1) Random experiment (or) Trial
2) Events (or) cases
3) Exhaustive no. of cases.
4) Favourable no. of cases.
5) equally likely cases.
6) Mutually Exclusive (or) disjoint cases
7) Independent event.

probability :-

In a trial there are 'n' outcomes which are equally likely and exhausting come out of which 'm' outcomes of favourable to a particular 'E' event then $P(E) = \frac{m}{n} = \frac{\text{Fav outcomes}}{\text{exhaustive outcomes}}$

Let $\bar{E}$ is a complimentary of (E) Then $\boxed{P(\bar{E}) = 1 - \frac{m}{n} = 1 - P(E)}$

Ex:-

if an unbaised (fair) coin is tossed 3 times find the probability of get-in

(i) Three heads

HHH	HTT
HHT	TTT
HTH	
TTH	
THT	

when The total possibilities is Three coins are tossed.

$2^n = 2^3 = 8$

$= \{HHH\}$

$P(E_1) = \frac{1}{8}$

2) atleast two heads

$P(E_2) = \frac{4}{8}$

P (2Heads) + P (3 Heads)

$3/8 + 1/8$

$= \frac{4}{8}.$

iii) probability of No heads

$P(E) = \frac{1}{8}$

2) when a die is Rolled twice calculate the proba -bility of happening

(i) Even number on both cases

The Total Possibilities ~~a~~ is a die is rolled Twice.

$6^n = 6^2 = 32$

(i) $P(E_1) = \frac{9}{36}$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(ii) Product of Two numbers is a perfect square.

1, 4, 9, 16, 25, 36

(1,1) (1,4) (2,2) (3,3) (4,4) (4,1) (5,5) (6,6)

$P(E_2) = 8/36$

(iii) sum of numbers is greater than 6.

$P(E_3) = 21/36$

(iv) ~~double~~ P

$P(E_4) =$

(v) probability

P(E

combinati

observatio

Pla

colour

suits D

Face ←
cards

3) if
the 6
gettin

(iv) double probability of getting double number.

$P(E_4) = 6/36$

(v) probability of getting sum 10 (or) 11

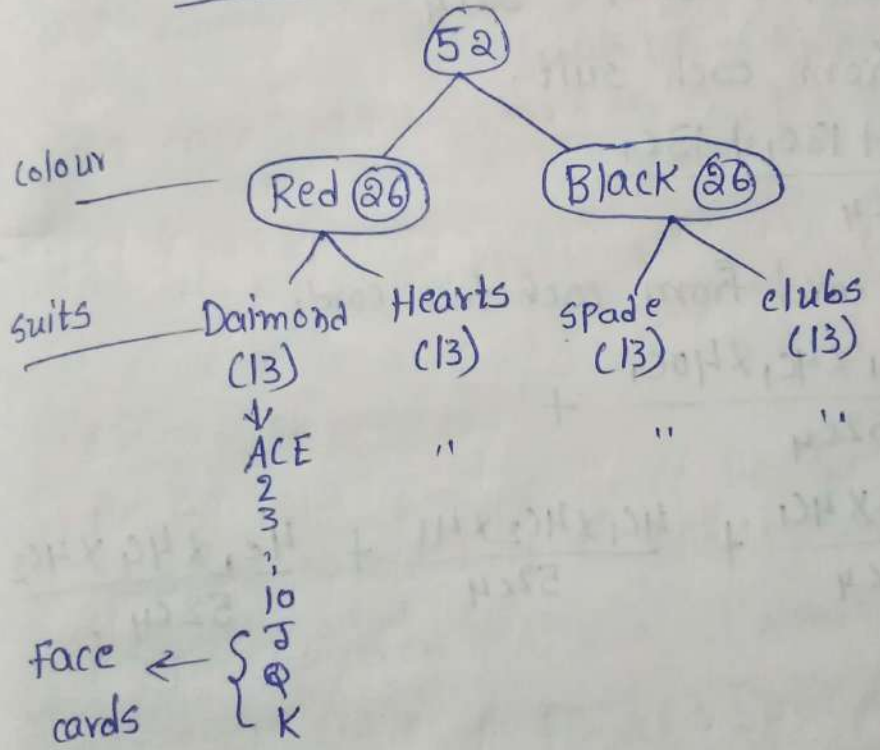
$P(E_5) = 5/36 = (5,5)\ (4,6)\ (6,4)\ (6,5)\ (5,6)$

combinations :-

selecting or observating from 'n' of observations

$$n_{c_r} = \frac{n!}{(n-r)!\,r!} \Rightarrow 4c_2 = \frac{4!}{(2!)\,2!}$$

Playing cards



3) if four cards are gone at random from the back of 52 cards find the probability of getting

(i) 2 are Red $ 2 are black

Total possible = $52C_4$

Favourable cards = $26C_2 \times 26C_2$

$$P(E) = \frac{26C_2 \times 26C_2}{52C_4}$$

(ii) All are diamond cards.

$$P(4\text{ diamonds}) = \frac{13C_4}{52C_4}$$

(iii) All the four cards are from same suits

Spade, clubs, Diamond, Hearts

$$\frac{13C_4}{52C_4} + \frac{13C_4}{52C_4} + \frac{13C_4}{52C_4} + \frac{13C_4}{52C_4}$$

(iv) one card from each suit.

$$\frac{13C_1 + 13C_1 + 13C_1 + 13C_1}{52C_4}$$

(v) Atleast one card from each face card.

$$\frac{4C_1 \times 4C_1 \times 4C_1 \times 40C_1}{52C_4} +$$

$$\frac{4C_2 \times 4C_1 \times 4C_1}{52C_4} + \frac{4C_1 \times 4C_2 \times 41}{52C_4} + \frac{4C_1 \times 4C_1 \times 4C_2}{52C_4}.$$

$$\frac{4C_1 \times 4C_1 \times 4C_1 \times 40C_1}{52C_4} + \frac{4C_2 \times 4C_1 \times 4C_1}{52C_4} + \frac{4C_1 \times 4C_2 \times 4C_1}{52C_4}$$

$$+ \frac{4C_1 4C_1 4C_2}{52C_4}$$

A box
if 4 boxs
that.
(i) 2 are
(ii) all ar
(iii) all ar
(iv) exact
(v) atleas
(i) $4C_2$ / 12
(ii) 5 / 1
(iii) 4 / 12
(iv) 4C
(v) 4C

Axiom
Let `S
possi
(i) gf
(ii) o
(iii) A
Ev
P(
(iv) for

A box contains 4 Red 3 white and 5 black balls. if 4 boxs are taken Randomly. find the probability that.

(i) 2 are Red and 2 are black
(ii) all are black.
(iii) all are of same colour.
(iv) exactly 2 of them are same colour.
(v) atleast one ball of each colour.

(i) $\frac{4C_2 \times 5C_2}{12C_4}$

(ii) $\frac{5C_4}{12C_4}$

(iii) $\frac{4C_4}{12C_4} + \frac{5C_4}{12C_4}$

(iv) $\frac{4C_2 \times 3C_1 \times 5C_1}{12C_4} + \frac{4C_1 \times 3C_2 \times 5C_4}{12C_4} + \frac{4C_1 \times 3C_1 \times 5C_2}{12C_4}$

(v) $\frac{4C_1 \times 3C_1 \times 5C_1 \times 12C_1}{12C_4}$

Axioms of probability:

Let 'S' is a sample space i.e the set of all possible outcomes of a trial.

(i) If $A \in S$, then $P(A) \geq 0$ [Non-negitivity]

(ii) $0 \leq P(A) \leq 1$ & $P(S) = 1$ [certainity]

(iii) $A_1, A_2, \ldots A_n$ or mutually exclusive (or) disjoint Events. then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

(iv) for two events A_1 and A_2

$P(A_1 \cup A_2) = P(A_1) + P(A_2)$

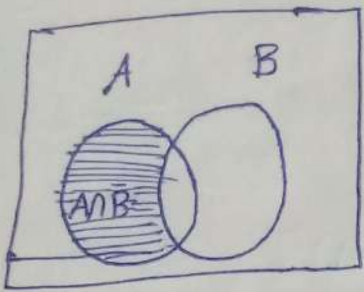
$\cancel{P(\bigcup_{i=1}^{2} A_i)} = \sum_{i=1}^{2} P(A_i)$

Formulas on the events of probability :-

let 's' is any sample space and A, B are any two events in 's'.

(i) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

P(only A but not B)



$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cap \bar{B} = \{1, 2\}$

(ii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

(P(only B but not A)

(iii) P(exactly one of the elements occur)

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

(iv) $P(\bar{A} \cap \bar{B})$

$= P(\overline{A \cup B}) = 1 - P(A \cup B)$

(v) $P(\bar{A} \cup \bar{B})$

$= P(\overline{A \cap B}) = 1 - P(A \cap B)$

(vi) $P(\bar{A}) = 1 - P(A)$

(ii) A & B are disjoint (Mutually Ex)

$* \; P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

iii) If A & B are independent

$* \; P(A \cap B) = P(A) \cdot P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

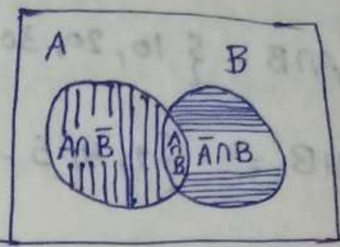
Addition theorm of probability :-

if A and B are any two events in S Then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

proof :-

if A & B are any two event in S.



we can have

$A \cup B = A \cup (\bar{A} \cap B)$

where A & $\bar{A} \cap B$ are mutually exclusive

$P(A \cup B) = P[A \cup (\bar{A} \cap B)]$

$= P(A) + P(\bar{A} \cap B)$

$= P(A) + [P(\bar{A} \cap B) + P(A \cap B) - P(A \cap B)]$

$= P(A) + P(B) - P(A \cap B)$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cap \bar{B} = \{1, 2\}$ $\bar{A} \cap B = \{5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

ii) $P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)$

$= P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(A \cap B).$

Remarks :-

i) if A and B are independent we have

$P(A \cap B) = P(A) \cdot P(B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B).$

ii) if A and B are mutually exclusive (disjoint) Then

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

iii) if a number is taken Randomly from the set $\{1, 2, 3 \ldots 30\}$ find the Probability of number is 2 or 5 ?

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\} = \frac{15}{30}$

$B = \{5, 10, 15, 20, 25, 30\} = \frac{6}{30}$

$A \cap B = \{10, 20, 30\} = \frac{3}{30}$

$A \cup B = \{2, 4, 5 \ldots 30\}$

$P(A) = \frac{15}{30}$

$P(B) = \frac{6}{30}$

$P(A \cap B) = \frac{3}{30}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{15}{30} + \frac{6}{30} - \frac{3}{30}$$

$$= \frac{18}{30}$$

if a number is taken Randomly from the set $\{1, 2, \cdots 300\}$. find the probability that it is a multiple of 6 (or) 10.

multiple of 6 — A

multiple of 10 — B

$P(A) = \frac{50}{300}$

$P(B) = \frac{30}{300}$

$P(A \cap B) = ?$

$\left[\frac{300}{6} = 50\right]$

$\left[\frac{300}{10} = 30\right]$

$\frac{15}{30}$

L.C.M of 6, 8, 10 is 30.

multiples of 30, $\frac{300}{30} = 10$ }

$P(A \cap B) = 10/300$

The failure of a electrical circuit depends on the failure or either compound A or compound B or both the circuit & probability failure is 0.4, compound B has a probability of failure is 0.2 esuming the following of A & B are independent what the probability of failure A.

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$

$P(A \cup B) = 0.4$

$P(B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$= P(A) + 0.2 - 0.2\,P(A)$

$0.4 = P(A) + 0.2 -$

~~0.4 - 0.2 = P(A) - 0.2 P(A)~~

~~0.2 = P(A) - 0.2~~

$0.4 - 0.2 = P(A) - 0.2\,P(A)$

$0.2 = 1\,P(A) - 0.2\,P(A)$

$0.2 = 0.8\,P(A)$

$\frac{0.2}{0.8} = P(A)$

$P(A) = 0.25$

A student passes a physics text is 2/3. He passes both physics and english is 14/45. The probability of He passes atleast one Test is 4/5. find the probability That He passes in English Test.

Physics 2/3

both $(A \cap B)$ 14/45

at

Physics $\rightarrow A$
English $\rightarrow B$

$P(A) = 2/3 \quad P(A \cap B) = \frac{14}{45}$

$P(A \cup B) = \frac{4}{5} \quad P(B) = ?$

Physics 2/3
Both $(A \cap B)$ $\frac{14}{45}$
at kastone $(A \cup B)$ $\frac{4}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{5} = \frac{2}{3} + P(B) - \frac{14}{45}$

$\frac{4}{5} - \frac{2}{3} + \frac{14}{45} = P(B) = \frac{20}{45}$

$\frac{12-10}{15} = \frac{2}{15} + \frac{14}{45}$

$\frac{6+14}{45} = \frac{20}{45}$

When two dice are thrown once find the probability that the sum of the numbers is 10 or 11.

$\frac{12-10}{15}$

$P(A) = P(\text{sum } 10) = \frac{3}{36} \; [(4,6)\ (5,5)\ (6,4)]$

$P(B) = P(\text{sum } 11) = \frac{2}{36} \; [(5,6)\ (6,5)]$

$P(A \cap B) = P(\text{sum } 10 \ \& \ 11) = 0$

$P(A \cup B) = P(\text{sum } 10 \text{ or } 11)$

$= P(A) + P(B) - \cancel{P(A \cap B)}$

$= P(A) + P(B)$ ✓

$= 5/36.$

Find the probability of Horse A wining a race is $\frac{1}{5}$. horse B is $\frac{1}{4}$. what is the probability that

(i) either of the horses win

(ii) None of the horses win in a particular ways.

$P(A) = \frac{1}{5}$

$P(B) = \frac{1}{4}$

$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{1}{5} \times \frac{1}{\ldots}$

$= \frac{1}{20}$

$P(A \cup B) = \ldots$

ii) $P(\bar{A} \cap \ldots$

$P(\bar{A} \cap \ldots$

person ... to 3 (4:... 'B' solvi... is the ... (if they ...

$P(A$ ...

$P(B$ ...

$= \frac{1}{5} \times \frac{1}{4}$

$= \frac{1}{20}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{5} + \frac{1}{4} - \frac{1}{20}$ $\frac{4+5-1}{20} = \frac{8}{20}$ $\frac{2}{5}$

$P(A \cup B) = \frac{2}{5}$

ii) $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

(or)

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - \frac{2}{5}$

$= \frac{5-2}{5} = \frac{3}{5}$

person 'A' solving a problem are odds against 4 to 3 (4:3) cases and odds in favour of person 'B' solving the same problem as 7 to 5 (7:5) what is the probability that the problem is solved? (if they try independently)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A)$ = Prob (Person A solving the problem)

$= \frac{3}{7}$

$P(B) = \frac{7}{12}$

P(The problem is solved) =

$P(A) + P(B) - P(A)\ P(B)$

$= \frac{3}{7} + \frac{7}{12} - \left(\frac{3}{7} \cdot \frac{7}{12}\right)$

$= 0.761$

$= \frac{3}{7} + \frac{5}{12} - P(A) \cdot P(B)$

$= \frac{36+35}{84} - \frac{3}{7} \cdot \frac{5}{12}$

$= \frac{71}{84} - \frac{15}{84}$

$= \frac{66}{84} = 0.761$

The odds that person "X" speaks the truth are 3:2, and the odds against person 'y' speakes the truth of 3:5. in what percentage of cases they likely to contradict each other instating the same fact?

$P(X) = \frac{3}{5}$ $P(\bar{X}) = \frac{2}{5}$

$P(Y) = \frac{5}{8}$ $P(\bar{Y}) = \frac{3}{8}$

$P(X \cap \bar{Y}) + P(\bar{X} \cap Y)$

$P(X) \cdot P(\bar{Y}) + P(\bar{X}) \cdot P(Y)$

$\frac{3}{5} \cdot \frac{3}{8} + \frac{2}{5} \cdot \frac{5}{8}$

$= \frac{9}{40} + \frac{10}{40} = \frac{19}{40}$

The probability then a husband who is 55 years old living till These 75 years is $\frac{8}{13}$ And the probability that his wife who is 48 years living till she is 68 years is 3/7 find the probability that

i) The couple will be alived 20 years more

ii) Atleast one of them will be alived 20 years more.

$P(A) = 8/13$

$P(B) = 3/7$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(i) Husband 'A' (55 to 75)

$P(A) = \frac{8}{13}$

wife 'B' (48 to 68)

$P(B) = \frac{3}{7}$

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{8}{13} \cdot \frac{3}{7}$$
$$= \frac{24}{91}$$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{8}{13} + \frac{3}{7} - \frac{24}{91}$$
$$= \frac{56+39}{91} - \frac{24}{91}$$
$$= \frac{25}{91} - \frac{24}{91}$$
$$= \frac{95-24}{91}$$
$$= \frac{71}{91}$$

Person 'x' is selected for interview for 3 Posts. First post there are 5 persons, for the second post there are 4 persons. for the three there are 6 persons. if the selection of each person is equally likely find the probability that the person 'x' will be selected for atleast one post?

$$P(A) = \frac{1}{5}, \quad P(B) = \frac{1}{4}, \quad P(C) = \frac{1}{6}$$

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$$
$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= 1 - (P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}))$$
$$= 1 - \left(\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{5}{6}\right)$$

$= 1 - \frac{60}{120}$

$= \frac{120-60}{120} = \frac{60}{120} = \frac{1}{2}$

conditional probability:

let A & B are two events in 's' The probability of happening of event 'B' after the occurance of the event 'A' then the probability of 'B' is called the conditional probability of B depending on the event A which is defined as $P(B/A) = \frac{P(A \cap B)}{P(A)}$

similarly the conditional probability of event A when the event B already occured

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \because P(B) > 0$$

multiplication Theoroem:

if A and B are any two events in s Then prove that $P(A \cap B) = P(A) \cdot P(B/A) \quad ; P(A) > 0$

$$= P(B) \cdot P(A/B) \quad ; P(B) > 0$$

Proof:- let A and B are any two events in 's' By the definition of probability we have

$$P(A) = \frac{n(A)}{n(S)} \; ; \; P(B) = \frac{n(B)}{n(S)} \; ; \; P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

Fore the conditional event B/A The sample space The number of events in A and the favourable number of cases are subset of A i.e The elements corresponding to $A \cap B$

$$\therefore P(B/A) = \frac{n(A \cap B)}{n(A)}$$

$$P(A/B) = \frac{n(A \cap B)}{n(B)}$$

Rewriting

$$P(A \cap B) = \frac{n(A)}{n(S)} \frac{n(A \cap B)}{n(A)}$$

$$= P(A) \cdot P(B/A)$$

$$P(A \cap B) = \frac{n(B)}{n(S)} \cdot \frac{n(A \cap B)}{n(B)}$$

$$= PB \cdot P(A/B)$$

The probability that a regularly scheduled flight departs on time is 0.83. The probability that it arrives on-time is 0.82 and the probability that it departs and arrives on-time is 0.78. find the probability that a flight

(i) arrives on-time given that it deparated on time.

(ii) departs on-time given that it has not arrived on time.

$A \to$ Departure

$B \to$ Arrival

$P(A) = 0.83 \quad P(B) = 0.82$

$P(A \cap B) = 0.78$

(i) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.78}{0.83}$

(ii) $P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$$= \frac{0.05}{0.18}$$

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$P(A\cup B) = \frac{3}{4}$; $P(A\cap B) = \frac{1}{4}$ $P(\bar{A}) = \frac{2}{3}$

(i) $P(B)$ (ii) $P(A/B)$

(iii) $P(\bar{A}/B)$ & $P(A/\bar{B})$ (iii) $P(B/\bar{A})$

$\cancel{P(A\cup B) = P(A)+}$

(i) $P(B) =$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$\frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$$

$$\frac{4}{4} - \frac{1}{3} = P(B)$$

$$P(B) = \frac{2}{3}$$

(ii) $$P(A/B) = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

(iii) $$P(\bar{A}/B) = \frac{P(\bar{A}\cap B)}{P(B)} = \frac{P(B) - P(A\cap B)}{P(B)}$$

$$= \frac{\frac{2}{3} - \frac{1}{4}}{\frac{2}{3}} = \frac{\frac{8-3}{\cancel{12}\ 4}}{\frac{2}{\cancel{3}}} = \frac{5}{8}$$

iv) $$P(A/\bar{B}) = \frac{P(A) - P(A\cap B)}{P(\bar{B})} = \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}} = \frac{\frac{4-3}{\cancel{12}\ 4}}{\frac{1}{\cancel{3}}}$$

$$= \frac{1}{4}$$

(v) $P(B/\bar{A}) = \dfrac{P(B \cap \bar{A})}{P(\bar{A})} = \dfrac{P(B) - P(B \cap A)}{\frac{2}{3}}$

$$= \dfrac{\frac{2}{3} - \frac{1}{4}}{\frac{2}{3}} = \dfrac{\frac{5}{4}}{\frac{2}{3}} = \frac{5}{8}$$

Independent Events:

The Event A and B are said to be independent if and only (iff) $P(A \cap B) = P(A) \cdot P(B)$

$P(H \cap H) = P(H) \cdot P(H)$

$= \frac{1}{2} \cdot \frac{1}{2}$

R	W
4	5

$P(A \cap B) = P$

pairwis Independent Events:

The Events $A_1, A_2 - - A_n$ are said to be pairwise Independent events.

$$P(A_i \cap A_J) = P(A_i) \cdot P(A_J) \quad [\forall i \neq J]$$

mutually independent events $A_1, A_2 \cdot A_n$ The events are mutually Independent we can have

$$P(A_i \cap A_J \cap A_K) = P(A_i) P(A_J) P(A_K)$$

~~mutually in~~

$$\vdots$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) P(A_2) \ldots P(A_n)$$

Results:

If A and B are ~~introduced~~ independent events Then show that

(i) A and $\bar{B}$

(ii) $\bar{A}$ and B

(iii) $\bar{A}$ and $\bar{B}$

are also independent.

given that

$P(A\cap B) = P(A) \cdot P(B)$

(i) we have to P.T

$P(A\cap \bar{B}) = P(A) \cdot P(\bar{B})$

LHS $P(A\cap \bar{B}) = P(A) - P(A\cap \bar{B})$

$= P(A) - P(A) \cdot P(B)$

$= P(A) \; (1 - P(B))$

$= P(A) \; P(\bar{B})$

(ii) $\bar{A}$ and B

$P(\bar{A}\cap B) = P(\bar{A}) \cdot P(B)$

LHS $P(\bar{A}\cap B) = P(B) - P(\bar{A}\cap B)$

$= P(B) - P(\bar{A}) \cdot P(B)$

$= (1 - P(A)) \; P(B)$

$= P(\bar{A}) \cdot P(B)$

(iii) we have to P.T

$P(\bar{A}\cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

LHS

$P(\bar{A}\cap \bar{B}) = P(\overline{A\cup B})$

$= 1 - P(A\cup B)$

$= 1 - (P(A) + P(B) - P(A\cap B))$

$= 1 - (P(A) + P(B) - P(A) \cdot P(B))$

$= 1 - (P(A) - P(B) \; (1 - P(A)))$

$= (1 - P(A)) \; (1 - P(B))$

$= P(\bar{A}) \; P(\bar{B})$

A Random
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(i) if a
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(iii) A

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A Random sample of 200% are classified by gender and name level of integration as possible.

Education		Male	Female	
	Elementary	38	45	83
	secondary	28	50	78
	college	22	17	39
		88	112	200

(i) if a Person is taken a Random find the Probability That

(i) The Person is male given that the percentage has secondary education

(ii) The Person does not have a college degree The person is a female.

(i) $P(\text{male}/\text{secondary}) = \frac{28}{78}$

(ii) $P\left(\frac{\text{does not have college Education}}{\text{female}}\right)$

$= \frac{95}{112}$

A married man watches a certain television show 0.4 and the probability that A married women watches the show is 0.5. The probability that a man watches the show given that his wife does. find The probability that

(i) a married couple watches the show

(ii) A wife watches the show given that are husband that

(iii) Atleast one of the couple watch the show

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(ii) A wife watches the show given that are husband that

(iii) Atleast one of the couple watch the show

8) A → man watches the show

B → women watches the show

$P(A) = 0.4 \qquad P(B) = 0.5$

$P(A/B) = 0.7$

(i) $P(A \cap B) = P(A) \cdot P(A/B)$

$= (0.5)(0.7)$

$= 0.35$

(ii) $P(B/A) =$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - 0.35$

A bag contains 4 red and 5 black balls and two balls are drawn inserts from the bag. find the probability That the two balls are red.

(i) Replaced

(ii) Not replaced

before drawing the second ball.

A → First ball is Red

B → second ball is Red.

(i) $P(A \cap B) = P(A)\, P(B)$

$= \frac{4C_1}{9C_1} \cdot \frac{4C_1}{9C_1}$

(ii) $P(A \cap B) = P(A)\ P(B/A)$

$$= \frac{4C_1}{9C_1} \cdot \frac{3C_1}{8C_1}$$

A bag contains 10 gold, 8 silver coins two successive drawings of 4 coins are made such that

(i) points are replaced before the second drawn

(ii) drawings are not replaced before the second case find the probability of 4 gold coins and second case of 4 silver coins.

$A \rightarrow 4$ Gold coins in first draw

$B \rightarrow 4$ silver coins in second draw

(i) $P(A \cap B) = P(A)\ P(B)$

$$= \frac{10C_4}{18C_4} \cdot \frac{8C_4}{18C_4}$$

(ii) $P(A \cap B) = P(A)\ P(B/A)$

$$= \frac{10C_4}{18C_4} \cdot \frac{8C_4}{14C_4}$$

Bag A contains 5 Red and 3 Red balls. bag b contains 4 Red and 4 black balls. one of the bags is selected at Random and a Ball is drawn ~~bt~~ from The bag find the ~~following~~ Probability that the

	R	B
$A \rightarrow$	5	3
$B \rightarrow$	4	4

A is first bag

B is second bag

$P(A \cap B) = P(A) + P(B)$

$$\left(\frac{1}{2} \cdot \frac{5}{8}\right) + \left(\frac{1}{2} \cdot \frac{4}{8}\right)$$

$$= \frac{5}{16} + \frac{4}{16}$$

$$= \frac{9}{16}$$

A boy goes to school by bus the probability that the bus is late is 0.1. if The bus is late The Probability That he is late to school is 0.8 if the bus is not late The probability that He is late to school. Then

(i) calculate the probability that bus is late and boy is late

(ii) The probability That the boy is late to school

(iii) The school works 56 dates in a turn how many dates do the boy expect to be late.

A - boy is late

B - bus is late

$P(A) =$

$P(B) = 0.1$

$P(A/B) = 0.8$

$P(A/\bar{B}) = 0.05$

(i) $P(A \cap B) = P(B) \cdot P(A/B)$

$= 0.1 \times 0.8$

$= 0.08$

(ii) $P(A) = P(B) \cdot P(A/B) + P(\bar{B})\, P(A/\bar{B})$

$= (0.1)(0.8) + (0.9)(0.05) = 0.125$

(iii) The no. of ~~class~~ days $=$ 56 (x late)

$\Rightarrow 56 \times P(A) = 56 \times 0.125$

$= 6.9$

H.W

Two-third $\left(\frac{2}{3}\right)$ of the student in a class are boys and the rest are girls.

It is Know that the probability of a girl getting first class is 0.25 and that of boy getting first class is 0.28. Find the probability that a student choosen at random will get

Baye's Theorem

if $E_1, E_2 \dots E_n$ are mutualy disjoint events with $P(E_i) \neq 0$, Then for any orbitary event 'A' such that $P(A) > 0$, which is a subset of $\cup_{i=0}^{n} E_i$.

we have $$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

Proof: Given that $A \subset \bigcup_{i=1}^{n} E_i$

$$A = A \cap \left(\bigcup_{i=1}^{n} E_i\right)$$

$$= A \cap (E_1 \cup E_2 \dots \cup E_n)$$

$$A = (A \cap E) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)$$

$$P(A) = P\big((A \cap E_1) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)\big)$$

since $A \cap E_i \subset E_i$ .. are also mutually Eclusive By

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$$

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

$$= \sum_{i=1}^{n} P(E_i)\, P(A/E_i)$$

$$P(E_i/A) = \cancel{P(E_i \cap A)}\ \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

Remarks !

(i) $P(E_i)$'s are 'prior' prob's

(ii) $P(A/E_i)$'s are likelihood prob's

(iii) $P(E_i/A)$'s are 'posterior' prob's

Ex-1

Box A contains 4 Red and 6 black balls, balls been Box B contains 5 Red and 5 black balls a box is selected random Then a ball is drawn from it.

i) what is the probability that the ball is red.

ii) if The ball is red find the probablity that it came from box B.

let box - A (E_1)

box - B (E_2)

$P(E_1) = \frac{1}{2}$ $\quad P(E_2) = \frac{1}{2}$ $\longrightarrow$ Prior

$A \rightarrow$ Red Ball

$P(A/E_1) = \frac{4}{10}$ $\quad P(A/E_2) = \frac{5}{10}$ $\rightarrow$ likelihood

(i) $P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$$= \left(\frac{1}{2} \times \frac{4}{10}\right) + \left(\frac{1}{2} \times \frac{5}{10}\right) = \frac{4}{20} + \frac{5}{20} = \frac{9}{20}$$

(ii) $P(E_2/A) = \frac{5/20}{9/20} = \frac{5}{9}$ $\rightarrow$ Posterior.

A factory has two machines m_1 and m_2. The machine m_1 produce 30% of the item and m_2 produce 70% of items Furthur 5% of the item produced by m_1 and only 1% per of produce if an item taken randomly from the total items of the factory and is found to be defective. Estimate the probability that it was produced by m_1, m_2.

Events m_1 and m_2

$$P(m_1) = \frac{30}{100} \qquad P(m_2) = \frac{70}{100}$$

$D \to$ defective Item

$$P(D/m_1) = \frac{5}{100}; \quad P(D/m_2) = \frac{1}{100}$$

$P(D) =$

$$= \left(\frac{30}{100} \times \frac{5}{100}\right) + \left(\frac{70}{100} \times \frac{1}{100}\right)$$

$$= \frac{150}{10000} + \frac{70}{10000} = \frac{220}{10000}$$

$P(m_1/D)$

$$\frac{4 \times 3 \times 2}{10}$$

$$= \frac{24}{10}$$

3 Machines A, B and C produced 30%, 40% and 30% of the items respectively the percentage of defectives from these machines are 2%, 3% and 2% respectively if an item is selected randomly from the Total output

i) what is the probability that it is defective

ii) if the item is defective find the probability that it was from machine A, it was not from machine C

Event A, B and C

$P(A) = \frac{30}{100}$ $\quad P(B) = \frac{40}{100}$ $\quad P(C) = \frac{30}{100}$

$P(\frac{d}{A}) = \frac{2}{100}$ $\quad P(\frac{d}{B}) = \frac{3}{100}$ $\quad P(\frac{d}{C}) = \frac{2}{100}$

(i) $P(D) = P(A) \cdot P(d/A) + P(B) \cdot P(d/B) + P(C) \cdot P(d/C)$

~~P(c). P(~~

$= \frac{30}{100} \times \frac{2}{100} + \frac{40}{100} \cdot \frac{3}{100} + \frac{30}{100} \cdot \frac{2}{100}$

$= \frac{60}{10000} + \frac{120}{10000} + \frac{60}{10000}$

$= \frac{240}{10000}$

(ii) $P(A/D) = \frac{60}{240}$ $\quad P(\frac{C}{D}) = 1 - P(\frac{C}{D})$

$= 1 - \frac{60}{240}$

$= \frac{180}{240}$

3 machine
450 units
Shows that
2% respecti
picked ran
machine is
unit

$P(m_1) =$

$P(\frac{d}{m}) =$

$=$

$=$

$=$

$P(\frac{m_1}{D_1}$

$\therefore$ The

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3 machine in a company are producing 300 units, 450 units and 250 units respectively Past experience shows that the defective percentages are 1%, 2% and 2% respectively from these machines. A unit is picked randomly observed as defective find which machine is more responsible of producing the defective unit

$P(\frac{d}{m_1}) = \frac{1}{100}$ $P(\frac{d}{m_2}) = \frac{2}{100}$ $P(\frac{d}{m_3}) = \frac{2}{100}$

$P(m_1) = \frac{300}{1000}$ $P(m_2) = \frac{450}{1000}$ $P(m_3) = \frac{250}{1000}$

$$P(\frac{d}{m}) = P(m) \times P(\frac{d}{m_1}) + P(m_2) \times P(\frac{d}{m_2}) + P(m_3) \times P(\frac{d}{m_3})$$

$$= \frac{3000}{10000} \times \frac{1}{100} + \frac{450}{1000} \times \frac{2}{100} + \frac{250}{100} \times \frac{2}{100}$$

$$= \frac{300}{10000} + \frac{900}{10000} + \frac{500}{10000}$$

$$= \frac{1700}{10000}$$

$P(\frac{m_1}{D}) = \frac{300}{170}$ $P(m_2) = \frac{900}{170}$ $P(m_3) = \frac{500}{170}$

$\therefore$ The 2nd machine is more responsible of defective item.

The Doctor will diagnoize a disease x correctly is 60%. The probability that a patient will die by his treatment after correct diagnosis is 40%. and the probability of death by wrong diagnosis is 70%. It is observed that a patient of this doctor died. find the probability that the dieseas was diagnise correctly.

$E_1 \rightarrow$ correct diagnosis

$E_2 \rightarrow$ wrong diagnosis

$P(E_1) = \frac{60}{100}$ $\quad P(E_2) = \frac{40}{100}$

D -> Patient Died

$P(D/E_1) = \frac{40}{100}$ $\quad P(D/E_2) = \frac{70}{100}$

$P(D) = \left(\frac{60}{100} \times \frac{40}{100}\right) + \left(\frac{40}{100} \times \frac{70}{100}\right)$

$\frac{2400}{10000} + \frac{2800}{10000} = \frac{5200}{10000}$

$P(E_1/D) = \dfrac{\frac{2400}{10000}}{\frac{520}{10000}}$

$= \frac{24}{52}$

3 machines A B C produce the items in the proportion 2 :3 :4. The defective probabilities are 0.1, 0.2 and 0.1 if a defective item selected random it was produced by machine A or B

$P(A) = \frac{2}{9}$ $\quad P(B) = \frac{3}{9}$ $\quad P(C) = \frac{4}{9}$

D- Defective

$P(D/A) = \frac{1}{10}$ $\quad P(D/B) = \frac{2}{10}$ $\quad P(D/C) = \frac{1}{10}$

$P(D) = \left(\frac{2}{9} \times \frac{1}{10}\right) + \left(\frac{3}{9} \times \frac{2}{10}\right) + \left(\frac{4}{9} \times \frac{1}{10}\right)$

$= \frac{2}{90} + \frac{6}{90} + \frac{4}{90} = \frac{2+6+4}{90}$

$= \frac{12}{90}$

$P(A/D) = \dfrac{\frac{2}{90}}{\frac{12}{90}} = \frac{2}{12}$

$$P(B/D) = \frac{6/90}{12/40} = \frac{6}{12}$$

$$P\left(\frac{A+B}{D}\right) = \frac{8}{12}$$

In a certain college 25% of boys 10% of girls are studying mathematics the girl constitue 60% of the total students. if a student is selected at random and found to be studying mathematics. find the probability that the student is a girl.

E_1 $\cancel{B} \rightarrow$ Boys $\quad$ E_2 $\cancel{G}$ - girls

$$P(E_1) = \frac{40}{100} \qquad P(E_2) = \frac{60}{100}$$

$m \rightarrow$ studying mathematics

$$\text{(ii)}\ P(D/E_1) = \frac{25}{100} \qquad P(D/E_2) = \frac{10}{100}$$

$$P(D) = \frac{40}{100} \times \frac{25}{100} + \frac{60}{100} \times \frac{10}{100}$$

$$= \frac{10\cancel{0}\cancel{0}}{100\cancel{0}\cancel{0}} + \frac{6\cancel{0}\cancel{0}}{100\cancel{0}\cancel{0}}$$

$$= \frac{10}{100} + \frac{6}{100} = \frac{16}{100}$$

if $P\left(\frac{D}{E_2}\right) = \dfrac{\frac{6}{100}}{\frac{16}{100}} = \frac{6}{16}$.

if a machine is correctly setup it will produce 90% good item if it is incorrectly setup it will produce 30% good items. past experience shows that 80% of all setups are done correctly. if after a setup is done the first item is produced good item. what is the probability the machine is correctly setup.

$P(E_1) = \frac{70}{100}$ $P(E_2) = \frac{30}{100}$

$P(\frac{d}{E_1}) = \frac{80}{100}$ $P(\frac{d}{E_2}) = \frac{20}{100}$

$P(D) = \frac{90}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{20}{100}$

$= \frac{7200}{10000} + \frac{600}{10000} = \frac{7800}{10000}$

$P(D/E_1) = \frac{7200}{7800} = \frac{72}{78}$

Random var

Mathemati

Discrete R.v

Probability

(pmf)

$\to$ (i)

$\to$ (ii)

$\boxed{\text{Expected (mean}}$

$\boxed{\text{varance (}}$

where

$E(X^2) =$

calculate

mass fu

X
P(X)

m

Random variables & Mathematical Exceptions

Random variable

Discrete R.V → Probability mass function (pmf)

- (i) $P(x_i) \geq 0$
- (ii) $\Sigma P(x_i) = 1$

continous Random variable → Probability Density Function (PDF)

- (i) $f(x) \geq 0$
- $\int f(x)dx = 1$

Expected value (mean (M)) $E(X) = \Sigma x P(x)$ $\quad$ $E(X) = \int_x x f(x) dx$

varance (σ^2) $V(X) = E(X^2) - [E(X)]^2$ $\quad$ $V(X) = E(X^2) - [E(X)]^2$

where

$E(X^2) = \Sigma x^2 . P(x)$ $\quad$ $E(X^2) = \int_x x^2 f(x) dx$

calculate the mean and varients of the following probability mass function

X	-1	0	1	2	3
P(X)	0.3	0.1	0.1	0.3	0.2

mean $E(X) = \Sigma x P(x)$

$= (-1)(0.3) + 0(0.1) + 1(0.1) + 2(0.3) + 3(0.2)$

$= -0.3 + 0.1 + 0.6 + 0.6$

$= 1$

$E(X^2) = \Sigma x^2 P(X)$

$= (-1)^2 (0.3) + 1^2 (0.1) + 2^2 (0.3) + (3^2 \times 0.2)$

$= 0.3 + 0.1 + 1.2 + 1.8$

$= 3.4$

varien $v(x) = E(x^2) - [E(x)]^2$

$E(x^2) = (1)(0.3) + (0)(0.1) + (1)(0.1) + 4(0.3) + 9(0.2)$

$= 0.3 + 0.1 + 1.2 + 1.8$

$= 3.4$

$\Rightarrow v(x) = 3.4 - (1)^2$

$= 2.4$

A random variable x has the following probability mass function.

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

Find the value of K, and mean, variance $E(X+1)^2$

consider $\Sigma P(X) = 1$

$0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$4K + 0.6 = 1$

$\Rightarrow 4K = 1 - 0.6 = 0.4$

$K = \frac{0.4}{4}$

$K = 0.1$

X	-2	-1	0	1	2	3
P(X)	0.1	0.1	0.2	0.2	0.3	0.1

mean :- $\Sigma X P(X)$

$= (-2)(0.1) + (-1)(0.1) + (0)(0.2) + (1)(0.2) + (2)(0.3) + (3)(0.1)$

$= (-0.2) + (-0.1) + 0 + 0.2 + 0.6 + 0.3$

$= 0.8$

Varience

$\Sigma x^2 (P(X)) - (\mu^2)$

$= (4)(0.1) + (1)(0.1) + 0(0.2) + (1)(0.2) + (4)(0.3) + 9(0.1)$

$= 0.4 + 0.1 + 0 + 0.2 + 1.2 + 0.9$

$= 2.8$

$E(X+1)^2$

let x den… appears wh… once deter… its mean

In th… (1,2,3,4,5,6) The

x	
P(x)	

$\Sigma(x) =$

$E(x^2)$

$= 2.8$

$$E(x+1)^2 = E(\cancel{2}\, x^2 + 2x + 1)$$
$$= E(x^2) + E(x) + 1$$
$$= 2.8 + 1.6 + 1$$
$$= 5.4$$

let x denotes the minimum of two numbers that appears when a dices is thrown once ~~is thrown~~ once determine the probability function. hence find its mean

In the experiment of random variable x takes the (1,2,3,4,5,6) The corresponding probability function is

x	1	2	3	4	5	6
$P(x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

$$E(x) = 1\left(\frac{11}{36}\right) + 2(9/36) + 3(7/36) + 4(5/36) + 5(3/36) + 6(1/36)$$

$$= \frac{11}{36} + \frac{18}{36} + \frac{21}{36} + \frac{20}{36} + \frac{15}{30} + \frac{6}{36}$$

$$= \frac{91}{36}$$

$\cancel{E}(x^2)$

* A Player Tosses 3 fair coins rupees Rs 500 if 3 heads of the appear Rs 300 if Two heads appear. And Rs 100 if 1 head appears. on the other hand he losses Rs 1500 if no head appears. determing the Expected value of the game, is the game favourable to the Player?

A)

	3H	2H	1H	No Heads
X	500	300	100	-1500
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

HHH
HHT
HTH
THH
TTH
THT
HTT
TTT

$E(x) = \sum x P(x)$

$= 500 \times \frac{1}{8} + 300 \times \frac{3}{8} + 100 \times \frac{3}{8} - 1500 \times \frac{1}{8}$

$= \frac{1700 - 1500}{8} = \frac{200}{8} = 25$

Three coins taken from a lot of 6 cars containing 2 defective cars then write the Probability distairbution of the Number of defective cars. also estimate the average number of afective cars.

X (def)	0	1	2
P (X)	$\frac{4C_3}{6C_3}$	$\frac{4C_2\,2C_1}{6C_3}$	$\frac{4C_1\,2C_2}{6C_3}$

$\frac{4}{20}$ $\frac{12}{20}$ $\frac{4}{20}$

mean $= \sum x\, P(x)$

$= 0\left(\frac{4C_3}{6C_3}\right) + 1\left(\frac{4C_2\,2C_1}{6C_3}\right) + 2\left(\frac{4C_1\,2C_2}{6C_3}\right)$

$= 0\left(\frac{4}{20}\right) + 1\left(\frac{12}{20}\right) + 2\left(\frac{4}{20}\right)$

$= \frac{20}{20} = 1$

Result on E
if X is
(i) E(a) = a
(ii) E(aX)
(iii) E(XI
(iv) E(aX
Results o
(i) V(a) =
(ii) V(aX) =
(iii) V(XI
(iv) V(aX+

Definiti
cumul
(CDF)
Defined
F(X) =

A Rand

X	
P(X)	

1) Fin
2) P(
3) me
4) E(
5) C

Result on $E(X)$

if X is a R.V, a and b are constants.

(i) $E(a) = a$; $E(5) = 5$

(ii) $E(aX) = aE(X)$; $E(4X) = 4E(X)$

(iii) $E(X \pm a) = E(X) \pm a$; $E(X \pm 3) = E(X) \pm 3$

(iv) $E(aX+b) = aE(X)+b$; $E(3X+4) = 3E(X)+4$

Results on $V(X)$:-

(i) $V(a) = 0$; $V(5) = 0$.

(ii) $V(aX) = a^2 V(X)$; $V(4X) = 16\,V(X)$

(iii) $V(X \pm a) = V(X)$; $V(X \pm 3) = V(X)$

(iv) $V(aX+b) = a^2 V(X)$; $V(3X+4) = 9\,V(X)$

Definition :-

cumulative distribution Function (or) Distribution Function (CDF)

Defined as

$$\boxed{F(X) = P(X \leq x)}$$

A Random variable has x following probability has

X	0	1	2	3	4
P(X)	2K	3K	2K	2K	K

1) Find the values of K

2) $P(X>1)$, $P(0<X\leq 3)$

3) mean and standard deviation of X

4) $E(2X-3)$, $V(3X+1)$

5) CDF $(F(X))$

1) $P(X) = 1$

$$2K + 3K + 2K + 2K + K = 1$$
$$10K = 1$$
$$K = 1/10$$

2) $P(X>1) = P(2) + P(3) + P(4)$
$= 2K + 2K + K$
$= 5K$
$= 5(0.1) = 0.5$

$P(0 < X \leq 3) = P(1) + P(2) + P(3)$
$= 2K + 3K + 2K$
$= 7K = 7(0.1)$
$= 0.7$

3)

X	0	1	2	3	4
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

mean $E(x) = \sum x P(x)$

$$= 0\left(\tfrac{2}{10}\right) + 1\left(\tfrac{3}{10}\right) + 2\left(\tfrac{2}{10}\right) + 3\left(\tfrac{2}{10}\right) + 4\left(\tfrac{1}{10}\right)$$
$$= 0 + \tfrac{3}{10} + \tfrac{4}{10} + \tfrac{6}{10} + \tfrac{4}{10}$$
$$= \frac{3+4+6+4}{10} = \frac{17}{10} = 1.7$$

~~varience~~ $E(x^2) = \sum x^2 P(x)$

$$= 0\left(\tfrac{2}{10}\right) + 1\left(\tfrac{3}{10}\right) + 4\left(\tfrac{2}{10}\right) + 9\left(\tfrac{2}{10}\right) + 16\left(\tfrac{1}{10}\right)$$
$$= 0 + \tfrac{3}{10} + \tfrac{8}{10} + \tfrac{18}{10} + \tfrac{16}{10}$$
$$= \frac{45}{10} = 4.5$$

varience $\sigma^2 = E(x^2) - (E(x))^2$

$$= \frac{45}{10} - \frac{289}{100}$$
$$= 4.5 - (1.7)^2$$
$$= 1.61$$

standard Deviation $(x) = \sqrt{v(x)}$

$$= \sqrt{1.61}$$
$$= 1.268$$

4) $E(2X-3) =$
$=$

$v(3x+1) =$
$=$

5)

X	0	1
P(x)	2K	3K
F(x)	2K	5K

Here P(x

then mir

4) $E(2X-3) = 2E(X) - 3$

$= 2(1.7) - 3$

$= 0.4$

$V(3X+1) = 9V(X)$

$= 9(1.61)$

$= 14.49$

5)

X	0	1	2	3	4
P(x)	2K	3K	2K	2K	K
F(x)	2K	5K	7K	9K	10K

$K = 0.1$

Here $P(X \leq 2), P(X \leq 3), P(X \leq 4)$ are $> \frac{1}{2}$

then minimum of m is 2.

III - sampling Distribution

unit - 3

Sample is defined as Part of a Population or subset of the population

Population - Parameter
sample - static

	Population (parameter)	sample (statistic)
size	N	n
mean	μ	$\bar{x}$
standard deviation	σ	s
varience	σ^2	s^2
proportion	P	p

A population consisting of five 5 observations which are $\{3, 7, 9, 11, 15\}$ consider a sample of size to drawn from the population without replacement. Then calculate

1) population mean (μ)
2) Population s.d (σ)
3) mean of sampling distribution of means ($\mu_{\bar{x}}$)
4) S.D of sampling distribution of means ($\sigma_{\bar{x}}$)

1) Population mean $= \mu = \frac{\sum x_i}{N} = \frac{45}{5}$

$= 9$

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2) $\sigma = \sqrt{\frac{1}{5}\left((3-9)^2 + (7-9)^2 + (9-9)^2 + (11-9)^2 + (15-9)^2\right)}$

$= \sqrt{\frac{1}{5}(36+4+0+4+36)}$

$= \sqrt{\frac{1}{5}(80)}$

$= \sqrt{16}$

$= 4$

$\therefore \sqrt{\frac{1}{N}\Sigma(x_i - \mu)^2}$

$K = {}^N C_n = {}^5 C_2 = 10$

Sample	sample values	sample mean ($\bar{x}$)	$(\bar{x}-\mu)^2$
1	3,7	5	$(5-9)^2 = 16$
2	3,9	6	$(6-9)^2 = 9$
3	3,11	7	$(7-9)^2 = 4$
4	3,15	9	0
5	7,9	8	1
6	7,11	9	0
7	7,15	11	4
8	9,11	10	1
9	9,15	12	9
10	11,15	13	16
		90	60

3) $\mu_{\bar{x}} = \frac{90}{10} = 9$

4) $\sigma_{\bar{x}} = \sqrt{\frac{1}{10}(60)}$

$= \sqrt{6}$

verification

1) $\mu_{\bar{x}} = \mu$

2) $\sigma_{\bar{x}} \neq$

but

$\sigma_{\bar{x}} =$

consider al

population

with replac

(i) Mean (

(ii) S.D (σ

(iii) mean of

(iv) S.D of

(i) popul

(ii) $\sigma =$

$=$

$N=$

The

verification

1) $\mu_{\bar{x}} = \mu = 9$

2) $\sigma_{\bar{x}} \neq \sigma$

but

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \left[\sqrt{\frac{N-n}{N-1}}\right]$$

$$= \frac{4}{\sqrt{2}} \left[\sqrt{\frac{5-2}{5-1}}\right]$$

consider all possible samples of size(2) from the population of 4 observation : {10, 12, 18, 20} using with replacement the calculate

(i) Mean (μ)

(ii) S.D (σ)

(iii) mean of "sampling distribution of means" ($\mu_{\bar{x}}$)

(iv) S.D of "sampling distribution of means" ($\sigma_{\bar{x}}$)

(i) population means (μ) = $\frac{10+12+18+20}{4} = \frac{60}{4}$

$= 15$

(ii) $\sigma = \sqrt{\frac{1}{4}\left((10-15)^2 + (12-15)^2 + (18-15)^2 + (20-15)^2\right)}$

$= \sqrt{\frac{1}{4}\left((5)^2 + (3)^2 + (3)^2 + (5)^2\right)}$

$= \sqrt{\frac{1}{4}(25+9+9+25)}$

$= \sqrt{\frac{1}{4}(68)}$

$= \sqrt{17}$

$$\boxed{\therefore \sqrt{\frac{1}{N}\Sigma(x_i-\mu)^2}}$$

$N = 4 \quad n = 2$

The possible sample 16 with replacement

$K = N^n = 4^2 = 16$

Sample No	sample value	Sample mean ($\bar{x}$)	$(\bar{x}-\mu)^2$
1	10, 10	10	$(10-15)^2 = 25$
2	10, 12	11	16
3	10, 18	14	1
4	10, 20	15	0
5	12, 10	11	16
6	12, 12	12	9
7	12, 18	15	0
8	12, 20	16	1
9	18, 10	14	9
10	18, 12	15	1
11	18, 18	18	0
12	18, 20	19	16
13	20, 10	15	0
14	20, 12	16	1
15	20, 18	19	16
16	20, 20	20	25
		240	136

3) $\mu_{\bar{x}} = \frac{240}{16} = 15$

4) $\sigma_{\bar{x}} = \sqrt{\frac{1}{16}(136)}$

$= \sqrt{8.5}$

<u>verification</u>

1) $\mu_{\bar{x}} = \mu = 15$

2) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$= \frac{\sqrt{17}}{2}$

Probability D

Discreate pro

* Binomial
* Poision

continous pr

* Normal
* Exponenti

Binomial

A d

follow a bi

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mass func

where n

$n \to$ no.

$P \to$ pro

$X \to$ po

if a fa

getting

(i) Ex

(ii) at

(iii) at

given t

Probability Distributions

Discreate probability distributions. They are 2 types follows

* Binomial
* Poision

continous probability distributions are 2 types. list in below.

* Normal
* Exponential.

Binomial

A discrete random variable x is assumed to follow a binomial distribution if it takes only non-negetive distinct values (Positive values) and its probability mass function is given by $P(X=x) = P(x) = nC_x p^x q^{n-x}$

for $X = 0, 1, 2, \cdots n$

where n and p are called parameters of binomial distribution

$n \rightarrow$ no. of trails

$p \rightarrow$ probability of sucess in a trial.

$X \rightarrow$ possible number of successes

if a fair coin is tossed 8 times and probability of getting head is $\frac{1}{2}$ Then find the probability of getting

(i) Exactly 4 heads

(ii) atleast 6 heads

(iii) at most 3 heads.

given that

no. of trails $n = 8$

$p =$ prob(sucess) $= p$(head)

$= 1/2$

$(q = 1 - \frac{1}{2} = \frac{1}{2})$

(i)
Let $x \to$ be the no. of sucesses out of 6 trails.

$$X = 0, 1, 2, 3, 4, 5, 6, 7, 8$$

$$P(x=4) = 8C_4 (\tfrac{1}{2})^4 (\tfrac{1}{2})^4$$
$$= 70 (\tfrac{1}{2})^8$$
$$= \frac{70}{256}$$

(ii) $P(X \geq 6) = P(x=6) + P(x=7) + P(x=8)$

$$= 8C_6 (\tfrac{1}{2})^6 (\tfrac{1}{2})^2 + 8C_7 (\tfrac{1}{2})^7 (\tfrac{1}{2})^1 + 8C_8 (\tfrac{1}{2})^8 (\tfrac{1}{2})^0$$
$$= (\tfrac{1}{2})^8 \, [28 + 8 + 1] = \frac{37}{256}$$

(iii) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$$= 8C_0 (\tfrac{1}{2})^0 (\tfrac{1}{2})^8 + 8C_1 (\tfrac{1}{2})^1 (\tfrac{1}{2})^7 + 8C_2 (\tfrac{1}{2})^2 (\tfrac{1}{2})^6$$
$$+ 8C_2 (\tfrac{1}{2})^3 (\tfrac{1}{2})^5$$
$$= \frac{1}{2^8} + \frac{8}{2^8} + \frac{28}{2^8} + \frac{56}{2^8}$$
$$= \frac{93}{256}$$

A and B play again whose chances of winning or in the ratio 3:2. if they play 4 games find the probability of A is atleast wining 3 games

g.T The no. of trails $n = 4$

prob of sucess $= P(A) = \frac{3}{5}$ $\left(q = 1 - \frac{3}{5} = \frac{2}{5}\right)$

$X \to$ No. of games A can win

$X = 0, 1, 2, 3, 4.$

$$P(X \geq 3) = P(3) + P(4)$$

$$= 4C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^1 + 4C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^0$$

$$= 4\left(\frac{3^3 \times 2}{5^4}\right) + 1\left(\frac{3^4}{5^4}\right)$$

$$= \frac{8 \times 3^3 + 3^4}{5} = \frac{810 + 81}{3125} = \frac{891}{3125} = \frac{216 + 81}{625}$$

$$= \frac{297}{625}$$

Remarks

$P(x) = nC_x p^x q^{n-x}, \quad x = 0, 1, 2, 3 \ldots n$

we have $\sum_{x=0}^{n} P(x) = 1$

$$\Rightarrow \sum_{x=0}^{n} nC_x \, p^x q^{n-x} = (q+p)^n = 1$$

$$\sum_{x=1}^{n} {}^{n-1}C_{x-1} \, p^{x-1} q^{n-x} = (q+p)^{n-1} = 1$$

$$nC_x = \frac{n}{x}\left[{}^{n-1}C_{x-1}\right] = \frac{n(n-1)}{x(x-1)}\left[{}^{n-2}C_{x-2}\right] \ldots$$

mean and varience of Binomial distribution:

mean μ $\quad E(x) = \sum_{x=0}^{n} x P(x)$

$$= \sum_{x=0}^{n} x \, nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n}{x} \left({}^{n-1}C_{x-1}\right) p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \left({}^{n-1}C_{x-1}\right) p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1} = np(1) = np$$

varriance σ^2 $\quad V(x) = E(x^2) - [E(x)]^2$

$$= \sum_{x=0}^{n} x^2 nC_x \, p^x q^{n-x} - (np)^2$$

$$= \sum_{x=0}^{n} [(x-1) + x] P(x) - (np)^2$$

$$=$$

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A binomial distribute of the mean and variences of are 2.49 and 1.44 respectively (i) $P(x \geq 5)$ and (ii) $P(1 < x \leq 3)$

it is given that mean $np = 2.4$ — ①

variance $npq = 1.44$ — ②

Substitute ② in ①

$npq = 1.44$

$2.4q = 1.44$

$q = \frac{1.44}{2.4}$

$\boxed{q = 0.6}$

$p = 1 - q = 1 - 0.6$

$\boxed{p = 0.4}$

from ①

$np = 2.4$

$n(0.4) = 2.4$

$n = \frac{2.4}{0.4} = 6$ $\boxed{n = 6}$

$P(X \geq 5) = P(X=5) + P(X=6)$

$= nC_x \, p^x \, q^{n-x}$

$= \cancel{6C_5 \, p^5 \; 6C_6 \; 0.6^{6-}}$

$= 6C_5 (0.4)^5 (0.6)^1 + 6C_6 (0.4)^6 (0.6)^0$

$= 6 \times (0.4)^5 \times 0.6 + 1 (0.4)^6 \cdot 1$

$= 6 \times 0.01024 \times 0.6 + 0.004096$

$= 3.6 \times 0.01020 + 0.004096$

$= 0.047$

(ii) $P(1 < x \leq 3$

The prob

attack the

is 1/5. if

(i) atleast 3

(ii) Exactly

(iii) All the

$P(X \geq 3$

(ii) $P(1 < x \leq 3) = P(2) + P(3)$

$= 6C_2 (0.4)^2 \cdot (0.6)^{6-2} + 6C_3 (0.4)^3 \cdot (0.6)^{6-3}$

$= 6C_2 (0.4)^2 \cdot (0.6)^4 + 6C_3 (0.4)^3 \cdot (0.6)^3$

The probability of a new generaled virus will attack the computer system and correct the file open is $1/5$. if 8 files are open find the probability that

(i) atleast 3

(ii) Exactly 5 files are corrected,

(iii) All the files are safe.

$nC_x p^x q^{n-x}$

$q = (1-P)$

$P(x \geq 3) = 1 - [P(x=0) + P(x=1) + P(x=2)$

~~$= 1 - [8C_0 (1/5)^0 + 8C_1 (1/5)^1 + 8C_2 (1/5)^2$~~

$= 1 - 8C_0 (1/5)^0 \left(\frac{4}{5}\right)^{8-0} + 8C_1 (1/5)^1 \left(\frac{4}{5}\right)^{8-1} + 8C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{8-2}$

$= 1 -$

$6C_1 \quad \frac{6 \times 5}{1 \times 1}$

$=$

(ii) $P(X=5) = 8C_5 (1/5)^5 (4/5)^3$

$= 56 \times (\frac{1}{5})^5 (\frac{4}{5})^3$

$= 2.8 \times 10^{-4}$

(iii) $P(X=0) = 8C_0 (\frac{1}{5})^0 (4/5)^8$

$= (4/5)^8$

$= 0.327$

3) one promient Phgsician claims ~~70% of lung clients says~~ that 70% of those lung cancers are chain smokers, if this assumption is crct ?

(i) find the probability that in 10 such cases les than of all chain smokers.

(ii) atleast 3 or chain smokers. $(q = 1-P)$

sol

$P = 0.7 \quad q = 0.3$

$n = 10$

X = no. of chain smokers in the samgole of 10 patients

$X = 0,1,2,3,4,5,6,7,8,9,10$

(i) P (less than 5 chain smokers)

$P(X \leq 5)$

$= P(0) + P(1) + P(2) + P(3) + P(4)$

$= 1 - nCx \, p^x q^{n-x}$

$= 1 - 10C_0 (0.7)^0 (0.3)^{10} + 10C_1 (0.7)^1 (0.3)^{10-1} + 10C_2 (0.7)^2 (0.3)^{10-2}$

$+ 10C_3 (0.7)^3 (0.3)^{10-3} + 10C_4 (0.7)^4 (0.3)^{10-4}$

$= 0.04734$

(ii) P(X

4) A tr... an ave... that in... arrive ... given tha...

probab...

P(

70 %
sumptn

than

(ii) $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - P(0) + P(1) + P(2))$

$= ~~1-P~~ \; nC_x \, p^x \, q^{n-x}$

$= 1 - (10C_0 (0.7)^0 (0.3)^{10-0} + 10C_1 (0.7)^1 (0.3)^{10-1}$
$+ 10C_2 (0.7)^2 (0.3)^{10-2})$

$= 0.998409$

4) A train arrives at a particular station on time on an average 8 out of 10 days. what is the probability that in a given b of 5 working days the train will not arrive on time in two days.

given that

tients

$n = 5$

probability the train will not come on time $P = \left(\frac{2}{10}\right) = 0.2$

$q = 1 - P$
$= 1 - 0.2$
$= 0.8$

P(train will not arrive on time in 2 days

$P(X=2) = 5C_2 (0.2)^2 (0.8)^3$

$= 60 \times (0.2)^2$

$= 0.2048$

$(0.7)^2_{10-2}$
(0.3)
-4

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Assuming that 4 in 10 automobile accidents due to speed violation. find the probability the average 6 automobile Accidents

(i) Exactly 3

(ii) Atleast 2

(iii) Atmost 5 Accidents are due to speed voilation.

$P = \frac{4}{10}$

$n = 6$

$q = \frac{6}{10}$

(i) $P(X=3) = 6C_3 \left(\frac{4}{10}\right)^3 \left(\frac{6}{10}\right)^3$

$= 20 \times \left(\frac{4}{10}\right)^3 \left(\frac{6}{10}\right)^3$

$= 0.27648$

(ii) $P(X \geq 2) = 1 - (P(0) + P(1))$

$= 1 - \left(6C_0 \left(\frac{4}{10}\right)^0 \left(\frac{6}{10}\right)^6\right) + 6C_1 \left(\frac{4}{10}\right)^1 \left(\frac{6}{10}\right)^5$

$= 1 - \left(\left(\frac{6}{10}\right)^6 + 6\left(\frac{4}{10}\right)\left(\frac{6}{10}\right)^5\right.$

$= 0.76672$

(iii) $P(X \leq 5) = 1 - [P(6)]$

$= 1 - \left[6C_6 \left(\frac{4}{10}\right)^6 \left(\frac{6}{10}\right)^0\right.$

$= 1\left[1\left(\frac{4}{10}\right)^6 \cdot 1\right]$

$= 0.995904$

6) out of
families
(i) 3 boys
(ii) 5 girls
(iii) 2 or 3
assume
$n = 5$
$P = \frac{1}{2}$
$X =$ no.
(i) P(X
(ii) P(X
(iii) P(
It i
crickb
memb
given

6) out of 800 families with 5 children how many families would you expect to have

(i) 3 boys

(ii) 5 girls

(iii) 2 or 3 boys

assume that equal prob for boys & girls)

$n = 5$

$P = \frac{1}{2}$ $q = 1 - \frac{1}{2}$ $q = \frac{1}{2}$

$\frac{5 \times 4 \times 3}{3 \times 2 \times 1}$ $\frac{60}{6} = 10$

x = no. of boys (or) remaing girls.

$nC_x \, p^x q^{n-x}$

(i) $P(X=3) = 5C_3 \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{5-3}$

$= 10 \times (0.5)^3 \cdot (0.5)^2$

$= 0.3125$

$= 0.3125 \times 800 =$ 25 families

(ii) $P(X=0) = 5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= \frac{1}{32} \times 800 = 25$ families

$\left(\frac{6}{10}\right)^5$

(iii) $P(X=2) + P(X=3) = P(2) + P(3)$

$= 5C_2 \left(\frac{1}{2}\right)^5 + 5C_3 \left(\frac{1}{2}\right)^5$

$= \frac{10}{32} + \frac{10}{32}$

$= \frac{20}{32} \times 800 = 500$ familis.

It is observed 80% of cricket fans watch the website crickbuzz.com. what is the probability atleast 80% of the members in a sample of 5 watch this website.

given that

$P = \frac{8\cancel{0}}{10\cancel{0}} \; \frac{8}{10} = 0.8$ $n = 5$

$q = 1 - P = 1 - 0.8$

$= 0.2$

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$P(X>=4) = P(4)+P(5)$

$= 5C_4 \left(\frac{8}{10}\right)^4 \left(\frac{2}{10}\right)^1 + 5C_5 \left(\frac{8}{10}\right)^5 \left(\frac{2}{10}\right)^0$

$= 5\left(\frac{8}{10}\right)^4 \left(\frac{2}{10}\right)+\left(\frac{8}{10}\right)^5$

$= \left(\frac{8}{10}\right)^4 \left[\frac{10}{10}+\frac{8}{10}\right]$

$= \left(\frac{8}{10}\right)^4 \left(\frac{18}{10}\right)$

~~The~~ $= \frac{8^4 \times 18}{10^5} = \frac{73728}{10^5} = 0.73728$ //.

The probability of a man hitting the target is $\frac{1}{4}$.

i) if he try 7 times what is the probability of hitting the target atleast twice.

ii) How many times must he try so that the probability of hitting the target atleast once is greather than $2/3$.

$P(x) = nC_x p^x q^{n-x}$ $\quad n=7, \quad p=\frac{1}{4}, \quad q=1-\frac{1}{4}=\frac{3}{4}$

i) $P(x \geq 2) = 1-[P(0)+P(1)]$

$= 1-\left[7C_0 \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{7-0}\right] + 7C_1 \left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^{7-1}$

$= 1-\left[\left(\frac{3}{4}\right)^7 + 7\left(\frac{3}{4}\right)^6 \left(\frac{1}{4}\right)\right]$

$= 1-\left(\frac{3}{4}\right)^6 \left[\frac{3}{4}+\frac{7}{4}\right]$

$= 1-\left(\frac{3}{4}\right)^6 \left(\frac{10}{4}\right) = 1-\frac{3^6 \times 10}{} = 0.5550$

ii) we have to find n

$P(X \geq 1) > \frac{2}{3}$

$1-P(X=0) > \frac{2}{3}$

$1-2/3 > P(X=0)$

$\frac{1}{3} > nC_0 \cdot p^0 q^n$

$\frac{1}{3} > nC_0 \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^n$

20% of large lot of mechanical components are found to be 5 batchs of components each are taken from this block. what is the probability that atleast 4 of these batches contains less than 2 defectives.

$P = 0.2 \quad q = 0.8 \quad n = 5$

$$P(x<2) = P(0) + P(1)$$

$$= 5C_0 (0.2)^0 \cdot (0.8)^{5-0} + 5C_1 (0.2)^1 \cdot (0.8)^{5-1}$$

$$= 0.737$$

$$P(x \geq 4) = P(4) + P(5)$$

$$= 5C_4 (0.2)^4 \cdot (0.8)^{5-4} + 5C_5 (0.2)^5 \cdot (0.8)^{5-5}$$

$$= 5C_4 (0.2)^4 \cdot (0.8)^1 + 5C_5 (0.2)^5 \cdot (0.8)^0$$

$$= 0.00672$$

* Fitting of binomial distribution :-

Fit a binomial distrib calculate the excepted frequency from the following data

x	0	1	2	3	4		
f	4	21	10	13	2	50	Σf
fx	0	21	20	39	8	80	Σfx

$n = 4$

$$\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{80}{50}$$

$$= 1.76$$

let mean $= np$

$1.76 = 4p$

$p = \frac{1.76}{4}$

$= 0.44$ $\qquad q = 0.56$

calculation of Binomial Expected frequency

$P(x) = 4C_x\, p^x q^{n-x}$

$P(x) = 4C_0 (0.44)^0 (0.56)^4$

x	$P(x) = 4C_x (0.44)^x (0.56)^{4-x}$	$f = 50 \times P(x)$
0	$P(0) = 1 \cdot 1 \cdot (0.56)^4 = 0.098$	$50 \times 0.98 = 4.91 \approx 5$
1	$4C_1 (0.44)^1 (0.56)^3 = \cancel{0.309}$ $P(1) = 4 \cdot (0.44)(0.56)^3 = 0.309$	$0.39 \times 50 = 15$
2	$4C_2 (0.44)^2 (0.56)^{4-2}$ $\frac{4 \times 3}{2 \times 1}$ 6 (	$0.36 \times 50 = 18$

3 $4C_3 (0.4$

4 $4C_4 (0.4$

1 A set of
following re

x 0
f 7
fx

calculate
case(i) :-
~~case(ii) :-~~

Sol

x	0
f	7
fx	0

(i) given
The P
Calculat

x	P(
0	P(
1	P(
2	P(
3	P(
4	P
5	
6	

3 $4C_3 (0.44)^3 (0.56)^1$

4 $4C_4 (0.44)^3 (0.56)^0$

A set of 6 coins are tossed 640 times with the following results?

x	0	1	2	3	4	5	6
f	7	64	100	210	132	75	12
fx							

calculate the expected frequiences following data, when ~~when~~
case(i) :- coins are unbaised ii) the nature of coins
~~case(ii) :- the nature of~~ coins not Known

Sol

x	0	1	2	3	4	5	6
f	7	64	140	210	132	75	12
fx	0	64	280	630	528	375	72

(i) given that coins are unbaised (fair)
The prob of head is $P = 1/2,\ q = 1/2,\ n = 6$
calculation expected mean

x	$P(x) = 6C_x (\frac{1}{2})^x (\frac{1}{2})^{6-x}$	$640 \times P(x)$
0	$P(0) = 6C_0 (\frac{1}{2})^0 (\frac{1}{2})^6 = \frac{1}{64}$	10
1	$P(1) = \frac{6}{64}$	60
2	$P(2) = \frac{15}{64}$	150
3	$P(3) = 20/64$	200
4	$P(4) = 15/64$	150
5	$P(5) = 6/64$	60
6	$P(6) = \frac{1}{64}$	10

mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1949}{640}$

$np = \frac{1949}{640}$

$P = \frac{1949}{6 \times 640}$

poison Distribution

A Discrete random variable is said to follow a poison Distribution. if it assumes only non-negetive Distint values (finite or infinite) and its probability mass function given by

$$\boxed{P(X=x) = P(x) = \frac{e^{-\tau}\,\tau^{x}}{x!}} \quad \text{for } x = 0,1,2 \ldots$$

* where τ is the parameter of the distribution

mean $= \tau$

varience $= \tau$

if x follows a poison Distribution such that ~~3/2 P(X=1)~~

$\frac{3}{2} P(X=1) = P(X=3)$

(i) The mean 'τ'

(ii) $P(X=3)$

(iii) $P(X \geq 1)$

$3/2 \; P(X=1) = P(X=3)$

$$\Rightarrow \frac{3}{2}\,\frac{\cancel{e^{-\tau}}\,\cancel{\tau^{1}}}{1!} = \frac{\cancel{e^{-\tau}}\,\tau^{\cancel{3}\,2}}{3!}$$

$$\Rightarrow \left(\frac{3}{2}\right)(3!) = \tau^2$$

$$\Rightarrow \left(\frac{3}{2}\right)(6) = \tau^2$$

$$\Rightarrow 9 = \tau^2 \qquad \lambda = \sqrt{9}$$

~~$\tau = \pm 3$~~ $\tau = \sqrt{9}$

$\boxed{\tau = \pm 3}$ $\qquad \tau = 3$

$\therefore \tau > 0$

(ii) $P(X=3)$

$= \frac{e^{-\tau}\,\tau ...}{3!}$

$= \frac{e^{-3}\,3 ...}{3!}$

$= \frac{e^{-3} \times 2 ...}{6}$

$= \frac{9e ...}{2}$

$= 0. ...$

2) at a c...

minute...

(i) atmost ...

ii) Exactly ...

given that ...

ave...

(i) $P(X$ ...

(ii) $P(x=3)$

$$= \frac{e^{-\tau}\tau^3}{3!}$$

$$= \frac{e^{-3}\,3^3}{3!}$$

$$= \frac{e^{-3}\times 27}{6}\;\frac{9}{2}$$

$$= \frac{9e^{-3}}{2}$$

$$= 0.224041 8077$$

2) at a check point counter customer arrives of 1.5 minute find the probability that ?

(i) atmost 3 arive

ii) Exactly 4 customer will arive in any even minute

given that

average customer per minute $\tau = x$ is number of customer per minute.

(i) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$$= \frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!} + \frac{e^{-1.5}(1.5)^3}{3!}$$

$$= e^{-1.5}\left(\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!}\right)$$

$$= e^{-1.5}\left(1 + 1.5 + \frac{2.25}{2} + \frac{3.375}{6}\right)$$

$$e^{-1.5}\left[1 + 1.5 + 1.125 + 0.5625\right]$$

$= e^{-1.5}\left[\frac{67}{66}\right]$

$= e^{-1.5}(4.1875)$

(ii) $P(X=4) = P(4)$

$$= \frac{e^{-1.5}(1.5)^4}{4!}$$

$$= 7.59375\, e^{-1.5}$$

Mean and varience of poisson Distribution

Remarks

1) we know that $\sum_{x=0}^{\infty} P(x) = 1$

$$\Rightarrow \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}$$

$$= e^{-\lambda}\left[1+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right]$$

$$= e^{-\lambda}e^{\lambda} = 1$$

(ii) $\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!} = e^{\lambda}$

$$\sum_{x=2}^{\infty}\frac{\lambda^{x-2}}{(x-2)!} = e^{\lambda}$$

mean $E(x) = \sum_{x=0}^{\infty} x\,P(x)$

$$= \sum_{x=0}^{\infty} x\,\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda}\sum_{x=0}^{\infty} x\cdot\frac{\lambda^1\lambda^{x-1}}{x(x-1)}$$

$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$

varience (σ^2)

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \sum_{x=0}^{\infty} x^2 \cdot P(x) - \lambda^2$$

$$= \sum_{x=0}^{\infty} (x(x-1)+x)\ P(x) - \lambda^2$$

$$= \sum_{x=0}^{\infty} x(x-1) P(x) + \sum_{x=0}^{\infty} x P(x) - \lambda^2$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^x}{x!} + \lambda - \lambda^2$$

$$= e^{\lambda} \sum_{x=0}^{\infty} x(x-1) \frac{\lambda^2 \lambda^{x-2}}{x(x-1)(x-2)!} + \lambda - \lambda^2$$

$$= x^2 = x^2 - x + x$$

$$= x(x-1) + x$$

* customers enters a large stone randomly at an average rate of 240 per hour. what is the probability that
i) during 1 minute interval no one will enter
ii) During two minute interval no one will enter

(i) During one minute interval

consider λ = avg customer per minute

$$= \frac{240}{60} = 4$$

$$P(X=0) = e^{-4}$$

$$= 0.08$$

(ii) During two-minute

$$\lambda = \frac{240}{60} \times 2 = 8.$$

$P(X=0) =$

2) In a book of 520 pages 390 typing errors occur assuming poisonus law for the number of errors for find the probability that (i) a Randomly selected will continue no ~~ever~~ error, atleast one error

(ii) A Random sample of 5 pages will contain no error.

Average mistakes for page $\lambda = \frac{390}{520}$

$= 0.75$

$P(X=0) = e^{-0.75}$

(ii) $P(X \geq 1) = 1 - P(0)$

$= 1 - e^{-0.75}$

Remarks

* Poison distribution is a special case of binomial distribution when

(i) The no. of trails n is large as $n \to \infty$

(ii) The prob of suces p is small, as $p \to 0$

(iii) mean $np = \lambda$ $\left(p = \frac{\lambda}{n}\right)$

* 2% of the items of a factory are defective find the probability that there will be ?

(i) 2 defec

(ii) atleast

given that

$n = 10$

mean

(i) P(X

(ii) at

P(

* It is

by a

sampl

(i) No

(ii) Ex

(iii)

given

(1) No

(2) 1

(3)

(i) 2 defectives

(ii) atleast 3 defective.

given that

$n = 100 \quad P = \frac{2}{100}$

mean $\lambda = np = 100 \times \frac{2}{100} = 2$

(i) $P(X=2) = \frac{e^{-2} \cdot 2^2}{2!} = 2e^{-2}$

(ii) at least 3 defectives

$P(X \geq 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - e^{-2}(1+2+2)$

$= 1 - e^{-2}(5)$

* It is given that 3% of Electric bulbs are manufactured by a company or defective find the probability that a sample of 100 bulbs will contain

(i) No defectives

(ii) Exactly one defective

(iii) Atleast one defective

given that

$n = 100, \quad P = \frac{3}{100}$

then mean $\lambda = np = 100 \times \frac{3}{100}$

$\boxed{\lambda = 3}$

(1) No defective $P(X=0) = \frac{e^{-3} 3^0}{1!} = 0.04178$

(2) 1 defective $P(X=1) = \frac{e^{-3} 3^1}{1!} = 0.149316$

(3) atleaste one $P(X=1) = 1 - P(0)$

$= 1 - \frac{e^{-3} 3^0}{1!} = 0.950212$

Fitting of Poison Distribution

Fit a poison distribution and calculate the Expected frequencies for the following distribution

x	0	1	2	3	4	5		
f	142	156	69	27	5	1	400	Σf
fx	0	156	138	81	20	5	400	Σfx

$$\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{400}{400} = 1$$

$$\text{mean } \lambda = 1$$

calculation of Expected frequencies

x	$P(x) = \frac{e^{-1} 1^x}{x!}$	$f = 400 \times P(x)$
0	$P(0) = e^{-1} = 0.3678$	$400 \times 0.3678 = 147.15$
1	$P(10) = \frac{e^{-1}.1}{1} = 0.3678$	$400 \times 0.3678 = 147.15$
2	$P(2) = \frac{e^{-1}}{2} = 0.1839$	$400 \times 0.1839 = \cancel{24.52}\ 73.56$
3	$P(3) = \frac{e^{-1}}{6} = 0.613$	$400 \times 0.613 = 24.52$
4	$P(4) = \frac{e^{-1}}{24} = 0.0532$	$400 \times 0.0532 = 6.13$
5	$P(5) = \frac{e^{-1}}{120} = 3.065 \times 10^{-3}$	$400 \times 3.065 \times 10^{-3} = 1.226$

poison distribution is a limiting case of bynomial distribution.

Proof

binomial distribution follows poison distribution when the number of trails, $n \to \infty$ (large)

2) P, The Prob of sucess is small (P→g

3) The mean $np = \lambda$ $\left(P = \frac{\lambda}{n}\right)$

consider $B(x; n, P) = nc_x \, p^x \, q^{n-x}$

$$= \frac{n!}{(n-x)!\,x!} \cdot \left(\frac{\lambda}{n}\right)^x \cdot \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n(n-1)(n-2)\ldots((n-(x-1))\cancel{(n-x)!}}{\cancel{(n-x)}!\,x!} \cdot \frac{\lambda^x}{n^x} \cdot \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}$$

$$= \frac{\frac{n}{n}\,\frac{(n-1)}{n}\ldots\frac{(n-(x-1))}{n}\cancel{n^x}}{x!} \cdot \frac{\lambda^x}{\cancel{n^x}} \cdot \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}$$

$$= \frac{1\left(1-\frac{1}{n}\right)\ldots\left(1-\frac{x-1}{n}\right)}{x!} \cdot \lambda^x \cdot \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}$$

if $B(x; n, P) = \frac{1 \cdot 1 \ldots 1}{x!} \cdot \lambda^x \, \frac{e^{-\lambda}}{1}$

$$= \frac{e^{-\lambda}\lambda^x}{x!} = P(x; \lambda)$$

$$\lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-x}$$

Normal distribution or Gaussian distribution

A continous random variable X that assum all the values in the range of $(-\infty, \infty)$ with mean μ and varience σ^2 when the probability density func of x given by

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma^2 > 0$

where μ and σ^2 are the parameters of normal distribution.

Remarks :-

we Know that total prob or area of $f(x)$ is 1

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

ii) If x is a normal variate with mean μ and variance σ^2 then $\boxed{z = \frac{x-\mu}{\sigma}}$ is called the standard normal variate with mean '0' and varience '1'.

In notations if x follows N

$\Rightarrow X \sim N(\mu, \sigma^2)$ then $z \sim N(0,1)$

iii) The prob density function of z is given by

$$\boxed{\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}} \quad ; -\infty < z < \infty$$

(i) Find the

(i) $P(0<z<1)$.

Note :- since

distribution

(ii) $P(z<1$

(iii) $P($

Normal distribution or Gaussian distribution

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(i) Find the

(i) P (0<z

Note :- si

distrib

P(

P(

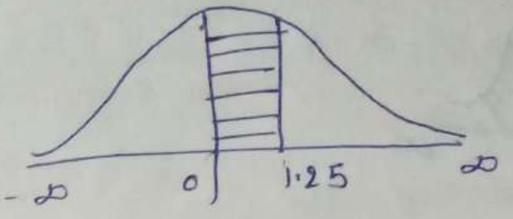
(ii) P(z

P(

(iii) P

(I) Find the area or prob value of z

(i) $P(0<Z<1.25)$



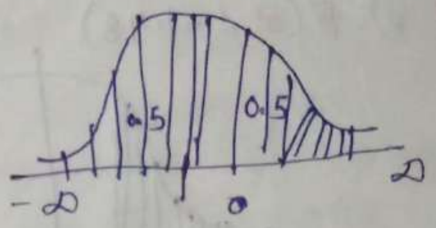
Z

$= 0.3944$

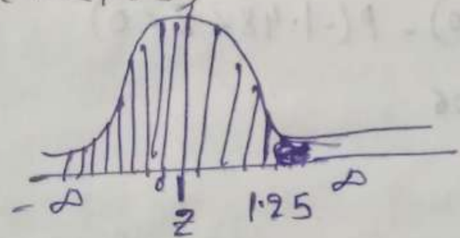
Note :- since Normal distribution is a symmetrical distribution then

$P(-\infty<Z<0)=0.5$

$P(0<Z<\infty)=0.5$



(ii) $P(Z<1.25)$

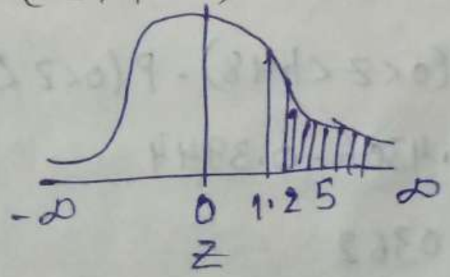


$P(Z<1.25) = P(-\infty<Z<0) + P(0<Z<1.25)$

$= 0.5 + 0.3944$

$= 0.8944$

(iii) $P(Z>1.25)$

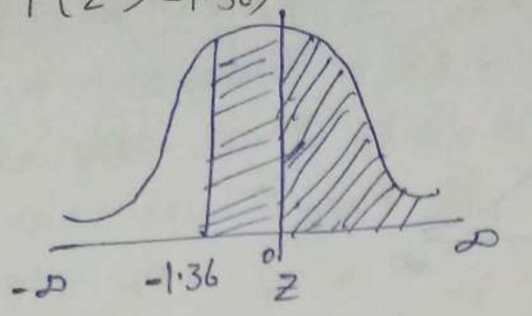


$P(Z>1.25) = P(0<Z<\infty) - P(0<Z<1.25)$

$= 0.5 - 0.3944$

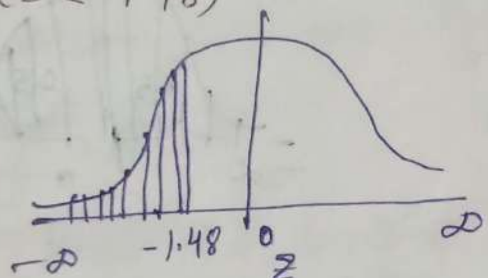
$= 0.1056$

iv) $P(Z > -1.36)$



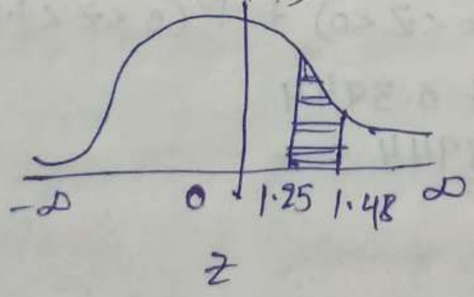
$$P(Z > -1.36) = P(-1.36 < Z < 0) + P(0 < Z < \infty)$$
$$= 0.4131 + 0.5$$
$$= 0.9131$$

v) $P(Z < -1.48)$



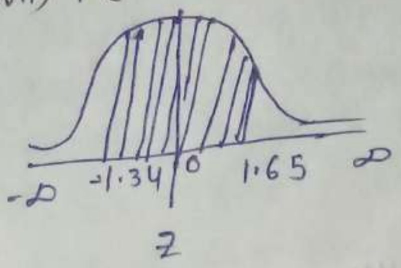
$$P(Z < -1.48) = P(-\infty < Z < 0) - P(-1.48 < Z < 0)$$
$$= 0.5 - 0.4306$$
$$= 0.0694$$

(vi) $P(1.25 < Z < 1.48)$



$$P(1.25 < Z < 1.48) = P(0 < Z < 1.48) - P(0 < Z < 1.25)$$
$$= 0.4306 - 0.3944$$
$$= 0.0362$$

vii) $P(-2.34 < Z < 1.65)$



$P(-2.34 < Z < 1.65)$

$= P(0 < Z < 2.34) + P(0 < Z < 1.65)$

$= 0.4904 + 0.4505$

$= 0.9409$

→ To measure the area or probability under the curve between 0 to x is given by

$$\boxed{P(0 < Z < x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-Z^2/2}\, dZ}$$

Application Questions :-

(Q) suppose that the weights of 800 male students are Normally distributed with mean 140 Pounds and SD 10 Pounds find the No. of students whose weights are

(i) More than 152 Pounds

(ii) less than 150 Pounds

(iii) B/w 138 and 148 Pounds.

given that $\mu = 140$, $\sigma = 10$

$x \leftarrow$ weight

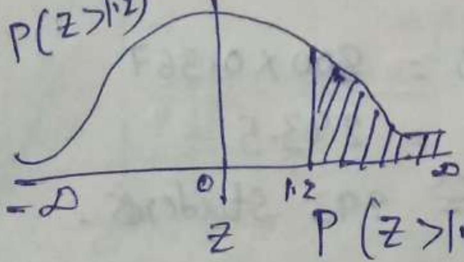
(i) $P(X > 152)$

at $X = 152$ $\quad Z = \frac{x-\mu}{\sigma} \Rightarrow Z = \frac{152-140}{10}$

$Z = \frac{12}{10}$

$\boxed{Z = 1.2}$

$\Rightarrow P(Z > 1.2)$



$P(Z > 1.2) = P(0 < Z < \infty) - P(0 < Z < 1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

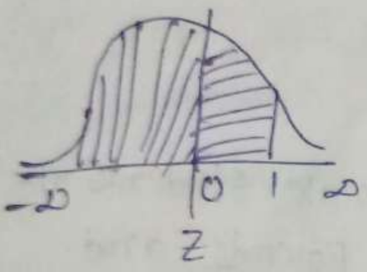
$\therefore$ No. of students $= 800 \times 0.1151$
$= 92.08$
$\Rightarrow \cong 92$ students

(ii) $P(X < 150)$

at $X = 150$ $\quad Z = \dfrac{X-\mu}{\sigma}$ $\quad Z = \dfrac{150-140}{10}$

$\boxed{Z = 1}$

$P(Z < 1)$



$P(Z<1) = P(-\infty < Z < 0) + P(0 < Z < 1)$
$= 0.5 + 0.3413$
$= 0.8413$

$\therefore$ No. of students $= 800 \times 0.8413$
$= 673.04$
$\cong 673$ students.

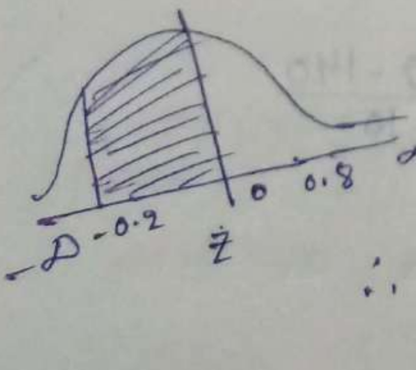
(iii) B/w 138 and 148

$P(138 < X < 148)$

at $X = 138$ $\quad Z = \dfrac{X-\mu}{\sigma} \Rightarrow Z = \dfrac{138-140}{10} \Rightarrow Z = -0.2$

at $X = 148$ $\quad Z = \dfrac{X-M}{\sigma} \Rightarrow Z = \dfrac{148-140}{10} \Rightarrow Z = 0.8$

$P(-0.2 < Z < 0.8) = P(0 < Z < 0.2) + P(0 < Z < 0.8)$
$= 0.0793 + 0.2881$
$= 0.3674$

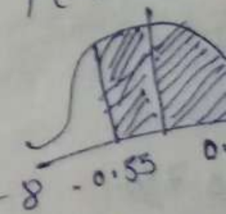


$\therefore$ No. of students $= 800 \times 0.367$
$= 293.5$
$\cong 293$ students.

Q) An electric
that have
800 hrs and
random sel
(i) B/w 778
(ii) More than
given $\mu = 800$
$X \leftarrow$ hou
(i) $P(778$
at $X = 7$

at $X = 83$

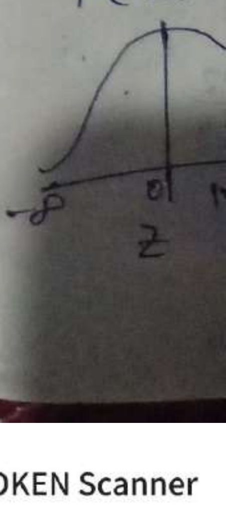
$P(-0.55 <$



$P(-0.5$

(ii) $P(X$
at X

$P(Z > 1$



Q) An electrical company manufactures light bulbs that have a life Normally distributed with mean 800 hrs and SD 40 hrs find the Prob that a random selected bulb has a life.

(i) B/w 778 and 834 hrs.

(ii) More than 850 hrs.

given $\mu = 800$ and $\sigma = 40$

$X \leftarrow$ hours

(i) $P(778 < X < 834)$

at $X = 778 \Rightarrow Z = \frac{x-\mu}{\sigma} = \frac{778-800}{40}$

$= \frac{-22}{40}$

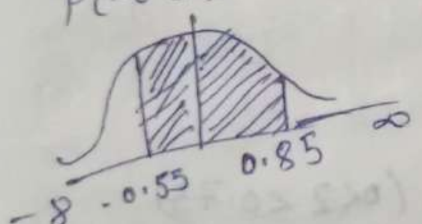
$= -0.55$

at $X = 834 \Rightarrow Z = \frac{x-\mu}{6} \Rightarrow Z = \frac{834-800}{40}$

$= \frac{34}{40}$

$= 0.85$

$P(-0.55 < Z < 0.85)$



$P(-0.55 < Z < 0.85) = P(0 < Z < 0.55) + P(0 < Z < 0.85)$

$= 0.2088 + 0.3023$

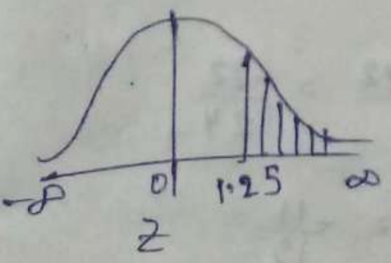
$= 0.5111$

(ii) $P(X > 850)$

at $X = 850 \Rightarrow Z = \frac{x-\mu}{\sigma} \Rightarrow Z = \frac{850-800}{40} = \frac{5}{4}$

$= 1.25$

$P(Z > 1.25)$



$P(Z > 1.25) = P(0 > Z > \infty) - P(0 < Z < 1.25)$

$= 0.5 - 0.3944$

$= 0.1056$

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Q) In an intelligence test conducted to 1000 children the arrange score is 42 and SD is 24. calculate the no. of children whose scores are

(i) exceding 60

(ii) B/w 20 and 40

(iii) Below 50

given $\mu = 42$, $\sigma = 24$

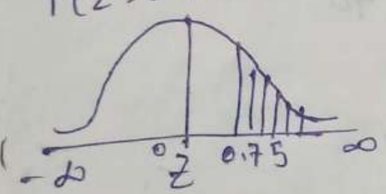
$X \leftarrow$ score

(i) $P(X > 60)$

at $X = 60 \Rightarrow Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{60-42}{24}$

$= \frac{18}{24} = 0.75$

$P(Z > 0.75)$



$P(Z > 0.75) = P(0 < Z < \infty) - (0 < Z < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

$\therefore$ No. of children $= 1000 \times 0.2266$

$\cong$ 226 children.

(ii) $P(20 < X < 40)$

at $X = 20$ $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{20-42}{24} = \frac{-22}{24}$

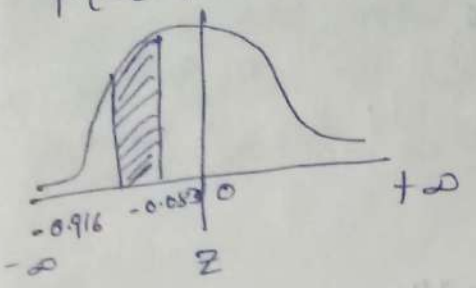
$= \frac{-11}{12}$

$= -0.916$

4\. at $X = 40$, $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{40-42}{24} = \frac{-2}{24} = \frac{-1}{12}$

$= -0.083$

$P(-0.916 < Z < -0.083)$



$P(-0.916 < Z < -0.083)$

$= P(0 < Z < 0.91) + P(0 < Z < 0.08)$

$= 0.3186 + 0.0319$

$= 0.3505$

$\therefore$ No. of children $= 1000 \times 0.3505$

$\cong 350$ children

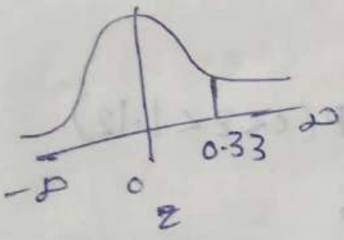
(iii) $P(X \leq 50)$

at $X = 50 \Rightarrow Z = \frac{x-\mu}{\sigma} \Rightarrow Z = \frac{50-42}{24}$

$= \frac{8}{24}$

$= 0.33$

$P(Z < 0.33)$



$P(Z < 0.33) = 0.1293$

$\therefore$ No. of children $= 1000 \times 0.1293$

$= 1.293$

$\cong 129$ children

The customer accounts of a departmental store have an average values of RS. 240/- and a standard deviation of Rs 80/- assuming that the accounts are normally distributed. Find the proportion of accounts.

(i) If over RS. 300/-

(ii) B/w 200 and 300/-

(iii) B/w 120 & 180 RS/-

given that

$\mu = 240$, $\sigma = 80$

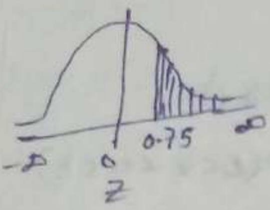
$P(X > 300)$

at $X = 300 \Rightarrow Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{300-240}{80}$

$= \frac{60}{80}$

$= 0.75$

$P(z > 0.75)$



$P(z > 0.75) = P(0 < z < \infty) - P(0 < z < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

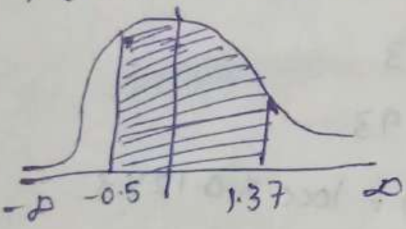
(ii) $P(200 < X < 330)$

at $X = 200$, $z = \frac{X - \mu}{\sigma} \Rightarrow z = \frac{200 - 240}{80} = \frac{-40}{80}$

$= -0.5$

at $X = 330$, $z = \frac{X - \mu}{\sigma} \Rightarrow z = \frac{330 - 240}{80} = \frac{19}{8} = 1.12$

$P(-0.5 < z < 1.37)$



$P(-0.5 < z < 1.12)$

$= P(0 < z < 0.5) + P(0 < z < 1.12)$

$= 0.1915 + 0.3686$

$= 0.5601$

(iii) $P(120 < X < 180)$

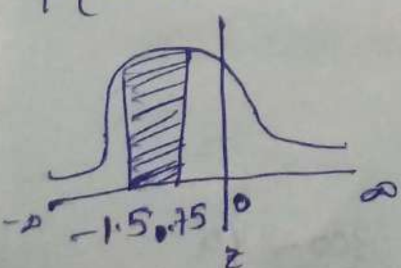
at $X = 120$, $z = \frac{X - \mu}{\sigma} \Rightarrow z = \frac{120 - 240}{80} = \frac{-120}{80}$

$= -1.5$

at $X = 180$, $z = \frac{X - \mu}{\sigma} \Rightarrow z = \frac{180 - 240}{80} = \frac{-160}{80}$

$= -0.75$

$P(-1.5 < z < 0.75)$



$P(-1.5 < z < 0.75)$

$= P(0 < z < 1.5) - P(0 < z < 0.75)$

$= 0.4332 - 0.2734$

$= 0.1598$.

Q) A soft drink machine is regulated so that it discharges an average of 200ml per cup. if the amount of drink is normally distributed with SD 15ml then determine the prob that a cup contains

(i) B/w 191 and 209 ml

(ii) The fraction of cups contain more than 234 ml.

(iii) The no. of cups will probably overflow if 200ml cups are used for next 1000 drinks.

given that

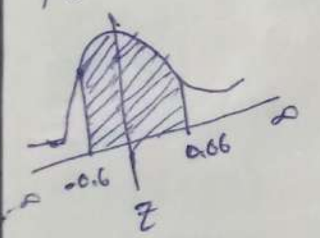
$\mu = 200, \quad \sigma = 15$

(i) $P(191 < 209)$

at $X = 191$, $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{191-200}{15} = \frac{-9}{15} = -0.6$

at $X = 209$, $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{209-200}{15} = \frac{9}{15} = 0.6$

$P(-0.6 < Z < 0.6)$



$P(-0.6 < Z < 0.6)$

$= 2(P(0 < Z < 0.6))$

$= 2 \times 0.2258$

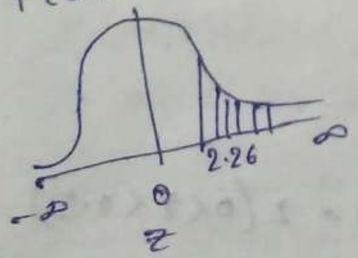
$= 0.4516.$

(ii) $P(X > 234)$

at $X = 234$, $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{234-200}{15} = \frac{34}{15}$

$= 2.26$

$P(Z > 2.26)$



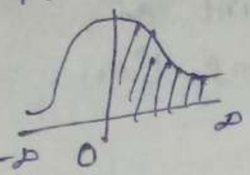
$P(Z > 2.26) = 0.4881$

(iii) $P(X > 200)$

at $X = 200$ $\quad Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{200-200}{15}$

$= 0$

$P(Z>0)$



$P(Z>0)$
$= 0.5$
$= 0.5 \times 1000$
$= 500$ cups

given that X is normally distributed with mean 10 and $P(X>12) = 0.1587$ find the probability that X falls in interval $P(9<X<11)$

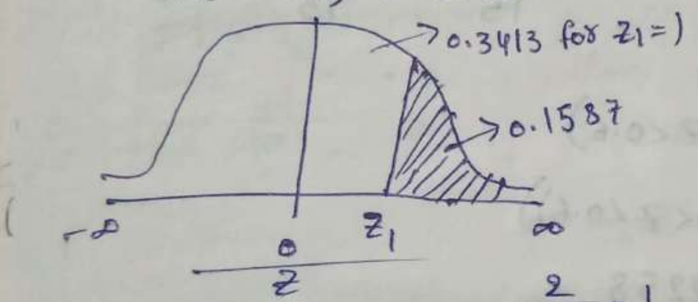
given that

$\mu = 10$ and $P(X>12) = 0.1587$

$\Rightarrow P(X>12)$

at $X=12 \Rightarrow Z = \dfrac{X-\mu}{\sigma} = \dfrac{12-10}{\sigma} = \dfrac{2}{6} = z_1$

$\rightarrow P(Z>z_1) = 0.1587$



$\dfrac{2}{\sigma} = 1$

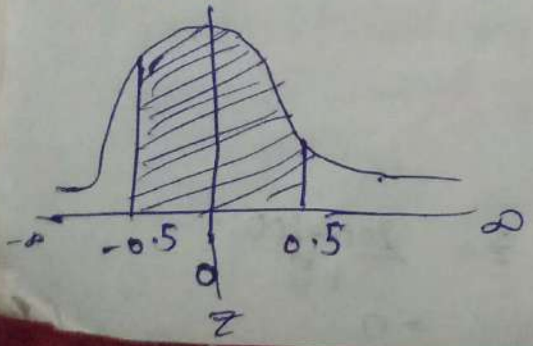
$\boxed{\sigma = 2}$

$P(9<X<11)$

at $X=9 \Rightarrow Z = \dfrac{X-\mu}{\sigma} \Rightarrow Z = \dfrac{9-10}{2}$
$= -0.5$

at $X=11 \Rightarrow Z = \dfrac{X-\mu}{\sigma} \Rightarrow Z = \dfrac{11-10}{2}$
$= 0.5$

$\Rightarrow P(-0.5<Z<0.5)$



$P(-0.5<Z<0.5) = 2(0<Z<0.5)$
$= 2 \times 0.1915$
$= 0.3830$

A random variable has normal distribution with mean 62.4 find the SD if the prob is 0.20 that it will take on a value >79.2

given that

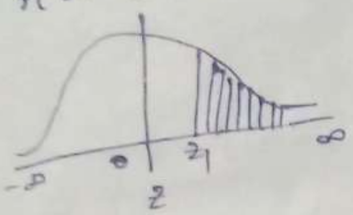
$\mu = 62.4 \quad P(X>79.2) = 0.20$

$P(X>79.2)$

at $X = 79.2 \Rightarrow Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{79.2 - 62.4}{\sigma}$

$= \frac{16.8}{\sigma} = Z_1$

$P(Z>Z_1) = 0.20$



$Z_1 = 0.84$

$\frac{16.8}{\sigma} = 0.84$

$\boxed{\sigma = 20}$

if x follows a normal distribution & it is given that 35% of observations are <45 and 5% of observation are >60, then find the mean & SD of distribution.

given that

$P(X<45) = 0.35$ and $P(X>60) = 0.05$

$\Rightarrow P(X<45)$

at $X = 45 \Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{45-\mu}{\sigma} = Z_1$

$P(Z<Z_1) = 0.35 \quad Z_1 = 0.38$

$\frac{45-\mu}{\sigma} = -0.38$

$\boxed{\mu - 0.38\sigma = 45}$ — ①

$P(X>60)$

at $X = 60, \Rightarrow Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{60-\mu}{\sigma} = Z_2$

$P(Z>Z_2) = 0.05$

$Z_2 = 1.64$

$\Rightarrow \frac{60-\mu}{\sigma} = 1.64$

$\boxed{\mu + 1.64\sigma = 60} \rightarrow$ ②

① - ②

$\mu + 1.64\sigma = 60$

$\mu - 0.38\sigma = 45$

(-) (+) (-)

$1.26\sigma = 15$

$\sigma = \frac{15}{1.26}$

$\boxed{\sigma = 11.9047}$

$\mu = 45 + 0.38(11.9047)$

$\boxed{\mu = 49.5237}$

$2.02\sigma = 15$

$\sigma = \frac{15}{20.2}$

$\boxed{\sigma = 7.4257}$

$\mu = 45 + 0.38(7.425)$

$\boxed{\mu = 47.8215}$

In a distribution excatly normal 10.03% of items are under 25Kg of weight and 19.97% of items are under 70 Kg's of weight. what are mean & S.D

$P(X<25) = 10.03\% = 0.1003$

$P(X<70) = 89.97\% = 0.8997$

$P(X<25)$

at $X=25$ $Z = \frac{25-\mu}{\sigma} = Z_1$

$P(Z<Z_1) = 0.1003$

$\boxed{Z_1 = -1.28}$

$\frac{25-\mu}{\sigma} = -1.28$

$= \boxed{\mu\ 1.28\sigma = 25}$

$P(X<70)$

at $X = 70$ $Z = \frac{70-\mu}{\sigma} = 0.8997$

$P(Z<Z_2) = 0.8997$

$\boxed{Z_2 = 1.28}$

$\frac{70-\mu}{\sigma} = 1.28 \Rightarrow \boxed{\mu + 1.28\sigma = 70} \longrightarrow (2)$

(2) + (1)

$\mu + 1.28\cancel{\sigma} = 70$

$\mu - \cancel{1}.28\sigma = 25$

$\mu = \frac{95}{2} \Rightarrow \boxed{\mu = 47.5}$

$\sigma = \frac{70-47.5}{1.28}$

$\sigma = \frac{22.5}{1.28}$

$\boxed{\sigma = 17.57}$.

i'tems are

Marks obtained by 1000 students are normally distributed with mean 78 and S.D 11 then determine.

(a) How many students got marsks above 90

(b) what was the highest mark obtained by the lowest 10% of students.

(c) within what limits did middle of 90% students

given that

$\mu = 78$

$\sigma = 11$

(i) $P(X>90)$

$X=90 \Rightarrow Z = \frac{90-78}{11} = \frac{12}{11} = 1.09$

$P(Z>1.01) = P(0<Z<\infty) - P(0<Z<1.01)$

$= 0.5 - 0.3621$

$= 0.1379$

(ii) $P(X<X_1) = 0.1$

at $X = X_1 \Rightarrow Z = \frac{X_1 - 78}{11} = Z_1$

$P(Z<Z_1) = 0.1$

$\boxed{z_1 = 1.28}$

$$\frac{x_1 - 78}{11} = 1.28$$

$\boxed{X = 9208}$

(ii) $P(x_1 < x < x_2)$

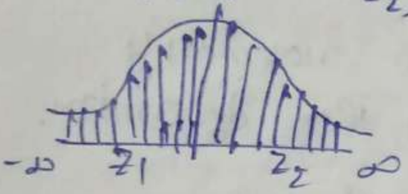
at $x = x_1$ $z = \frac{x_1 - 78}{11} = z_1$

at $x = x_2$ $z = \frac{x_2 - 78}{11} = z_2$

$\Rightarrow P(z_1 < z < z_2) = 0.9$

$\rightarrow P(z_1 < z < z_2) = \left[P(-\infty < z < 0 - P(z_1 < z < 0)\right]$

$+ \left[P(0 < z < \infty) - P(0 < z < z_2)\right]$



$= 2 \times 0.5$

Binomial approximation to Normal distribution :-

Bianamial follows normal distribution by conducting

$np = \mu$, $npq = \sigma^2$

The probability that a patient recover from a rare blood ~~D~~ diease is 0.4 if 100 people have this disease. What is prob that more than 45 will survive.

given that $P = 0.4$ and $n = 100$

$q = 0.6$

$\mu = nP \Rightarrow \mu = 100 \times 0.4$

$\boxed{\mu = 40}$

$\sigma^2 = npq \Rightarrow \sigma^2 = 100 \times 0.4 \times 0.6$

$\sigma^2 = 24$

$\sigma = \sqrt{24}$

$\boxed{\sigma = 4.89}$

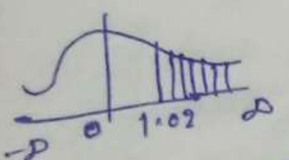
$\Rightarrow P(X > 45)$

at $X = 45$ $Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{45-40}{4.89} = \frac{5}{4.89} = 1.02$

$P(Z > 1.02) = P(0 < Z < \infty) - P(1.02 < Z < \infty)$

$= 0.5 - 0.346$

$= 0.1539$



Q) if 20% of residents in US prefer a white telephone over any other color availability. What is the prob that among next 1000 installed.

(i) Between $(170 < X < 185)$ will be white.

(ii) atleast 210 but not more than 225 will be white.

given $P = 0.2$, $n = 1000$

$q = 0.8$

$\mu = nP = 1000 \times 0.2$

$\boxed{\mu = 200}$

$\sigma^2 = npq = 1000 \times 0.2 \times 0.8$

$\sigma^2 = 160$

$\sigma = \sqrt{160}$

$\sigma = 4\sqrt{10}$

$= 4 \times 3.16$

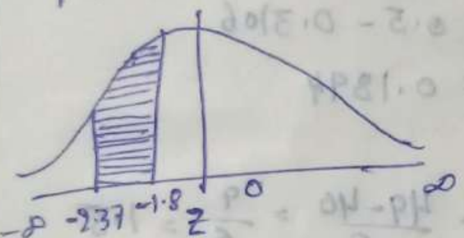
$\boxed{\sigma = 12.64}$

(i) $P(170 < X < 185)$

at $X = 170 \Rightarrow Z = \dfrac{170-200}{12.64} = \dfrac{-30}{12.64} = -2.37$

at $X = 185 \Rightarrow Z = \dfrac{185-200}{12.64} = \dfrac{-15}{12.64} = -1.185$

$P(-2.37 < Z < -1.8)$



$= P(-2.37 < Z < -1.18)$

$= P(0 < Z < 2.37) - P(0 < Z < 1.18)$

$= 0.4911 - 0.3810$

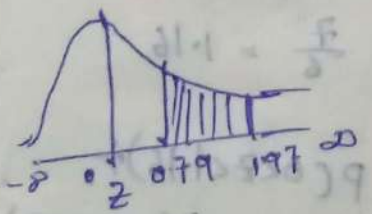
$= 0.1101$

ii) $P(210 < X < 225)$

at $X = 210$ $\quad Z = \dfrac{210-200}{12.64} = \dfrac{10}{12.64} = 0.791$

at $X = 225$ $\quad Z = \dfrac{225-200}{12.64} = \dfrac{22}{12.64} = 1.97$

$P(0.79 < Z < 1.97)$



$P(0.79 < Z < 1.97) = P(0 < Z < 1.97)$

$- P(0 < Z < 0.79)$

$= 0.4756 - 0.2852$

$= 0.904$

(8) statistics released by the national highway authority and national safety council that on an average weekend night 1 out of 10 drivers is drunk, if 400 drivers are randomly checked in a weekend the prob of drivers identified as drunk, will be

(i) less than 32
(ii) more than 49
(iii) atleast 35 but less than 47.

$P = 0.1$ and $n = 400$

$\mu = nP = 0.1 \times 400 = 40$

$\boxed{\mu = 40}$

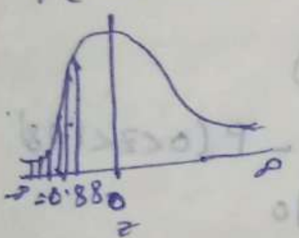
$\sigma^2 = npq = 0.1 \times 400 \times 0.9$

$\sigma^2 = 36 \quad \boxed{\sigma = 6}$

(i) $P(X < 32)$

for $X = 32$, $z = \frac{X - \mu}{\sigma} = \frac{32 - 40}{6} = \frac{-8}{9} = -0.88$

$P(Z < -0.88)$

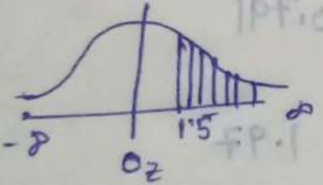


$P(Z < -0.88) = P(-\infty < Z < 0) - P(-.88 < z ...$
$= 0.5 - 0.3106$
$= 0.1894$

(ii) $P(X > 49)$

for $X = 49$, $Z = \frac{49 - 40}{6} = \frac{9}{6} = 1.5$

$P(Z > 1.5)$



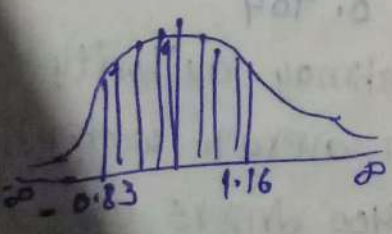
$P(Z > 1.5) = P(0 < Z < \infty) - P(0 < z < 1.5)$
$= 0.5 - 0.4332$
$= 0.0668$

(iii) $P(35 < X < 47)$

at $X = 35 \Rightarrow Z = \frac{35 - 40}{6} = \frac{-5}{6} = -0.83$

at $X = 47 \Rightarrow Z = \frac{47 - 40}{6} = \frac{7}{6} = 1.16$

$P(-0.88 < Z < 1.16)$



$P(-0.83 < Z < 1.16) = P(0 < Z < 1.16) + P(0 < Z < 0.83)$

$= 0.3770 + 0.2967$

$= 0.6737$

Exponential

A continou
Exponential
in the range
function f(

where theta(
distribution

Mean & vari

mean E(x)

Varience

Exponential Distribution

A continous random variable X is said to follow Exponential Distribution that executes the values in the range (0 to ∞) and its probability density function $f(x) = \theta . e^{-\theta x}$, $x > 0$, $\theta > 0$

where theta (θ) is the parameter of exponential distribution

Mean & varience of exponential distribution

$$\text{mean } E(x) = \int_0^\infty x f(x) dx = \int_0^\infty x . \theta e^{\theta x} dx$$

$$= \cancel{\int_0^\infty x f(x)}$$

$$= \theta \int_0^\infty x e^{-\theta x} dx = \theta \left[x \int e^{-\theta x} dx - \left[\int 1 . \int e^{-\theta x} dx \right] dx \right]_0^\infty$$

$$= \theta \left[\frac{x . e^{-\theta x}}{-\theta} - \int \frac{1 . e^{-\theta x}}{-\theta} dx \right]_0^\infty$$

$$= \theta \left[\frac{x . e^{-\theta x}}{-\theta} + \frac{1}{\theta} \left[\frac{e^{-\theta x}}{-\theta} \right] \right]_0^\infty$$

$$= \theta \left[(0+0) - \left(0 + \frac{1}{\theta} \left(\frac{1}{-\theta}\right)\right) \right]$$

$$= \theta \left(+ \frac{1}{\theta^2} \right) = \frac{1}{\theta}$$

Varience

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \int_0^\infty x^2 f(x) dx - \left(\frac{1}{\theta}\right)^2$$

$$= \int_0^\infty x^2 . \theta . e^{-\theta x} dx - \frac{1}{\theta^2}$$

$$= \theta \int_0^\infty x^2 . e^{-\theta x} dx - \frac{1}{\theta^2}$$

$$= \theta\left[x^2 . \int e^{-\theta x} dx - \left[\int 2x . \int e^{-\theta x} dx\right] dx\right] - \frac{1}{\theta^2}$$

$$= \theta\left[x^2 \frac{e^{-\theta x}}{\theta} \quad 2\int x . \frac{e^{-\theta x}}{-\theta} dx\right]_0^{\infty} - \frac{1}{\theta^2}$$

$$= \theta\left[x^2 \frac{e^{-\theta x}}{-\theta} + \frac{2}{\theta}\int x e^{-\theta x} dx\right]_0^{\infty} - \frac{1}{\theta^2}$$

$$= \theta\left[x^2 . \frac{e^{-\theta x}}{-\theta} + \frac{2}{\theta}\left[\frac{x e^{-\theta x}}{-\theta} + \frac{1}{\theta}\left[\frac{e^{-\theta x}}{-\theta}\right]\right]\right]_0^{\infty} - \frac{1}{\theta^2}$$

$$= \theta\left[\left[(0 + \frac{2}{\theta}(0+0))\right] + \left[0 + \frac{2}{\theta}\left(0 + \frac{1}{\theta}\left(\frac{1}{-\theta}\right)\right)\right]\right]$$

$$= \theta\left[0 - (0 + \frac{2}{\theta}\left(\frac{1}{-\theta^2}\right) - \frac{1}{\theta^2}\right.$$

$$= \theta . \left[+\frac{2}{\theta^3}\right] - \frac{1}{\theta^2}$$

$$= \frac{2}{\theta^2} - \frac{1}{\theta^2}$$

$$= \frac{1}{\theta^2}$$

unit - III

central limit theorem (CLT) :-

if x is a normal varient follows a normal distribution with mean meuw variance sigma square(σ^2) then the standard normal varient. $\boxed{z = \frac{x-\mu}{\sigma}}$

*The sample mean normal varient that follows normal distribution with mean, μ and variance $\frac{\sigma^2}{n}$ then the corresponding standard normal varient

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

A random sample of size 64 is taken from a normal population with mean 51.4 and standard deviation 6.8. what is the probability that the mean of the sample (i) will be less than 50.6

(ii) b/w 50.5 and 52.5

$\mu = 51.4$, $\sigma = 6.8$, $n = 64$

Population mean $(\mu) = 51.4$

standard deviation $(\sigma) = 6.8$

consider the mean of the sample is $\bar{x}$, we have to find (i) $P(\bar{x} < 50.6)$:

$$\bar{x} = 50.6, \quad z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{50.6 - 51.4}{\frac{6.8}{\sqrt{64}}}$$

$$= \frac{-0.8}{0.85} = -0.94$$

$$= -0.941$$

P($\bar{x}$ < 50

RBas

(ii) P (5

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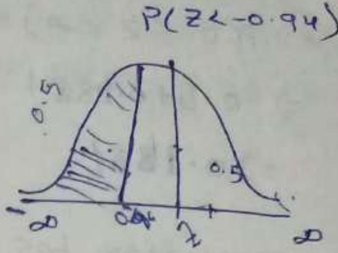
$P(\bar{x} < 50.6) = P(Z < -0.94)$

$P(\cancel{0 x}) = 0.5 - P(-0.94 < Z < 0)$

$= 0.5 - 0.3264$

$= 0.1736$

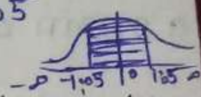


(ii) $P(50.5 < \bar{x} < 52.3)$

at $\bar{x} = 50.5 \rightarrow Z = \dfrac{50.5 - 5.14}{\frac{6.8}{8}} \rightarrow Z = \dfrac{-0.9}{0.85} = -1.05$

at $\bar{x} = 52.3 \Rightarrow Z = \dfrac{52.3 - 5.14}{\frac{6.8}{8}} \Rightarrow Z = \dfrac{0.9}{0.85} = 1.05$

$P(-1.05) < Z < 1.05) = 0.3531 + 0.3531 = 0.7062$



is 155 cente meteres and standard deviation is 15cm, what is the probability that the mean type of A sample of 36 students from this college is

(i) less than 157 cm

(ii) more than 153 cm

mean height → 155 cm $\mu = 155$ cm $n = 36$

S. D of height → 15 cm $\sigma = 15$ cm

$P(\bar{x} < 157)$

at $\bar{x} = 157$ $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

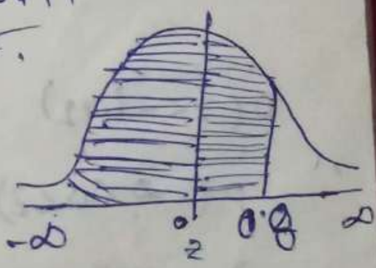
$= \dfrac{157 - 155}{15/\sqrt{36}}$

$= 0.8$

$\Rightarrow P(-\infty < Z < 0) + P(0 < Z < 0.8)$

$\Rightarrow 0.5 + \cancel{0.3849}\ 0.2881$

$= \cancel{0.8849}\ 0.7881$



$P(\bar{x} > 153)$

at $\bar{x} = 153$ $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{153 - 155}{15/\sqrt{36}}$

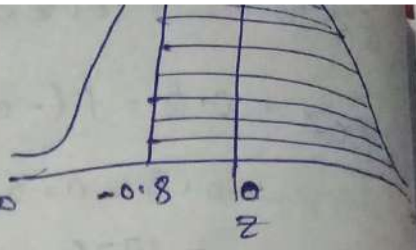
$= -0.8$

~~$P(\bar{x} > 153) = P(Z > 0.8)$~~

$\Rightarrow P(0 < z < \infty) + P(-\infty < z < -0.8)$

$\Rightarrow 0.5 + 0.2881$

$\Rightarrow 0.7881$



A population has a mean of 0.1 and ~~standard~~ And standard deviation 2.1 find the probability that a mean of sample size 900 will be negetive.

$\mu = 0.1$

$\sigma = 2.1$

$n = 900$

$P(\bar{x} < 0)$

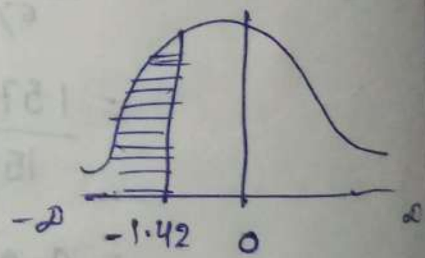
at $\bar{x} = 0 \Rightarrow z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{0 - 0.1}{2.1/\sqrt{900}}$

$z = -1.42$

$P(\bar{x} < 0) = P(z < -1.42)$

$= -1.42$



$P(z < -1.42)$

$\therefore P(-\infty < 0 < z) - P(0 > z > -1.43)$

$\Rightarrow 0.5 - 0.4267$

$= 0.0791$

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A normal Population has mean 100 and varience 25 How large the random sample be considered when we want the standard error of the sample mean to be 1.5

$\mu = 100$

varience $\sigma^2 = 25$

standard $= \sqrt{25} = 5$

$\bar{x} = 1.5$

$P(\bar{x} = 1.5)$

standard error $(\sigma) = \frac{\sigma}{\sqrt{n}}$

$1.5 = \frac{\sigma}{\sqrt{n}} \Rightarrow 1.5 = \frac{25}{\sqrt{100}} = 1.5 = \frac{25}{10} =$

$n \times 2.25 = 25$

$n = \frac{25}{2.25}$

$\boxed{n = 11}$

mean voltage as a battery is 15 volts and S.D is 0.2 volts find the probability that, 4 such batteries connected in series ~~will have a combined~~ will have a combined voltage of 60.8 or more volts

$\mu = 15$

$\sigma = 0.2$

$n = 4$

we have to find

$P(\Sigma x_i > 60.8)$

$P\left(\left(\frac{\Sigma x_i}{n}\right) > \frac{60.8}{4}\right)$

$\Rightarrow P(\bar{x} > 15.2)$

at $\bar{x} = 15.2 \Rightarrow z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

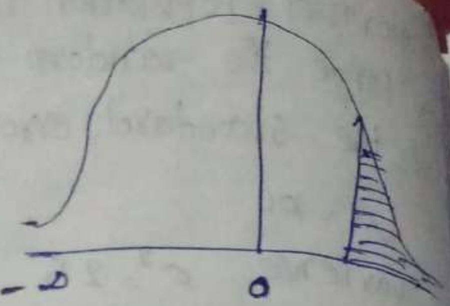
$= \frac{15.2 - 15}{\frac{0.2}{\sqrt{4}}}$

$\boxed{z = 2}$

$\Rightarrow P(0<z<\infty) - P(0>z>2)$

$\Rightarrow 0.5 - 0.4772$

$\Rightarrow 0.0228$



if the distribution of weights of travelling by air between hyderabad and NewDelhi as a mean of 163 pounds and standard deviation of 18 pounds what is the probability that the combined 36 men traveling on a plane between two citys is more than 6000 pounds

$$P(\Sigma x_i > 6000)$$

$$P\left(\frac{\Sigma x_i}{n} > \frac{6000}{36}\right)$$

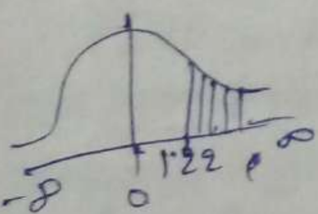
$$P(\bar{x} > 166.6)$$

at $\bar{x} = 166.6$ $\quad z = \dfrac{166-163}{18/\sqrt{36}}$

$$z = 1.2$$

$P(\bar{x} > 166.6) = P(z > 1.2)$

$= P(0<z<\infty) - P(0<z<1.22)$



$= 0.5 - 0.3888$

$= 0.1112$

1) mean breakin
with a stan
sample mu
chance in l
is less than

2) A random
large a sa
is in b/w

1) $\mu = 5$
$\sigma =$
$n =$

$P(\bar{x}$
at

$P(\bar{x} < 5$

$\Rightarrow P(0$

$\Rightarrow \frac{\bar{x}}{\sigma}$

$\frac{-3}{8}$

1) mean breaking strength of copper wire is 575 lbs with a standard deviation of 8.3 lbs. How large a sample must be used in order that there will be 1 chance in 100 that the mean breaking strength of sample is less than 572 lbs

2) A random sample of (normal $\mu = 30, \sigma^2 = 12$) then How large a sample should be taken if the sample mean is in b/w 25 and 35 with Probability 0.95

1) $\mu = 575$

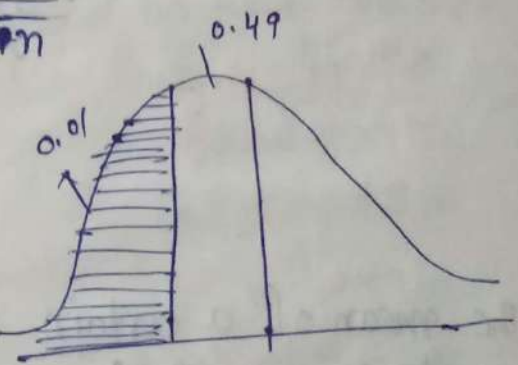
$\sigma = 8.3$

$n = ?$

$P(\bar{x} < 572) = \frac{1}{100} = 0.01$

at $\bar{x} = 572 \Rightarrow z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{572 - 575}{8.3/\sqrt{n}}$



$P(\bar{x} < 572) = P(Z < z_1)$

$\Rightarrow P(0 < Z < z_1) = 0.5 - 0.01$

$= 0.49$

$z_1 = -2.33$

$\Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = z_1$

$\frac{-3\sqrt{n}}{8.3} = -2.33$

$\boxed{n = 41.5}$

2) $\mu = 30$

$\sigma^2 = 12$

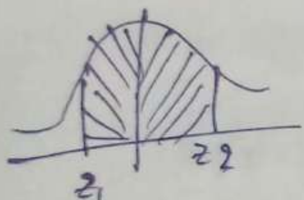
$\sigma = \sqrt{12} = 3.46$

$P(25 < \bar{x} < 35) = 0.95$

at $\bar{x} = 25$ $\quad Z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{25-30}{\frac{\sqrt{12}}{\sqrt{n}}} = \dfrac{-5\sqrt{n}}{\sqrt{12}} = Z_1$

$\bar{x} = 35 \Rightarrow Z = \dfrac{35-30}{\frac{\sqrt{12}}{\sqrt{n}}} = \dfrac{5\sqrt{n}}{\sqrt{12}} = Z_2$

$P(Z_1 < Z < Z_2) = 2 \times P(0 < Z < Z_1) = 0.95$



$P(0 < Z < Z_1) = 0.475$

$Z_1 = -1.96 \qquad Z_2 = 1.96$

$\dfrac{-5\sqrt{n}}{\sqrt{12}} = -1.96 \Rightarrow \sqrt{n} = \dfrac{1.96 \times \sqrt{12}}{5}$

$n = (1.3579)^2$

$n = 1.84$

$\boxed{n \cong 2}$

The mean of a certain normal population is equal to standard error of the mean of the samples of 64 from that distribution find the prob that the mean of sample size 36 will be negative.

given $\mu = \dfrac{\sigma}{\sqrt{n}} \Rightarrow$ $\quad \boxed{\sigma = 8\mu}$

$n = 64$

$P(\bar{x} < 0)$

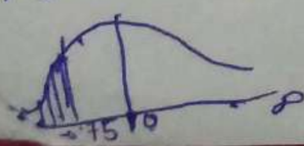
$\bar{x} = 0 \Rightarrow Z = \dfrac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow Z = \dfrac{0-\mu}{\frac{8\mu}{6}} = \dfrac{-6}{8} = \dfrac{-3}{4} = -0.75$

$P(Z < -0.75)$

$= P(-\infty < Z < 0) - P(0 < Z < 0.75)$

$= 0.5 - 0.20734$

$= 0.2266$



5.1 Estimat...

estimating the ... statistics.

Methods of E...

1) Point Estim...

2) Internal E...

Internal Esti...

using the s... to an exact ... the parameter ... which are a... confidence I... large

A random s... of standard ... 2.08. const... Population m...

$n = 81$, σ ...

at 95%.

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ME ...

ME

ME

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5.1 / 5.2 Estimation

The concept of estimation is a process of estimating the population parameters using the sample statistics.

Methods of Estimation

1) Point Estimation
2) Interval Estimation

Interval Estimation :-

To Estimate any population parameter using the sample statistic when it is not possible to an exact point estimate then the prefer estimating the parameter value as an interval with some confidence which are also called confidence intervals.

confidence Intervals for 'μ' using $\bar{x}$ in large sample

large sample is $n \geq 30$

A random sample of size 81 is taken from a population of standard deviation 0.9, if the sample mean is 2.08. construct a 95% confidence interval for the population mean.

$n = 81, \ \sigma = 0.9, \ \bar{x} = 2.08$

at 95% , $Z_{\alpha/2} = 1.96$

$$\boxed{\mu \in \left(\bar{x} \mp Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)}$$

$$\mu \in \left(2.08 \mp 1.96\left(\frac{0.9}{9}\right)\right)$$

$$\mu \in (2.08 \pm 0.196)$$

$$\mu \in (2.08 - 0.196, \ 2.08 + 0.196)$$

$$\mu \in (1.884, \ 2.276)$$

construct 99% confidence interval for μ,

Given $n = 81$, $\sigma = 0.9$, $\bar{x} = 2.08$

at 99%, $Z_{\alpha/2} = 2.58$

$$\mu \in \left(\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$\mu \in \left(2.08 \pm 2.58\left(\frac{0.9}{9}\right)\right)$$

$$\mu \in (2.08 \pm 0.258)$$

$$\mu \in (2.08 - 0.258,\ 2.08 + 0.258)$$

$$\mu \in (1.822,\ 2.338)$$

Confidence Interval for μ:-

$$\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

where $E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$ is called the maximum error, defined as the maximum difference between $\bar{x}$ and μ

then $\mu \in (\bar{x} \pm E)$

$Z_{\alpha/2}$ values

Level %	$Z_{\alpha/2}$
90%	1.645
95%	1.96
98%	2.33
99%	2.58

Here at 95% the tabular value of $Z_{\alpha/2}$ is identified as consider $\frac{0.95}{2} = 0.475$

corresponding to 0.475 Prob that $Z_{\alpha/2}$ in table is 1.96

if the mean b
is 505lbs wit
observations,
confidence int
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given that
for 90%. Z_{α}
$\Rightarrow \mu \in (\bar{x}$
$\mu \in (505$
$\mu \in (50$
$\mu \in (50$
$\mu \in (50$
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for 95%.
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To estima
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99%. confid
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Given

If the mean breaking ~~Statemer~~ strength of copper wire is 505 lbs with a S.D of 15 lbs for a sample of 49 observations, then estimate the 90% and 95% confidence interval for the mean breaking strength of entire population.

given that $\bar{x} = 505$ and $\sigma = 15$, $n = 49$

for 90%. $Z_{\alpha/2} = 1.645$ [∴ for large sample $\sigma = 1$]

$\Rightarrow \mu \in \left(\bar{x} \pm \frac{Z_{\alpha/2}}{2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$

$\mu \in \left(505 \pm 1.645\left(\frac{15}{7}\right)\right)$

$\mu \in (505 \pm 0.235(15))$

$\mu \in (505 \pm 3.525)$

$\mu \in (505 - 3.525,\ 505 + 3.525)$

$\mu \in (501.475,\ 508.525)$

for 95%, $Z_{\alpha/2} = 1.96$

$\mu \in \left(\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$

$\mu \in \left(505 \pm 1.96\left(\frac{15}{7}\right)\right)$

$\mu \in (505 \pm 0.28(15))$

$\mu \in (505 \pm 4.20)$

$\mu \in (505 - 4.2,\ 505 + 4.2)$

$\mu \in (500.8,\ 509.2)$

To estimate the mean amount spent per customer for dinner at a city hotel, data was collected from a sample of 49 customers shows the average amount of Rs. 185/- with a SD of Rs 25/- construct 98% and 99% confidence intervals for the average amount spent by the population of customers.

Given that $\bar{x} = 185$, $\sigma = 25$, $n = 49$

at 98%, $Z_{\alpha/2} = 2.33$

$\Rightarrow \mu \in \left(\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$

$\mu \in \left(185 \pm 2.33\left(\frac{25}{7}\right)\right)$

$\mu \in (185 \pm 0.333(25))$

$\mu \in (185 \pm 8.325)$

$\mu \in (185 - 8.325,\ 185 + 8.325)$

$\mu \in (176.675,\ 193.325)$

at 99% $Z_{\alpha/2} = 2.58$

$\mu \in \left(\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$

$\mu \in \left(185 \pm 2.58\left(\frac{25}{7}\right)\right)$

$\mu \in (185 \pm 0.368(25))$

$\mu \in (185 \pm 9.200)$

$\mu \in (185 - 9.200,\ 185 + 9.200)$

$\mu \in (175.8,\ 114.2)$

Q) Assuming that the population SD is 20, how large a random sample be taken to a assert with prob 0.95 that the sample mean will not differ from the true mean by more than 3.

Given that $\sigma = 20$, for 0.95 or 95% $Z_{\alpha/2} = 1.96$

$E = 3$ W.K.T $E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$3 = 1.96\left(\frac{20}{\sqrt{n}}\right)$

$\sqrt{n} = \frac{1.96 \times 20}{3}$

$\sqrt{n} - 0.653 \times 20$

$\sqrt{n} = 1.306 \times 10 = 13.06$

$\sqrt{n} = 13.06$

$\boxed{n = 170.5636}$

It is desired to estimate the mean no.of continues use with a contain computer with first require. if it can be assumed that $\sigma = 48$ hours. How large sample B meeden so that we can Assert with 90% confidence that a sample mean is off by almost 10 hours.

at 90%. $Z_{\alpha/2} = 1.645$

$\sigma = 48$

maximum error $E = 10$

$$\boxed{E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$10 = 1.645\left(\frac{48}{\sqrt{n}}\right)$$

In a study of an automobile insurance of a random sample of 80 vehicle repair costs had a mean of Rs. 472.36 and a standard deviation of RS 62.35 if $\bar{x}$ is an estimate, with what confidence we can measure maximum error does not exceed RS. 10

given that $n = 80$

$\bar{x} = 472.36$

$\sigma = 62.35$

maximum Error $E = 10$

$$\mu = \bar{x} + \left[Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right]$$

$$= \bar{x} + E$$

$$E = Z_{\alpha/2}\left(\frac{62.35}{\sqrt{80}}\right)$$

$$\frac{10 \times \sqrt{80}}{62.35} = Z_{\alpha/2}$$

$$Z_{\alpha/2} = 1.43$$

corresponding to 1.43 of z probability is 0.4236

$\therefore$ The confidental $= 2 \times 0.4236$
$= 84.72\%$.

The efficiency expert of a computer company tested 40 injuries to estimate the average time it takes to assemble a particular component, getting a mean of 12.73 minutes and a standard deviation of 2.06 minutes.

(i) determine the maximum error with 99% conflicts

(ii) construct 95% confidence interval for the two average time

(iii) with what confidence we can say that the sample mean does not filter from the true mean by more than 30sec.

$n = 40$

sample mean $\bar{x} = 12.73$

standard deviation $\sigma = 2.06$

$Z_{\alpha/n} = 2.58$

$\boxed{Z\alpha/n = 258 \text{ constant}}$

$\mu = \bar{x} + \left[Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right]$

$\bar{x} + E$

$= 12.73 \pm \left(2.58 \cdot \frac{2.06}{\sqrt{40}}\right)$

$= (12.73 \pm 0.84,\ 12.73 - 0.84)$

$= (13.57,\ 11.89)$

(ii) $\mu = \bar{x} + \left[Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right]$

$= \bar{x} + E$

$= 12.73 \pm \left[1.96\left(\frac{2.06}{\sqrt{40}}\right)\right]$

$= 12.73 \pm (1.96)(0.32)$

$= 12.73 \pm (0.62)$

$= (12.73 + 0.62,\ 12.73 - 0.62)$

$= (13.36, 12.1)$

(iii) $E = 30sec = 0$

w.k.T

$E = Z$

$0.5 =$

$0.5 =$

$Z\alpha/2 =$

$\boxed{Z\alpha/2}$

$P(1.53) = .0$

$=$

$=$

$=$

confidence

confidence

the sample

The confide

$= (13.36, 12.1)$

(iii) $E = 30 sec = 0.5$

w.k.t

$$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$0.5 = Z_{\alpha/2}\left[\frac{2.06}{\sqrt{40}}\right]$$

$$0.5 = Z_{\alpha/2}(0.325)$$

$$Z_{\alpha/2} = \frac{0.5}{0.325}$$

$$\boxed{Z_{\alpha/2} = 1.53}$$

$P(1.53) = 0.4370$

$= 0.4370 \times 2$

$= 0.874 \times 100 (\%)$

$= 87.4\ \%$ //

<u>confidence interval of Population</u>

confidence interval of Population proposition (P) using the sample proposition 'p'

The confidence interval is given by $p \pm Z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}}$

$$\boxed{\hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}}$$ $\hat{p}$ $\hat{q}$ are estimates

A sample of maximum Pressure 12 trails with a stamping machine yielded a mean of 21.2 thousand psi and standard deviation of 1.2 thousand psi obtain a 95% and 98% verified interval for the population mean of maximum.

At 98% $t_{\alpha/2} = 2.72$, 95% $t_{\alpha/2} = 2.20$

The total value of t at $n-1 = 11$ degree of freedom.

$s = 1.2, n = 12, \bar{x} = 7.2$, At 95% = 2.20

$\mu \in (\bar{x} \pm t_{\alpha/2} (\frac{s}{\sqrt{n}})$

$\mu \in (7.2 \pm 2.20 (\frac{1.2}{\sqrt{12}}))$

$\mu \in (7.2 \pm 0.762)$

$\mu \in (7.962, 6.438$

At 98% = 2.72

$\mu \in (\bar{x} \pm 2.72 (\frac{s}{\sqrt{n}})$

$\mu \in (7.2 \pm 2.72 (\frac{1.2}{\sqrt{12}}))$

$\mu \in (7.2 + 0.942)$

$\mu \in (8.142, 6.258)$

A sample 11 rats from a central Population an average blood vescosity of 3.92 with a S.D of 0.61, estimate ~~interval~~ limits for the 95% time, 99% confidence interval limits for the Population.

At 95% = 2.23 $n = 11$ $s = 0.61$

$\bar{x} = 3.92$

$\mu \in (3.92 \pm 2.23 (\frac{0.61}{\sqrt{11}}))$

$\mu \in (3.92 \pm 0.410)$

$\mu \in (4.330, 3.50)$

at 99% = 3.17

$\mu \in \left(3.92 \pm 3.17 \left(\frac{0.61}{\sqrt{11}}\right)\right)$

$\mu \in (3.92 \pm 0.583)$

$\mu \in (4.503,\ 3.337)$

* Find 95% confidence interval for the mean of a normality distribute Population from which the following sample mistake.

mean $= \frac{15+17+10+18+16+9+7+11+13+14}{10}$

$\bar{x} = 13$

$S.D = \sqrt{\frac{1}{n-1} \Sigma (x_i - \bar{x})^2}$

$= \sqrt{\frac{1}{9} (15-13)^2 + (17-13) + (10-13)^2 + (18-13)^2 + (16-13)^2 + (9- \ldots + (7-13)^2 + (11-13)^2 + (13- \ldots + (14-13)^2}$

$= \sqrt{\frac{1}{9} (4+16+9+25+9+16+36+4+0+1)}$

$= \sqrt{\frac{120}{9}}$

$s = 3.65$

$\mu \in \left(\bar{x} \pm 2.26 \left(\frac{3.65}{\sqrt{10}}\right)\right)$

$\mu \in \left(13 \pm 2.26 \left(\frac{3.65}{\sqrt{10}}\right)\right)$

$\mu \in (13 \pm 2.60)$

$\mu \in (15.60,\ 10.4)$

4. Tes

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4. Tests of Hypothesis

→ Hypothesis
→ Null Hypothesis
→ Alternate Hypothesis
→ one-tail and Two Tail tests
→ Two types of errors
→ critical Region (level of significance)

From the problem context, identify the parameters of interest and then the listed below.

step 1 : state the Null Hypothesis ~~is~~ H_0.

step 2 : specify an ~~alternate~~ appropriate alternate Hypothesis H_1.

step 3 : choose the level of significance α.

step 4 : ~~cho~~ calculate The appropriate test statistic ~~value~~ $\cancel{Z = \frac{t - E(t)}{S.E(t)}}$

step 5 : compare the calculated test statistic value with critical region value at level.

~~step 6 : conclusion : If~~ $\cancel{|Z|_{cal} > Z}$ ~~table value~~.

~~step 6~~. conclusion : Reject H_0 when the calculated test statistic value is greater than the critical region value otherwise we fail to Reject H_0

1) Null Hypothesis $H_0 = \mu = 5.7$

2) Alternate Hypothesis $H_1 : \mu \neq 5.7$ (Two-trial)

$\mu > 5.7$
$\mu < 5.7$ } one-trail

3) level of significance (α)

4) calculate test statistic value $Z = \frac{t - E(t)}{S.E(t)}$

5) conclusion:

If $|Z|_{cal} > Z_{table\ value}$

1) Z-test for single mean

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$\boxed{\text{Model-1 (Z-Test for single mean)}}$

4) An Ambulance service.

mean $\mu = 8.9$, standard deviation $\sigma = 1.6$

Sample size $n = 50$, sample mean $\bar{x} = 9.3$

we have to test

Null Hypothesis $H_0 : \mu = 8.9$

Alternate Hypothesis $H_1 : \mu \neq 8.9$ (Two-tailed test)

level of significance $\alpha = 0.01$

Test statistic $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{9.3 - 8.9}{1.6/\sqrt{n}}$

$= \frac{0.4}{0.2263}$

$= 1.768$

The calculated value of $Z = 1.768 < 2.58$

$\mu = 20,000$

samp

we have to tes

N

The

Two ta
one t

$\mu = 20,000$ Standard deviation $\sigma = 3900$,

sample size $n = 100$, sample mean $\bar{x} = 23.500$

we have to test

Null Hypothesis $H_0 : \mu = 20,000$

Alternate Hypothesis $H_1 : \mu > 20,000$ (one-tailed test)

significance $\alpha = 0.05$

Test stastic $\quad z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{23500 - 20000}{3900/\sqrt{100}}$

$= \dfrac{3500}{390}$

$= 8.974$

The calculated value of $z = 8.974 > 1.645$ (significance value or table value z)

	1%	5%	10%
Two tailled	2.58	1.96	1.645
one tailed	2.33	1.645	1.28

Z - Test for difference of means :-

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - E(\bar{x}_1 - \bar{x}_2)}{S.E(\bar{x}_1 - \bar{x}_2)} =$$

$$\boxed{Z = \frac{(\bar{x}_1 - \bar{x}_2) - (x_1 - x_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n_2}}}}$$

3. Z-test for single proportion

Z-test for single proportion

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} \quad \text{(under } H_0\text{)} \quad \text{where } Q = 1-$$

Example 3 ; In a sample of 1,000 people in mahara

Sol :- given the samples size $n = 1,000$

No. of rice eaters in sample $X = 540$

sample proportion of rice eaters $p = \frac{X}{n} = \frac{540}{1000}$

whe have to test

$= 0.54$

Null Hypothesis $H_0 : P = 0.5$ $(Q = 1-P = 0.5)$

Alternate Hypothesis $H_1 : P \neq 0.5$ (Two - tailed test)

level of significance $\alpha = 0.01$

Test statistic $Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$

$$= \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}}$$

$$= \frac{0.04}{\sqrt{0.0025}}$$

$$= 2.52$$

4. Z-test for Dif

$$Z = \frac{(P_1 - P_2) - E}{S.E(P_1 -}$$

$$\boxed{\Rightarrow Z = \frac{(P_1 -}{\sqrt{PQ}}}$$

capital $P = \frac{n_1 P}{}$

$\boxed{P = }$

$Q = 1 - P$

1) $n_1 = 100$

$P_1 = \frac{63}{100}$

Null Hypothesis

Alternate Hypoth

level of

Test stati

$$= \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}}$$

$$= \frac{0.04}{\sqrt{0.0025}}$$

$$= 2.52$$

4. Z-test for Difference of Proportions

$$Z = \frac{(P_1 - P_2) - E(P_1 - P_2)}{S.E\,(P_1 - P_2)} = \frac{(P_1 - P_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$$

$$\boxed{\Rightarrow Z = \frac{(P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_2} + \frac{1}{n_2}\right)}}}$$

$Q = 1 - p$

capital $P = \dfrac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$$\boxed{P = \frac{x_1 + x_2}{n_1 + n_2}}$$

$Q = 1 - P$

1) $n_1 = 100$, $x_1 = 63$, $n_2 = 125$, $x_2 = 59$

$P_1 = \frac{63}{100} = 0.63 \qquad P_2 = \frac{59}{125} = 0.472$

Null Hypothesis $H_0 : P_1 = P_2$ (proportion of residents in city and suburbans are same)

Alternate Hypothesis $H_1 : P_1 \neq P_2$ (Two-tailed test)

Level of significance $\alpha = 0.05$

Test statistic $z = \dfrac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ (under $H_0 : P_1 = P_2$)

An unbaised estimate of Population portion P based on samplesis

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$= \frac{63 + 59}{100 + 125} = \frac{122}{225}$$

$$= 0.542$$

$$Z = \frac{(P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.63 - 0.472}{\sqrt{0.542 \times 0.458\left[\frac{1}{100} + \frac{1}{125}\right]}}$$

$$= \frac{0.158}{\sqrt{0.00447}}$$

$$= 2.363$$

since $Z = 2.363 > 1.96$, the value is significant, so we reject the null Hypothesis H_0.

Part - II

4. Tests of Hypothesis

1. t - test for single mean

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

given that

$\mu = 18.5$

$n = 14$

sample mean $\bar{x} = 17.85$

standard deviation $s = 1.955$.

we have to test

Null Hypothesis $H_0 : \mu = 18.5$

Alternat Hypothesis $H_1 : \mu \neq 18.5$ (Two-tailed test)

level of significance $\alpha = 0.05$

Test static $t = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right|$

$= \left|\frac{17.85 - 18.5}{1.955/\sqrt{14}}\right|$

$= \left|\frac{-0.625}{0.5225}\right|$

$= 1.244$

sample size $n = 22$ sample mean $\bar{x} = 153.7$

Standard deviation $s = 17.2$ comparative (EXP) mean $\mu = 146.3$

we have to test

Null Hypothesis $H_0 : \mu = 146.3$

Alternate Hypothesis $H_1 : \mu \neq 146.3$ (~~Two~~ one-tailed test)

level of significance $\alpha = 0.05$

Test statistic $t = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right|$

$= \left|\frac{153.7-146.3}{17.2/\sqrt{22}}\right|$

$= \left|\frac{7.4}{3.667}\right|$

$= 2.0179$

2. t- test for Difference of Means

$\Rightarrow t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$ (under H_0)

which follow a Student's t- distribution with (n_1+n_2-2) degrees of freedom

where $s^2 = \frac{1}{n_1+n_2-2}\left[(n_2-1)s_1^2 + (n_2-1)s_2^2\right]$

Example

$n_1 = 21$ $\quad \bar{x}_1 = 420$

$s_1 = 4$ $\quad n_2 = 21$

$\bar{x}_2 = 426$ $\quad s_2 = 3$

we have to test

Null Hypothesis $H_0: \mu_1 = \mu_2$

Alternate Hypothesis $H_1: \mu_1 \neq \mu_2$ (Two - tailed test

Level of significance $\alpha = 0.05$

test statistic $t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{s^2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$

$s^2 = \frac{1}{n_1+n_2-2}$

$= \frac{1}{40}$ (64

$= 12.50$

$t =$ 4

$=$

$= 5.$

since calcula

significant.

4. F- test fo

Example 1 :-

given th

we have

Level o

$$s^2 = \frac{1}{n_1+n_2-2}\left[(n_2-1)s_1^2 + (n_2-1)s_2^2\right]$$

$$= \frac{1}{40}(6400+3600)$$

$$= 12.50$$

$$t = \frac{420-426}{\sqrt{12.5\left[\frac{1}{21}+\frac{1}{21}\right]}}$$

$$= \frac{6}{1.09}$$

$$= 5.5045$$

since calculated value of $t = 5.5045 > 2.021$, it is highly significant.

4. F-test for Equality of population Variances

Examp 1 :- pumpkins were grown

given that $n_1 = 11$, $n_2 = 9$; $s_1 = 0.8$, $s_2 = 0.5$

we have to test

$H_0 : \sigma_1^2 = \sigma_2^2$

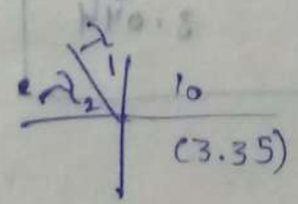
$H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance $\alpha = 0.05$

$s_1^2 = 0.64$

$s_2^2 = 0.25$

$$F = \frac{s_1^2}{s_2^2} = \frac{0.64}{0.25} = 2.56$$



$2.56 < 3.35$

no reason to reject

5. χ^2 - test for GOODNESS of FIT

$$\chi^2 = \sum_{i=0}^{m} \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

6. χ^2 - Test for Independence

$$\chi^2 = \sum \left(\frac{(O_i - E_i)^2}{E_i} \right)$$

	A	B
1	5	15
2	15	10
3	20	5
	40	30

Ex

The given attributes are Independent = H_0

Alternate Hypothesis H_1 : Number of accidents depend on age of drivers.

Level of significance $\alpha = 0.05$

O_i	E_i	$(O_i - E_i)^2$	$\left(\frac{O_i - E_i}{E_i}\right)^2$
5	$\frac{20 \times 40}{70} = 11.42$	41	3.59
15	$\frac{25 \times 40}{70} = 14.28$	0.5184	0.03
20	$\frac{25 \times 40}{70} = 14.28$	32.7	2.28
15	$\frac{20 \times 30}{70} = 8.57$	41.3	4.81
10	$\frac{25 \times 30}{70} = 10.71$	0.504	0.04
5	$\frac{25 \times 30}{70} = 10.71$	32.604	3.044

Unit - 5

Correlation & Regression

Karl pearson's coefficient of correlation

$$r_{xy} = r = \frac{cov(x, y)}{\sigma_x \cdot \sigma_y}$$

$$= \frac{\left(\frac{\Sigma xy}{n}\right) - (\bar{x}\,\bar{y})}{\sqrt{\left(\frac{1}{n}\Sigma x^2\right) - \bar{x}^2}\ \sqrt{\left(\frac{1}{n}\Sigma x^2\right) - y^2}}$$

Ex

x	y	xy	x^2	y^2
3	1	3	9	1
5	3	15	25	9
4	2	8	16	4
2	4	8	4	16
6	5	30	36	25
20	15	64	90	55

$n = 5$, $\Sigma x = 20$, $\Sigma y = 15$, $\bar{x} = \frac{20}{5} = 4$, $\bar{y} = \frac{15}{5} = 3$

$\Sigma xy = 64$, $\Sigma x^2 = 90$, $\Sigma y^2 = 55$,

$$r = \frac{\frac{64}{5} - (4 \times 3)}{\sqrt{\frac{90}{5} - 4^2}\ \sqrt{\frac{55}{5} - 3^2}}$$

$$= \frac{12.8 - 12}{\sqrt{2}\ \sqrt{2}}$$
$$= \frac{0.8}{2}$$
$$= 0.4$$

form this correlation data we have

$\bar{x} = 4$

$\bar{y} = 3$

$\sigma_x = \sqrt{2} = 1.414$

$\sigma_y = \sqrt{2} = 1.414$

$r = 0.4$

1) regresson line of y on x

$y = a + bx$

equation is

$$(y - \bar{y}) = \left(r \cdot \frac{\sigma_y}{\sigma_x}\right)(x - \bar{x})$$

y on x

$\therefore y - 3 = (0.4)\left(\frac{1.414}{1.414}\right)\boxed{(x-4)}$

$\Rightarrow y - 3 = (0.4)(x - 4)$

$\Rightarrow y - 3 = 0.4x - 1.6$

$y = 0.4x - 1.6 + 3$

$\Rightarrow \boxed{X = 0.4X + 1.4}$

2) Regression line of x on y

$(x = a + by)$

equation is

$$(x - \bar{x}) = r \cdot \frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

Estimate the most likely value of y.

Estimod the value of x when $y = 2.5$

$x = a + by$

The fo...
certain ...
obtain
regulation

1) Deter...
lines ...

The following table gives the age of cars of a certain Model and their anual maintainance cost obtain The ~~Pottat~~ correlation efficient of two regulation lines.

1) Determine the coefficient of coqulation and two lines of regression

$$\Sigma X = 50$$
$$\Sigma Y = 60$$
$$\bar{x} = 5$$
$$\bar{y} = 6$$
$$\Sigma XY = 350$$

var of $x = 4$ $(\sigma^2 x = 4)$

var of $y = 9$ $(\sigma^2 y = 9)$

$$\boxed{r = \frac{cov(x,y)}{\sigma_x \sigma_y}}$$

$$r = \frac{\left(\frac{\Sigma XY}{n}\right) - (\bar{x}\,\bar{y})}{\sqrt{\left(\frac{1}{n}\Sigma (x^2)\right) - \bar{x}^2}\sqrt{\left(\frac{1}{n}\Sigma y^2\right) - \bar{y}^2}}$$

$$= \frac{\left(\frac{350}{10}\right) - (5\times6)}{\sqrt{4} \quad \sqrt{9}}$$

$$= \frac{35-30}{6} = \frac{5}{6} = 0.8333$$

$\bar{X} = 5$ $\sigma_x = 2$

$\bar{y} = 6$ $\sigma_y = 3$

$r = 0.833$

2) Regreesion line of Y on X

$(Y = a + bX)$

equation is

$$Y - \bar{y} = r\left(\frac{\sigma_y}{\sigma_x}\right)(X - \bar{x})$$

$$Y - 6 = 0.833\left(\frac{3}{2}\right)(X - 5)$$

$y - 6 = (1.245)(x - 5)$

$y - 6 = 1.245x - 6.225$

$y = 1.245x - 6.225 + 6$

$y = 12.45x - 0.225$

ii) Regression line of x on y

equation is $x = a + by$

$(x - \bar{x}) = (r \cdot \frac{\sigma x}{\sigma y})(y - \bar{y})$

$(x - 5) = 0.55y - 3.3$

$\boxed{x = 0.55y + 1.7}$

From the following information about advertising expenditure and sales. What should be the advertising budget if the company sales target of rupees 120 lakhs.

	Advertising exp (lakhs)	Sales (lakhs)
mean	10	90
S.D	3	12

$\bar{x} = 10 \quad \bar{y} = 90$

$\sigma x = 3. \quad \sigma y = 12$

$r = 0.8$

we have to find $x = ?$

at $y = 120$

x on y regression line

$x - \bar{x} = r \cdot \frac{\sigma x}{\sigma y}(y - \bar{y})$

$x - 10 = (0.8)(\frac{3}{12})(120 - 90)$

$x - 10 = 0.2(30)$

$x = 6 + 10$

$x = 16$

* The following data is obtained from Ten observations that is $\Sigma X = 250$, $\Sigma Y = 300$, $\Sigma x^2 = 6500$, $\Sigma y^2 = 10000$
$\Sigma XY = 7900$

(i) $r \times y$

(ii) Two regression lines.

* given that the means of X and Y are 65 and 67 with standard deviations 2.5 and 3.5 respectively and $r = 0.8$
(i) determine two regression line and estimate the value of Y when $X = 68$.

* Estimate the most likely production corresponding to a rainfall of 40 inches from the following data

	Rainfall (in inches)	Production (in quintalls)
Avg	30	50
S.D	5	10

The correlation coefficient is 0.8

Regression - coefficience

Regression-coefficient of y on x

equation is

$$y - \bar{y} = \left(r \cdot \frac{\sigma_y}{\sigma_x}\right)(x - \bar{x}) \qquad y \text{ on } x.$$

$r \cdot \frac{\sigma_y}{\sigma_x}$ is called regression coefficient of

denoted by $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

Regression- coefficient of x on y

$$x - \bar{x} = ($$

Properties

* the Geometric mean of the two regression is the cogulation co-relation $\sqrt{ab}$

$$\sqrt{b_{yx} \quad b_{xy}}$$

$$= \sqrt{r \cdot \frac{\sigma_y}{\sigma_x} \cdot r \frac{\sigma_x}{\sigma_y}}$$

$$= \sqrt{r^2} = \pm r$$

* The two regression coefficient must be of same sign.

3) * if both are positive the coagulation is positive if both are Negitive then coagulation is Negetive

4) if one or
1. The oth
5) The Arithe
greater tha

Spearman's Rank correlation

To find correlation b/w the order of the observations of the observations of x and y the coefficient is given by ρ - "row'.

$$\rho = 1 - \left[\frac{6(\Sigma d_i^2)}{n(n^2-1)}\right]$$

* $d_i = R_x - R_y$

* where R_x and R_y are Ranks of x and y series

1) calculate The correlation coefficient for the following data of Ranks of 10 sets of data.

x	y
6	5
5	8
3	4
10	7
2	10
4	2
9	1
7	6
8	9
	3

Sol :- The ranks are alread given so assume R_x and R_y

R_x	R_y	d_i^2
6	5	$(6-5)^2 = 1$
5	8	$(5-8)^2 = 9$
3	4	1
10	7	9
2	10	64
4	2	4
9	1	64
7	6	1
8	9	1
1	3	4

$\Sigma d_i^2 = 158$

$$\rho = 1 - \left[\frac{6 \times (\Sigma d}{n(n^2-}\right.$$

$$= 1 - \frac{6 \times 158}{10 \times 9}$$

$= 1 -$

A Random ... subjets marke... Rank correlatio...

Roll No	Math
1	85
2	60
3	73
4	40
5	90
6	94
7	82

observations
ent is

$$\rho = 1 - \left[\frac{6 \times (\Sigma d_i^2)}{n(n^2-1)}\right]$$

$$= 1 - \frac{6 \times 158}{100 \times 99}$$

$$= 1 -$$

A Random sample of 7 students choosen in two subjects marks in maths and computers calculate the rank correlation coefficient for the data.

y series
for the

Roll No	Maths	computer
1	85	93
2	60	75
3	73	65
4	40	50
5	90	80
6	94	91
7	82	84

n 60
x and Ry

Roll No	Maths	comp	Rx	Ry	d_i^2
1	85	93	3	1	4
2	60	75	6	5	1
3	73	65	5	6	1
4	40	50	7	7	0
5	90	80	2	4	4
6	94	91	1	2	1
7	82	84	4	3	1

$$e = 1 - \left[\frac{6(\Sigma d_i^2)}{n(n^2-1)}\right]$$

$$= 1 - \left[\frac{6(12)}{7(7^2-1)}\right]$$

$$= 1 - \frac{72}{336}$$

$$= 1 - 0.214$$

$$\boxed{e = 0.7857}$$

Repeated Ranks

Repeated Ranks or tied ranks

Ex 1: find the rank correlation coefficient for the following data which represents the marks of 10 students in two subjects X and Y.

X	Y
68	62
64	58
75	68
50	45
64	81
80	60
75	68
40	48
55	50
64	70

X	Y	R
68	62	
64	58	
75	68	
50	45	
64	81	
80	60	
75	68	
40	48	
55	50	
6.4	70	

Formula

where

T_x o

m

X series

75

64

T

X	Y	Rx	Ry	d_i^2
68	62	4	5	1
64	58	6	7	1
75	68	2.5	3.5	1
50	45	9	10	1
64	81	6	1	2.5
80	60	1	6	25
75	68	2.5	3.5	1
40	48	10	9	1
55	50	8	8	0
64	70	6	2	16
				72

for the students

Formula for Repeated Ranks

$$e = 1 - \left[\frac{6(\Sigma d^2 + T_X + T_Y)}{n(n^2-1)} \right]$$

where T_X and T_Y are correction Factors

$$T_X \text{ or } T_Y = \frac{\Sigma m_i(m_i^2 - 1)}{12}$$

$m_i \rightarrow$ number of times a value is Repeated.

X series

$\Sigma d_i^2 = 72$

$n = 10$

$75 \rightarrow$ 2 times ($m_i = 2$)

$64 \rightarrow$ 3 times ($m_i = 3$)

$$T_X = \frac{2(2^2-1)}{12} + \frac{3(3^2-1)}{12}$$

$$= 0.5 + 2$$

$$= 2.5$$

Y-series

68 → 2 times ($m_i = 2$)

$$T_y = \frac{2(2^2-1)}{12} = 0.5$$

$$e = 1 - \left[\frac{6(\Sigma d^2 + T_x + T_y)}{n(n^2-1)}\right]$$

$$= 1 - \left[\frac{6(72 + 2.5 + 0.5)}{10(10^2-1)}\right]$$

$$= 1 - \left[\frac{6(75)}{970}\right]$$

$$= 1 - [0.4545]$$

$$e = 0.5455$$

* show that IQ of 10 fathers and their eldest sons. calculate the Rank correlation between them

Father's IQ	Son's IQ	R_x	R_y	d_i^2 $(R_x - R_y)^2$
91	102	10	8	4
97	94	9	10	2
102	105	8	7	1
103	115	6.5	3	12.5
103	113	6.5	4.5	4
105	99	5	9	16
110	113	3.5	4.5	1
114	112	2	6	16
110	120	3.5	1.5	4
124	120	1	1.5	0.25

X-series

$\Sigma d_i^2 = 59.5$ $n=10$

$103 \rightarrow 2$ times $(m_i = 2)$

$110 \rightarrow 2$ times $(m_i = 2)$

$$T_X = \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}$$

$$= 0.5 + 0.5$$

$$= 1$$

Y-series

$120 \rightarrow 2$ times $(m_i = 2)$

$113 \rightarrow 2$ times $(m_i = 3)$

sons.

Calculate Rank correlation

Price of Share A	Price of Share B	R_X	R_Y	$(R_X - R_Y)^2$ d_i^2
160	292	7	1	36
164	280	6	2	16
172	260	3	4	1
182	234	1	6	25
166	266	5	3	4
170	254	4	5	1
178	230	2	7	25

$\Sigma d_i^2 = 108$

$$P = 1 - \left[\frac{6(\Sigma d_i)^2}{n(n^2-1)}\right]$$

$$= 1 - \left[\frac{6 \times 108}{7(7^2-1)}\right]$$

$$= 1 - \frac{648}{336}$$

$$= 1 - 1.928$$

$$= -0.928$$

5-unit Formulas

$$r = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\left(\frac{\Sigma xy}{n} - \bar{x}\bar{y}\right)}{\sqrt{\frac{1}{n}(\Sigma x^2) - \bar{x}^2} \cdot \sqrt{\frac{1}{n}(\Sigma y^2) - \bar{y}^2}}$$

Regression lines

(y on x) $\Rightarrow y = a + bx$

equation is $y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x}(x - \bar{x})$

(x on y) $\Rightarrow x = a + by$

equation is $x - \bar{x} = r \cdot \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

Regression coefficients

y on x $\Rightarrow b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

x on y $\Rightarrow b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$