, c \in S$ then

$a+(b+c) = (a+b)+c$

$a*(b*c) = (a*b)*c$

$1, -1, i \in G$

$1*(-1*i) = (1*-1)*i$

$-i = -i$

∴ it satisfies Associative property.

3. Identity elements

for any 2 element $a, e \in S$ then

$a+e = e+a = a$ Ident $+ = 0$

$a*e = e*a = a$ Ident $* = 1$

Here im Assuming to take $-i = a$ $-i \in G$

$-i*1 = 1*-i = -i$

Result $= -i$ 'a' values also $a = -i$

So, it satisfies Identity element

4. Inverse element

$a \in S$ there exists $a^{-1} \in S$

$a+a^{-1} = a^{-1}+a = e$

$a*a^{-1} = a^{-1}*a = e$

$\boxed{+=0 \quad *=1}$

$a \in S \Rightarrow$ Here im Assuming to take $i = a$ $i \in G$

$a*a^{-1} = e$

$a^{-1} = \frac{e}{a}$

2) Associative property

$1, 2, 3 \in G$

$1+(2+3) = (1+2)$

$1+5 = 6$

$6 = 6$

from 2nd problem $G = \{0, 1, 2, 3, 4\}$

Also satisfies

3 Identity element

Here im Assuming $a = 3$

$3+0 = 0+3 = 3$

Result value 3 assuming also

So it satisfies

4. Inverse element

$a = 2$

$a+a^{-1} = e$ $+=0$

$a^{-1} = e-a$ → doing $+=$

$a^{-1} = e-2$

$a^{-1} = 0-2$

$a^{-1} = -2$ Substituting

$a+a^{-1} = a^{-1}+a = e$

$2+(-2) = (-2)+2 = 0$

$0 = 0$

The Addition $+=0$ Result also

So, it satisfies Inverse

$= \frac{1}{i}$ substituting

$a * a^{-1} = a^{-1} * a = e$

$i * \frac{1}{i} = \frac{1}{i} * i =$

$1 = 1$

$e = 1$

The multiplicat I The result also I

so, it satisfies Inverse element

4. commutative property

for any 2 element $a, b \in S$

$a+b = b+a$

$a*b = b*a$

here we taking $1, -1 \in S$

$1*-1 = -1*1$

$-1 = -1$

LHS = RHS

so it satisfies commutative property.

5. commutative property

Taking 2 element $2+3 \in S$

$a+b = b+a$

$2+3 = 3+2$

$5 = 5$

LHS = RHS

so it satisfies commutative property

Group in Algebraic system

Ex 1: show that set of all non zero real numbers

Ex2: In a group $(G, *)$ prove that

$(a*b)^{-1} = b^{-1} * a^{-1} \ \forall \ a, b \in G$

$\langle S, * \rangle \qquad S = \{1, 2, 3 \ldots\}$

1.) closure property
2) Associative property
3) Identity property
4) Inverse property

if It satisfies Then it is called Group in Algebraic system.

closure property

for any 2 elements $a, b \in S$

$a+b \in S$

$a*b \in S$

Here we taking elements 1, 2 from set $1, 2 \in S$

$1 * 2 \in S$

$= 2$

2 is present is set s so it satisfies

Associative property

for any 3 elements $a, b, c \in S$ then

$a + (b + c) = (a + b) + c$

$a * (b * c) = (a * b) * c$ → we need to perform given question mentioned *

Here we taking 1,2,3 from set $1, 2, 3 \in S$

$1 * (2 * 3) = (1 * 2) * 3$

$1 * (6) = 3 * 3$

$6 = 6$

LHS = RHS

∴ So Associativ proproperty is satisfied

Indentity element

for any 2 element $a, e \in S$ then

$a + e = e + a = a$ $\quad + = 0$

$a * e = e * a = a$ $\quad * = 1$

Here we taking 3 from set $3 \in S$

$3 * 1 = 1 * 3 = 3$

Here taking value 3 and Result also 3 so it satisfies

Inverse property

for any element $a \in S$ there exists $a^{-1} \in S$

$a * a^{-1} = a^{-1} * a = e$ $\quad + = 0$

$a + a^{-1} = a^{-1} + a = e$ $\quad * = 1$

Here we taking value 5 from set $5 \in S$

$a * a^{-1} = e$

$5 * a^{-1} = 1$

$a^{-1} = 1/5$

substituting in formula

$a * a^{-1} = a^{-1} * a = e$

$5 * 1/5 = 1/5 * 5 = 1$

$1 = 1 = 1$

Here it is also satisfies

Ex2: In a group $(G, *)$ P.T $(a*b)^{-1} = b^{-1} * a^{-1} \forall a, b \in G$

Inverse element

$\cancel{(a*b)^{-1} = b^{-1} * a^{-1}}$

$a * a^{-1} = a^{-1} * a = e$

$(a*b)^{-1} = b^{-1} * a^{-1}$

$(a*b) * (b^{-1} * a^{-1}) = e$

$a * (b * b^{-1}) * a^{-1}$

$a * e * a^{-1}$

$e * a * a^{-1}$

$e * e$

$= e$

same

So, it satisfies

Sub group in Algebraic system

Ex: let $\langle G, * \rangle$ is a group $G = \{1, -1, i, -i\}$ & let $\langle H, * \rangle$ is a sub group of G check wheather $\{1, -1\}$ is a subgroup or not?

$\langle G, * \rangle \rightarrow$ group

$\langle H, * \rangle \rightarrow$ subgroup of G

1. closure
2. Associative
3. Identity
4. Inverse

If These satisfies Then it is sub group

1. closure property

taking elements in subgroup

$a, b \in H$

if any 2 element $a, b \in S$ then

$a*b \in S$

$a*b \in H$

Here we take $1, -1$ $1, -1 \in H$

$a*b \in H$

$1 * -1 \in H$

$= -1$

-1 is present in set H so it satisfies

2. Associative property

for any 3 elements $a, b, c \in S$ then

$a*(b*c) = (a*b)*c$

Here sub group have only 2 element so 1 is taken two times as a, c

$1, -1, 1 \in H$

$1*(-1*1) = (1*-1)*1$

$1*-1 = -1*1$

$-1 = -1$

LHS = RHS

So Associative property satisfies

3. Identity element

for any 2 element $a, e \in H$

$a * e = e * a = a$ $\quad + = 0$

$e = 1 \quad * = 1$

Here we taking $a = -1$ from set H $-1 \in H$

$-1 * 1 = 1 * -1$

$-1 = -1$

Here taking value -1 and result also -1

so, it satisfies

4. Inverse element

let $a \in H$ There exist $a^{-1} \in H$

$a * a^{-1} = a^{-1} * a = e$ $\quad + = 0$

$* = 1$

taking value $a = 1$ from set H $1 \in H$

$a * a^{-1} = e$

$1 * a^{-1} = 1$

$a^{-1} = \frac{1}{1}$

substituting from formula

$a * a^{-1} = a^{-1} * a = e$

$1 * \underset{+}{1} = 1 * \underset{+}{1} = 1$

$1 = 1 = 1$

So inverse also satisfies

So, $\langle H, * \rangle$ of $G = \{1, -1\}$ is a sub group of $(G, *)$ of $G = \{1, -1, i, -i\}$

Semigroup Homomorphism

definition

Consider 2 semigroups $\langle S, * \rangle$ and $\langle T, + \rangle$ let f be a function from S to T i.e $f : S \to ST$ then f is called as Homomorphism from S to T is it satisfies the condition $f(a*b) = f(a) + f(b)$ where $a, b \in S$ & $f(a)\ f(b) \in T$.

Ex1 :- consider 2 semigroups $\langle R^+, * \rangle$, $\langle R, + \rangle$ let be function $f : R^+ \to R$ is $f(x) = \log_e x$ P.T f is homomorphism

formula for semigroup homomorphism $f(a*b = f(a) + f(b)$

$$f(a*b) = \log_e a*b$$
$$= \log_e a + \log_e b$$
$$= f(a) + f(b)$$

$\therefore \log_a m*n = \log_a m + \log_a n$

Lagrange's Theorem

one order of subgroup of a group divides order of groups let G be a group, H be a subgroup of group then $|H|$ divides $|G|$ where $|G|$ means order of group
$|H|$ means order of subgroup

Ex : Find all subgroups of group $\langle G, * \rangle$ where $G = \{1, -1, i, -i$

Basics of counting

sum of rule

out of all the Tasks we perform only one task

$T_1 \rightarrow$ 'm' ways

$T_2 \rightarrow$ 'n' ways

sum rule = $m+n$ ways

product of rule

out of all the Tasks we perform both the tasks

$T_1 \rightarrow$ 'm' ways

$T_2 \rightarrow$ 'n' ways

product rule = $m * n$ ways

sum rule

EX1: suppose these are 16 boys & 18 girls in a class. How many ways to select one of these students either a boy or girl as a class Representative.

Sol

T_1 = No. of ways for selects a boy as CR = 16 ways

T_2 = No. of ways for selects a girl as CR = 18 ways

$m+n$

$16+18$

$= 34$ ways

34 ways to select a student as CR.

Scanned with OKEN Scanner

Ex2: suppose a hostel library has 12 books on maths 10 books on physics, 16 books on chemistry 11 books on social How many ways to select whishes to choose one of book for study.

T_1 = selecting maths book for study = 12 ways
T_2 = selecting physics books for study = 10 ways
T_3 = selecting chemistry books for study = 16 ways
T_4 = selecting social book for study = 11 ways

~~m+n~~ sum of rule = m+n one of book for study so its sum of rule

12+10+16+11
= 59 ways

Ex3:- find out how many way for selects a prime number less than 10 & even number less than 10

out of numbers selects prime num or even number less than 10 so sum of rule

2, 3, 5, 7 → prime num < 10
= 4 ways

even num < 10 = 0, 2, 4, 6, 8
= 5 ways

Ignore 2 because present in both so.

4+5-1
= 9-1
= 8

Product rule

Ex1: suppose a person has 5 shirt & 7 ties How many ways a person can choose a shirt & tie

we perform product rule because Inorder to perform both tasks

T_1 = choose a 5 shirts
T_2 = choose a 7 ties

product of rule = m*n
= 5*7
= 35 ways

There 35 ways to choose a shirt & tie.

Ex2: How many ways to construct a sequence of 5 letters in which first 3 letters are english and next 2 are digits.

we perform product rule because Inorder to perform both tasks i.e

Here 2 ways to solve this problem
* Repetation allowed
* Repetation not allowed

Repetation allowed 1st letter → A have 26 ways

26 26 26 10, 10

next two are digits

Repetation not allowed

26

second letter not same as first letter 26 so $26-1 = 25$

26 25 24 10 9

→ next two digits so 10 second digit -1 $= 9$

Combinatorics

combinatorics is an important part of discrete mathematics that resolves counting Problems without acoutually enumerating all possible cases

combinatorics deals with counting the number of ways of arranging or choosing objects from finite set according to certain specified rules.

In other words, combinatrics is concerned with Problems of permutations and combinations.

Permatations

An ordered Arrangement of r elements of a set of containg n elements is called an r- Permutation of n elements and is denoted by $P(n, r)$ (or) ^nP_r where $r \leq n$

* necessary condition should be satisfied is $r \leq n$

$$P(n, r) = n(n-1)(n-2)\ldots(n-r+1)$$

$$\boxed{\begin{aligned} \therefore P(n, r) &= \frac{n!}{(n-n)!} \\ P(n, n) &= n! \end{aligned}}$$

→ This formula is used to ~~select~~ arrange n elements from set of n elements

Combinations

An ordered selection of r elements of a set containing n distinct elements is called an r-combination of n elements and is denoted by $C(n,r)$ (or) ${}^nC_r$ (or) $\binom{n}{r}$

$$\boxed{\begin{aligned} \therefore\ C(n,r) &= \frac{n!}{r!(n-r)!} \\ C(n,n) &= 1 \end{aligned}}$$

This formula is used to choose n elements from all elements in set of elements

Formulas

① Number of permutations of n distinct (different) objects (without duplicatn)

The number of different arrangements (permutations) of n distinct objects, taken all at a time is

$$\boxed{P(n,n)} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = \boxed{n!}$$

Then we mind it $n!$ is used for without duplication of n distinct objects.

③ Number of permutations of n objects (with duplication)

it is required to find the number of permutation that can be formed from a collection of n objects of which n_1 are one type, n_2 are of a second type, ... n_K are of Kth type with $n_1 + n_2 + \dots + n_K = n$, then the number of permutations of n objects are

$$\frac{n!}{n_1! * n_2! * \dots n_K!}$$

② Number of permutations of r objects among in distinct objects

Suppose we are given n distinct objects and wish to arrange r of the objects denoted by

$$P(n,r) = \frac{n!}{n-r!} \quad \text{or } {}^nP_r = P(n,r)$$

Circular permutation

permutation in a circle are called circular permutation. i.e we are ~~giving~~ given n number of objects These n number of objects are arranged in a circular or round table.

The Total number of ways of arranging the n persons in a circle $= (n-1)!$

Problems

1) How many way are there to sit 10 boys and 10 girls arround a circular table?

1) Here 10 Boys and 10 girls sit around a circular table

Total number of persen $n = 10+10$
$= 20$

Total no. of ways of circular permutations are $= (n-1)!$
$= (20-1)!$
$= 19!$

2) How many different arrangements of letters in the word BOUGHT?

The given word BOUGHT contains 6 letters that are distinct (without duplicate)

The no. of arrangements of letters word BOUGHT $= P(n,n) = n!$
$= 6!$
$= 6\times5\times4\times3\times2\times1$
$= 720$

2) How many way are there 3 persons sit arround a table

No. of persons $= 3$ $\therefore n=3$

Total number of ways of arranging 3 Person in arround a round table is $= (n-1)!$
$= (3-1)!$
$= 2!$
$= 2$

4) How many different strings of length 4 can be formed using the letter of the word PROBLEM?

Sol

The given word "PROBLEM" has 7 letters $\therefore n=7$

The Number of different strings of length 4 can be formed by using letter of word PROBLEM.

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$= \frac{7!}{(7-4)!}$$

$$= \frac{7!}{3!} = \frac{7\times6\times5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}}$$

$= 840$

Ex:5 How many words of 3 distinct letters can be formed from the letter of the word PASCAL

The given word PASCAL contain 6 letters
$n = 6$

The number of 3 distinct letters can be formed from letter word PASCAL

$$P(n, r) = P(6, 3) = \frac{6!}{(6-3)!}$$

$$= \frac{6!}{3!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

$= 120$ ways

Ex:6 Find the No. of permutations of the letter of the word ENGINEERING

The given word is ENGINEERING have 11 letters, out of which 3 are e, 3 are N 2 are I, 2 are G, 1 is R

∴ Total numbers of permutations of the letters of the word ENGINEERING

$$= \frac{n!}{n_1! * n_2! * n_3! * n_4! * n_5!}$$

$$= \frac{11!}{3! * 3! * 2! * 2! * 1!}$$

$=$

$= 270200$

Ex7: Find the Number of permutations of the letters of the words (i) SUCCESS

No. of letter in word SUCESS
$= 7$ ∴ $n = 7$

out of 7 letters, 3 are S,
2 are C, 1 are E,

∴ Total No. of permutation of letter SUCESS $= \frac{n!}{n_1! * n_2! * n_3!}$

$$= \frac{7!}{3! * 2! * 1! * 1!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{3! * 2 * 1 * 1}$$

$$= \frac{7 \times 6 \times 5 \times 4}{2}$$

$= 420$

Ex 8: Find the no. of permutation of the letters of word STRUCTURES

No. of letters in word STRUCTURES
$n = 10$

out of 10 letters 2 are S,
2 are T, 2 are R
2 are U, 2 are C
1 are E,

∴ Total no. of permutation $= \frac{n!}{n_1! * n_2! * n_3! * n_4! * n_5! * n_6!}$

$$= \frac{10}{2! * 2! * 2! * 2! * 1! * 1!}$$

$$\frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2!\times2!\times2\times!\,2!\times1!\times1!}$$

$$\frac{10\times9\times8\times7\times6\times5\times4\times3}{2\times2\times2\times!\times1}$$

$= 226800$

Find the No. of permutation of letter of word MATHEMATICS

No. of letter of word MATHEMATICS = 11 $n = 11$

out of which 2 are M, 2 are A, 2 are T, 1 are I, 1 are H, 1 are I, 1 are C, 1 are S

Total no. of permutation $= \frac{n!}{n_1!\times n_2!\times n_3!\times n_4\times n_5!\times n_6!\times n_7\times n_8!}$

$$= \frac{11!}{2!\times2!\times2!\times1!\times1!\times1!\times1!\times1!}$$

$$= \frac{11\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2!\times2!\times2!\times1!\times1!\times1!\times1!\times1!}$$

$= 4989,600$

(10) Find the no. of permutations of the letters of the word MASSASAUGA. In how many of these all four A's together? How many of them begins with S?

The given word "MASSASAUGA" has 10 letters

$\therefore n = 10$

out of 10 letters, 4 are A, 3 are S, and 1 each are M, U and G

$\therefore$ The required no. of permutations

$$= \frac{n!}{n_1!\times n_2!\,n_3!\times n_4!\times n_5!}$$

$$= \frac{10!}{4!\times3!\times1!\times1!\times1!}$$

$= 25,200$

In a permutation, all A's are to be together, we treate all of A's as are single letter. Then the letters to be permutated read (AAAA) S, S, S, M, U, G and the No. of permutation are

$$\frac{7!}{1!\,3!\times1!\times1!\times1!}$$

$= 840$

$\frac{n!}{n_1\times n_2!\times n_3!\,n_4!\,n_5}$

II

For permutations beging with S, there occur 9 open positions to fill, where 2 are S, 4 are A, and 1 each are M, U, G. The No. of such permutations are set

S -> fixed 10-1

S - - - - - -

$$= \frac{9!}{2!\times4!\times1!\times1!\times1!}$$

$$= \frac{9!}{2!\times4!\times1!\times1!}$$

1st letter fixed so 1S are minuesd so S = 2!

$= 7560$