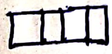
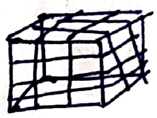
# UNIT - 1

## Linear Algebra

1) Matrix operations
2) Special matarices
3) Matrics Decomposition Techniques

## Data Structures

1) Scaler → Single variable 
2) vector → Array 1D 
3) Matrix → 2D Array 
4) Tensor → nD Array (or) Matrix 

## Operations

1) Norms
2) Trace
3) Determinant
4) Eigen values
5) singular values

**Scalar :-**
The scalar is a just a single number

**vector** – is a array of number

→ vector can be understood as identifying point in space with each element giving the coordinate along different axis

**Matrix**

→ It is a 2D array of number too

→ Each Element is identity by x indexes instance of one

## Tensor

→ Tensor is an array of a number on a regular grid with a variable number on axis.

## Operations on matrics

1) Matrix Addition
2) Multiplication
3) Transpose
4) Inverse
5) Broad casting
6) Hadamard Product
7) Dot product
8) Trace
9) Norm

## Broad casting

Adding a vector with a matrix

$m \times n$

$n \times 1$

⇓

$$\boxed{A_{ij} + b_j = c_{ij}}$$

## Hadamard product:-

Element wise multiplication

$$\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 2\times1 & 4\times2 \\ 1\times5 & 5\times6 \end{bmatrix}$$

$$\boxed{C = A \odot B}$$

$m \times n$ $m \times n$ $m \times n$

## Trace

Sum of all the principal diagonal elements is called trace.

## Norm :-

Mapping from a vector or tensor to a scaler is called Norm.

→ To estimate how big a vector (or) tensor

→ To estimate the closeness b/w two tensors

→ Norm the way of measuring the length of vectors, Matrics (or) tensors.

## properities of Norms :-

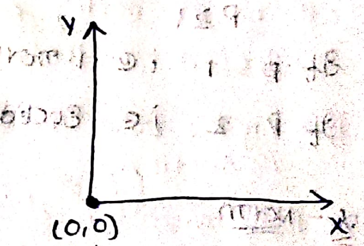
1) Triangular inequality ⇒ $f(x), f(y)$
$f(x+y) \leq f(x) + f(y)$

2) Positive definiteness → If $f(x) = 0$, then $x = 0, \forall x \in X$

3) Linearity

## Linearity :-

→ For $\forall a \in \mathbb{R}, f(ax) = |a| f(x)$

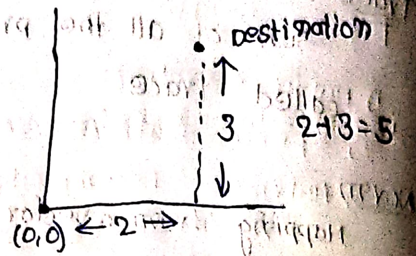



→ one value $f(x) = 0$ at only origin

## Different types of Norms

1) Euclidean Norm
2) $L_1$-norm (or) (Manhattan Distance)
3) P-norm
4) $\infty$-norm
5) Frobenius Norm

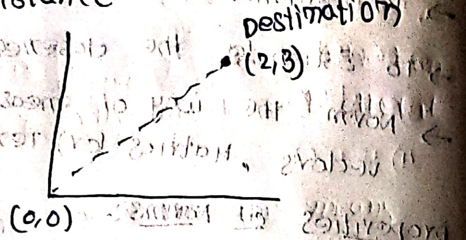
**1-norm**




$$||a|| = (2+3)^1 = 5$$

**Euclidean Norm**

- It is the shortest distance




$$||a||_2 = \sqrt{2^2+3^2} = \sqrt{13}$$

**P- Norm**

$$||v||_p = \left(|v_1|^p + |v_2|^p + \dots + |v_n|\right)^{1/p}$$

$P \geq 1$

If $p = 1$ i.e 1-norm

If $P = 2$ i.e Euclidean Norm

**$\alpha$ - Norm**

$$||v||_\alpha = \max\left(|v_1|, |v_2|, \dots |v_n|\right)$$

$$\vec{v} = \begin{bmatrix} -5 \\ 8 \\ 2 \end{bmatrix} \quad ||v||_\alpha = 8$$

8 is highest among them because

**Frobenius Norm**

→ the input is matrix

$$||A||_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2}$$

Eg :-

$$\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} = \sqrt{2^2+3^2+5^2+1^2}$$

$$= \sqrt{4+9+25+1}$$

$$= \sqrt{39}$$

**Example**

Find the norm of the following vector

$$A = [3, 4, 5]$$

i, Euclidean norm

ii, 1-Norm

iii, Infinity Norm

i,

$$||a||_2 = \sqrt{3^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$$

ii) 1- Norm

Given vector A = [3, 4, 5]

$$||a|| = (3+4+5)^1 = 12$$

iii) $\alpha$ - Norm

$$||V||_\alpha = \max\left(|v_1|, |v_2|, \dots |v_n|\right)$$

$$\vec{v} = [3, 4, 5]$$

$$||V||_\alpha = 5$$

**Example 2**

Given that $A = \begin{bmatrix} 2 \\ 1 \\ 7 \\ -2 \end{bmatrix}$

i,

$$||a||_2 = \sqrt{2^2+1^2+7^2+(-2)^2} = \sqrt{4+1+49+4} = \sqrt{58}$$

ii, $||a|| = [2+1+7-2]^1 = 8$

iii, $||V||_\alpha = \max(|v_1|, |v_2| \dots |v_n|)$

$$||V||_\alpha = 7$$

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## Special Matrics

1) Diagonal Matrix
2) Symmetric Matrix $\Rightarrow A = A^T$
3) Upper Triangular
4) Lower Triangular
5) orthogonal Matrix $\Rightarrow$
7) or

### Orthogonal Matrix

- A vector $x$ and vector $y$ orthogonal to each other if

$x^T y = 0$

→ If both vector have non-zero norm, this means 90° to each other

### Orthonormal:-

→ If vector are orthongonal with a unit norm they are called as orthonormal.

→ othogonal vector or matrics are used to develop advanced methods.

1) Singular value Decomposition (SVD) for dimensionality reduction.
2) Regularization of a convolution layer, in CNN
3) Haar Transform ⇒ Image Process applications

## Eigen Decomposition

- Decomposition:- Breaking an object into parts
- Eigen Decomposition
  Decompose a matric into a set of eign vector or eigen value.

## Eigen vector :-

The Eigen value A is a non zero vector $N$ such that multiplication by A alter only the scale &

$$\boxed{Av = \lambda v}$$

- Scalar $\lambda$ is known as Eigen value corresponding

$$\boxed{A = V \operatorname{diag}(\lambda) V^{-1}}$$

to Eigen vector A conihic can be difference as

→ Positive definite $\lambda > 0$ 3 → only positive

→ Positive semi definite 0-2 0 → No negative

→ Negative Definite → All negatives

→ Negative semi definite → Negatives + zero values

## Singular value Decomposition

Let A be any $m \times n$ matrix

→ There are orthogonal matrics U, V and a diagonal matrix $\Sigma$ such that

$$\boxed{A = U \Sigma V^T}$$

→ The column of $\boxed{U}$ are the eign vectors of $AA^T$

→ The column of $\boxed{V}$ are the eigen vector of $A^TA$

→ The diagonal element of $\Sigma$ are the singular values

$$\boxed{\sigma_i = \sqrt{\lambda_i}}$$

what the Relationship b/w $u^i$ and $v_i$

① $Av_i = \sigma_i u_i = A^T u_i = \sigma_i v_i$

② $\|Av_i\| = \sigma_i = \|A^T u_i\|$

$$B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

1) $AA^T$
2) $|AA^T - \lambda I| = 0$

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**Diagnolization Algorithm**

n Square Matrix $A$

**Step1:-** Find the characteristic polynomial $\Delta(t)$ of $A$.

**Step2:-** Find the roots of $\Delta(t)$ to obtain the eignvalues of $A$.

**Repeat:** i, & ii, for each eignvalue $\lambda$ of $A$

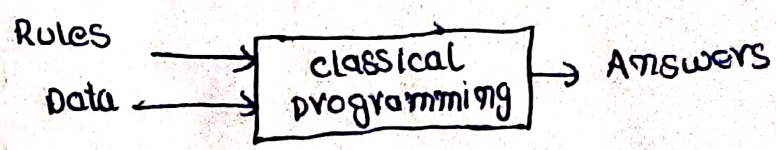
**Step3:-** i, Form the matrix $\boxed{M = A - \lambda I}$ by subtraction

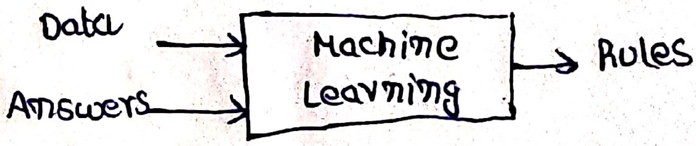
20/12/23

# UNIT-2

## Definition (machine learning)

A machine learning system is trained rather than explicity programmed. It's presented with many examples relevant to a ask, and its finds statistical structure in these examples that eventually allows the system to come up with rules for automating the task




## How Deep Learning works ?