UNIT IV: Electric Traction-II

INTRODUCTION

- The movement of trains and their energy consumption can be most conveniently studied by means of the speed–distance and the speed–time curves.
- The motion of any vehicle may be at constant speed or it may consist of periodic acceleration and retardation.
- The speed–time curves have significant importance in traction. If the frictional resistance to the motion is known value, the energy required for motion of the vehicle can be determined from it. Moreover, this curve gives the speed at various time instants after the start of run directly.

TYPES OF SERVICES

There are mainly three types of passenger services, by which the type of traction system has to be selected, namely:

1. Main line service
2. Urban or city service
3. Suburban service

UNIT IV: ELECTRIC TRACTION-II

Main line service.

- In the main line service, the distance between two stops **is usually more than 10 km**. High balancing speeds should be required. Acceleration and retardation are not so important.

Urban or city service

- In the urban service, the distance between two stops is very less and **it is less than 1 km**. It requires high average speed for frequent starting and stopping.

Suburban service

- In the suburban service, the distance between two stations is **between 1 and 8 km**. This service requires rapid acceleration and retardation as frequent starting and stopping is required

TRAIN MOVEMENT

- The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various times after the start of the run and the later gives speed at various distances from the starting point. Out of the two, speed/time curve is more importance because

1. Its slope gives acceleration or retardation as the case may be.
2. Area between it and the horizontal (i.e time) axis represents the distance travelled.
3. Energy required for propulsion can be calculated if resistance to the motion of train is known.

	Mainline service	*Suburban service*	*Urban service*
Distance between stops in km	More than 10	1–8	1
Maximum speed in kmph	160	120	120
Acceleration in kmphp	0.5–0.9	1.5–4	1.5–4
Retardation in kmphp	1.5	3–4	3–4
Features	Long free-run period, coasting and acceleration braking periods are small	No free-running period, coasting period is long	No free-running period, coasting period is small

TYPICAL SPEED/TIME CURVE

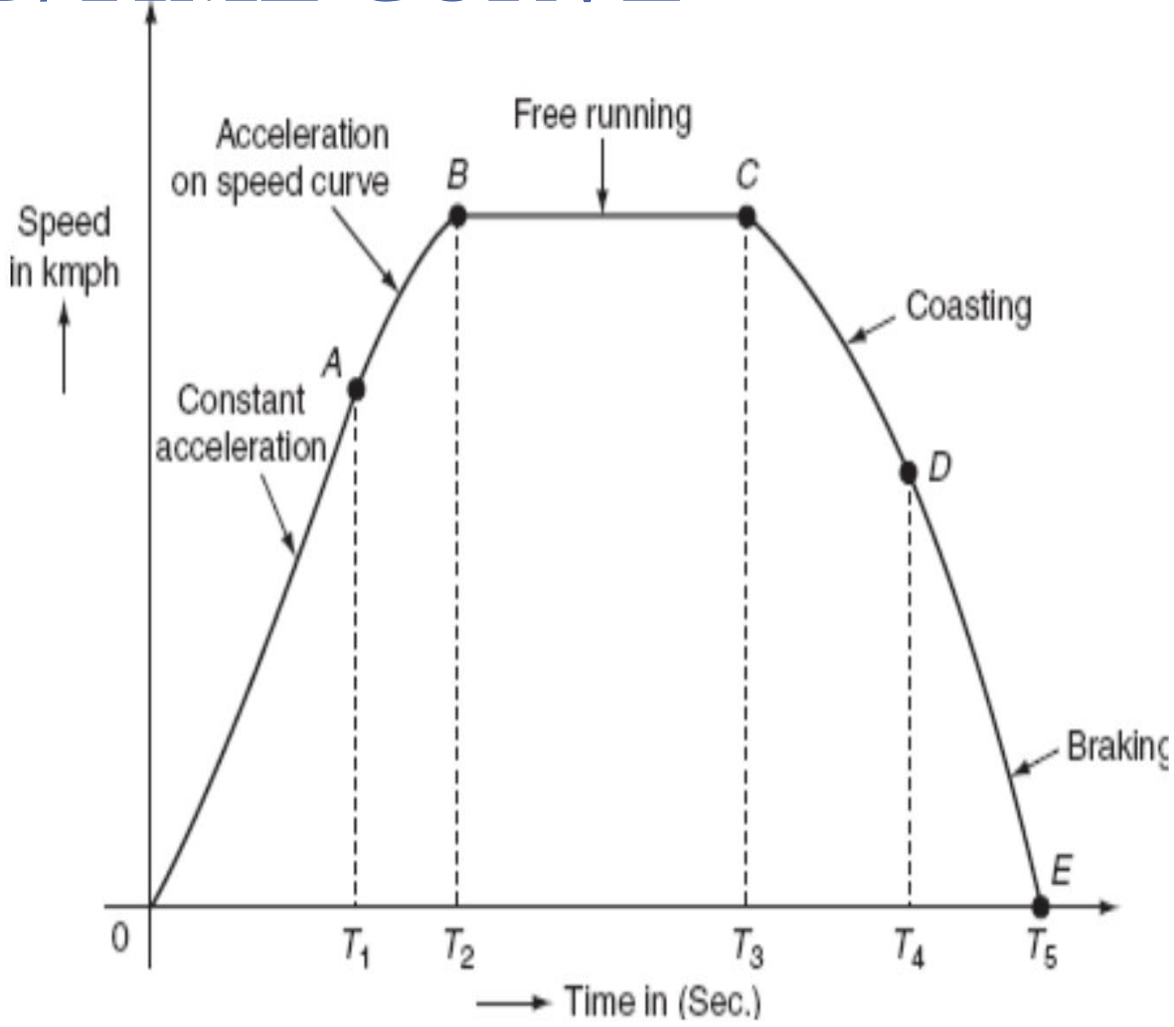
- The curve that shows the instantaneous speed of train in kmph along the ordinate and time in seconds along the abscissa is known as ‘speed–time’ curve.
- The curve that shows the distance between two stations in km along the ordinate and time in seconds along the abscissa is known as ‘speed–distance’ curve.
- The area under the speed–time curve gives the distance travelled during, given time internal and slope at any point on the curve toward abscissa gives the acceleration and retardation at the instance, out of the two speed–time curve is more important. Speed–time curve for main line service
- Typical speed–time curve of a train running on main line service is shown in Figure below. It mainly consists of the following time periods:

    1. Constant accelerating period
    2. Acceleration on speed curve
    3. Free-running period
    4. Coasting period
    5. Braking period

TYPICAL SPEED/TIME CURVE

Constant acceleration

- During this period, the traction motor accelerate from rest.
- The curve ‘OA’ represents the constant accelerating period.
- During the instant 0 to T1, the current is maintained approximately constant and the voltage across the motor is gradually increased by cutting out the starting resistance slowly moving from one notch to the other.
- Thus, current taken by the motor and the tractive efforts are practically constant and therefore acceleration remains constant during this period.
- Hence, this period is also called as notch up accelerating period or rheostatic accelerating period. Typical value of acceleration lies between 0.5 and 1 kmph.
- Acceleration is denoted with the symbol ‘α’.



Speed-time curve for mainline service

TYPICAL SPEED/TIME CURVE

Acceleration on speed-curve

- During the running period from T1 to T2, the voltage across the motor remains constant and the current starts decreasing, this is because cut out at the instant ‘T1’.
- According to the characteristics of motor, its speed increases with the decrease in the current and finally the current taken by the motor remains constant.
- But, at the same time, even though train accelerates, the acceleration decreases with the increase in speed.
- Finally, the acceleration reaches to zero for certain speed, at which the tractive effort excreted by the motor is exactly equals to the train resistance. This is also known as decreasing accelerating period. This period is shown by the curve ‘AB’.

Free-running or constant-speed period

- The train runs freely during the period T2 to T3 at the speed attained by the train at the instant ‘T2’. During this speed, the motor draws constant power from the supply lines. This period is shown by the curve BC.

TYPICAL SPEED/TIME CURVE

Coasting period

- This period is from T3 to T4, i.e., from C to D. At the instant ‘T3’ power supply to the traction, the motor will be cut off and the speed falls on account of friction, windage resistance, etc.
- During this period, the train runs due to the momentum attained at that particular instant. The rate of the decrease of the speed during coasting period is known as coasting retardation. Usually, it is denoted with the symbol ‘β_c’.

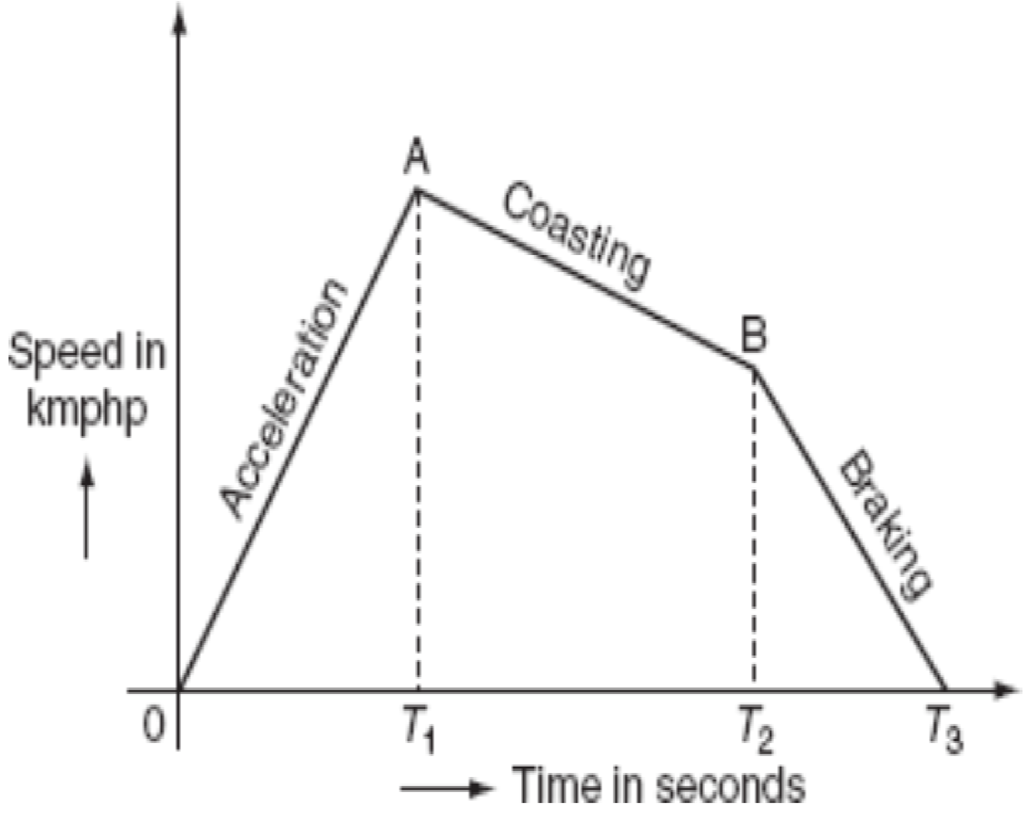
Braking period

- Braking period is from T4 to T5, i.e., from D to E. At the end of the coasting period, i.e., at ‘T4’ brakes are applied to bring the train to rest.
- During this period, the speed of the train decreases rapidly and finally reduces to zero. In main line service, the free-running period will be more, the starting and braking periods are very negligible, since the distance between the stops for the main line service is more than 10 km.

SUBURBAN SERVICE

- Speed–time curve for suburban service In suburban service, the distance between two adjacent stops for electric train is lying between 1 and 8 km.
- In this service, the distance between stops is more than the urban service and smaller than the main line service. The typical speed–time curve for suburban service
- The speed–time curve for urban service consists of three distinct periods. They are:

1. Acceleration
2. Coasting
3. Retardation

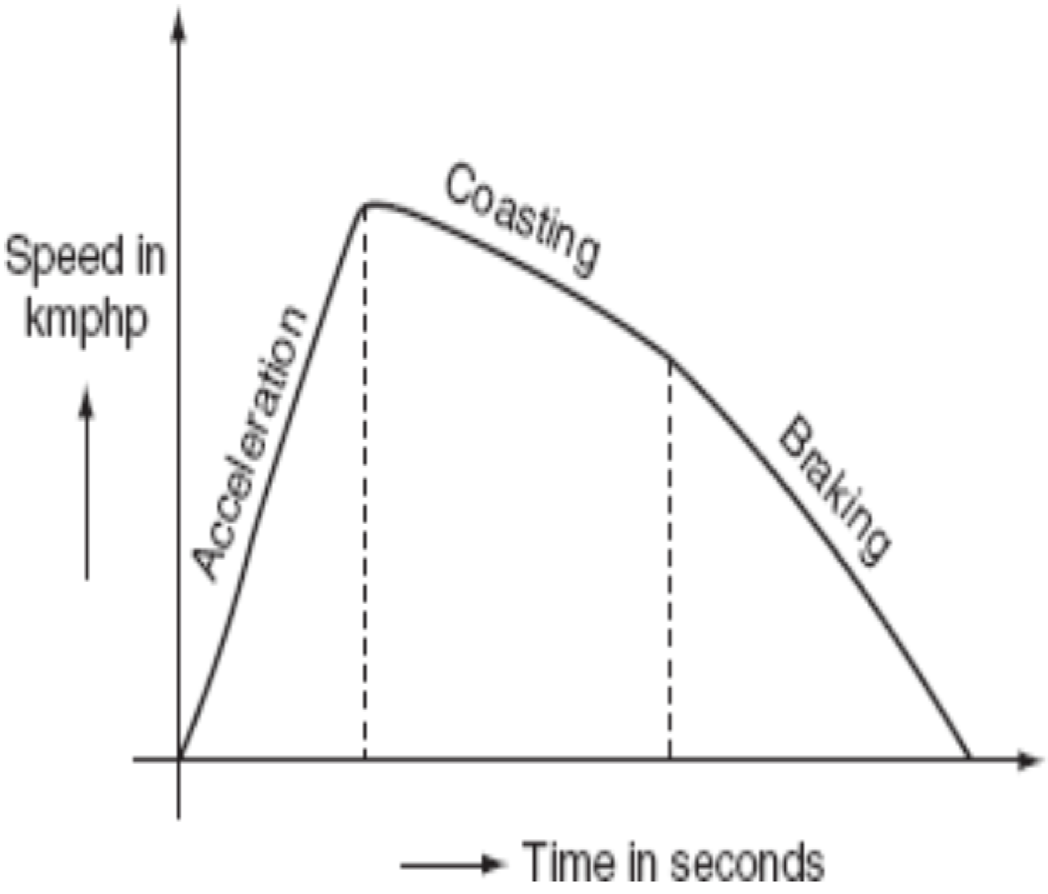


Typical speed–time curve for suburban service

- For this service, there is no free-running period. The coasting period is comparatively longer since the distance between two stops is more.
- Braking or retardation period is comparatively small. It requires relatively high values of acceleration and retardation. Typical acceleration and retardation values are lying between **1.5 and 4 kmphps** and **3 and 4 kmphps**, respectively.

URBAN SERVICE

- Speed–time curve for urban or city service The speed–time curve urban or city service is almost similar to suburban service and is shown in Figure.
- In this service also, there is no free-running period. The distance between two stop is less about 1 km. Hence, relatively short coasting and longer braking period is required.
- The relative values of acceleration and retardation are high to achieve moderately high average between the stops. Here, the small coasting period is included to save the energy consumption.
- The acceleration for the urban service lies between **1.6 and 4 kmphps.** The coasting retardation is about **0.15 kmphps** and the braking retardation is lying between **3 and 5 kmphps**.



Typical speed–time curve for urban service

SOME DEFINITIONS

Crest speed

- The maximum speed attained by the train during run is known as crest speed. It is denoted with ‘Vm’.

Average speed

- It is the mean of the speeds attained by the train from start to stop, i.e., it is defined as the ratio of the distance covered by the train between two stops to the total time of rum. It is denoted as Va.

$$Average\ speed = \frac{distance\ between\ stops}{actual\ time\ of\ run}$$

$$V_a = \frac{D}{T}$$

- where Va is the average speed of train in kmph, D is the distance between stops in km, and T is the actual time of run in hours.

SOME DEFINITIONS

Schedule speed:

- The ratio of the distance covered between two stops to the total time of the run including the time for stop is known as schedule speed. It is denoted with the symbol ‘Vs’. where Ts is the schedule time in hours.

Schedule Time: It is defined as the sum of time required for actual run and the time required for stop.

$$T_s = T_{run} + T_{stop}$$

FACTORS AFFECTING THE SCHEDULE SPEED OF A TRAIN

The factors that affect the schedule speed of a train are:

1. Crest speed
2. The duration
3. The distance between the stops
4. Acceleration
5. Braking retardation

Crest speed

- It is the maximum speed of train, which affects the schedule speed as for fixed acceleration, retardation, and constant distance between the stops.
- If the crest speed increases, the actual running time of train decreases. For the low crest speed of train it running so, the high crest speed of train will increases its schedule speed.
- Duration of stops: If the duration of stops is more, then the running time of train will be less; so that, this leads to the low schedule speed.
- Thus, for high schedule speed, its duration of stops must be low. Distance between the stops If the distance between the stops is more, then the running time of the train is less; hence, the schedule speed of train will be more.

Acceleration

- If the acceleration of train increases, then the running time of the train decreases provided the distance between stops and crest speed is maintained as constant. Thus, the increase in acceleration will increase the schedule speed.

Braking retardation

- High braking retardation leads to the reduction of running time of train. These will cause high schedule speed provided the distance between the stops is small.

Simplified Trapezoidal and Quadrilateral Speed Time Curves

- Simplified speed–time curves gives the relationship between acceleration, retardation average speed, and the distance between the stop, **which are needed to estimate the performance of a service at different schedule speeds.**
- So that, the actual speed–time curves for the main line, urban, and suburban services are approximated to some from of the simplified curves.
- These curves may be of either trapezoidal or quadrilateral shape. Analysis of trapezoidal speed–time curve Trapezoidal speed–time curve can be approximated from the actual speed–time curves of different services by assuming that:
- The acceleration and retardation periods of the simplified curve is kept same as to that of the actual curve.
- The **running and coasting periods** of the actual speed–time curve **are replaced by the constant periods**.
- This known as **trapezoidal approximation**, a simplified trapezoidal speed–time curve is shown in Figure.

Calculations from the Trapezoidal Speed–time Curve

Let **D be the distance between the stops** in km,

T be the actual running time of train in second,

α be the acceleration in km/h/sec,

β be the retardation in km/h/sec,

Vm be the maximum or the crest speed of train in km/h,

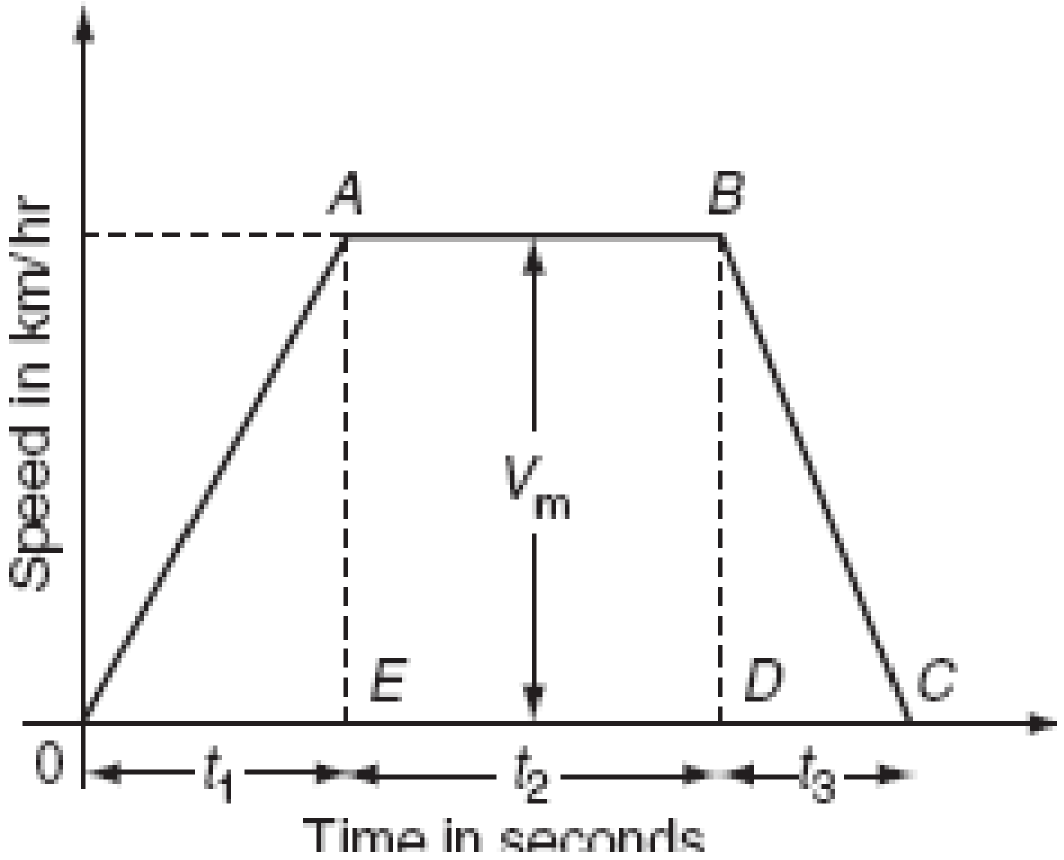
and **Va be the average speed** of train in km/h.

$$Actual\ running\ time\ of\ train, T = t_1 + t_2 + t_3$$

$$Time\ for\ acceleration, t_1 = \frac{V_m - 0}{\alpha} = \frac{V_m}{\alpha}$$

$$Time\ for\ retardation, t_3 = \frac{V_m - 0}{\beta} = \frac{V_m}{\beta}$$

$$= T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$$



Trapezoidal speed–time curve

Calculations from the Trapezoidal Speed–time Curve

- **Area under the trapezoidal speed–time curve** gives the total distance between the two stops (D)

The distance between the stops (D) = **area under triangle OAE + area of rectangle ABDE + area of triangle DBC**

= The distance travelled during acceleration + distance travelled during free running period + distance travelled during retardation.

- The distance travelled during acceleration = average speed during accelerating period × time for acceleration

$$= \frac{0+V_m}{2}\ x\ t_1\ (k\,m/h\ x\ \text{sec})$$

$$= \frac{0+V_m}{2}\ x\ \frac{t_1}{3600}\ km$$

Calculations from the Trapezoidal Speed–time Curve

The distance travelled during free-running period = average speed × time of free running

$$= V_m \, x \, t_2 \;\; km/h \, x \sec$$

$$= V_m \, x \, \frac{t_2}{3600} \, km$$

The distance travelled during retardation period = average speed × time for retardation

$$= \frac{0+V_m}{2} \, x \, t_3 \; (km/h \, x \sec)$$

$$= \frac{0+V_m}{2} \, x \, \frac{t_3}{3600} \, km$$

The distance between the two stops is:

$$D = \frac{V_m}{2} \, x \, \frac{t_1}{3600} + V_m \, x \, \frac{t_2}{3600} + V_m \, x \, \frac{t_3}{3600}$$

Solving quadratic Equation, we get:

$$V_m^2 X - V_m T + 3600D = 0$$

Calculations from the Trapezoidal Speed–time Curve

$$D = \frac{V_m t_1}{7200} + \frac{V_m}{3600}[T - V_m(t_1 + t_2)] + \frac{V_m t_3}{7200}$$

$$D = \frac{V_m^2}{7200\alpha} + \frac{V_m}{3600}\left[T - V_m\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right] + \frac{V_m^2}{7200\beta}$$

$$3600\ x\ D = \frac{V_m^2}{2\alpha} + \frac{V_m^2}{\beta} - V_m^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)V_m T$$

$$3600\ x\ D = V_m^2\left(\frac{1}{2\alpha} - \frac{1}{\alpha}\right) + V_m^2\left(\frac{1}{2\beta} - \frac{1}{\beta}\right) + V_m T$$

$$3600\ D = \frac{-V_m^2}{2\alpha} - \frac{V_m^2}{2\beta} + V_m T$$

$$V_m^2\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - V_m T + 3600D = 0$$

Calculations from the Trapezoidal Speed–time Curve

$$\text{Let} \quad \frac{1}{2\alpha} + \frac{1}{2\beta} = X = \frac{\alpha + \beta}{2\alpha\beta}$$

$$V_m^2 X - V_m T + 3600D = 0$$

Solving quadratic Equation, we get:

$$V_m = \frac{T + \sqrt{T^2 - 4\ x\ X\ x\ 3600D}}{2\ x\ X}$$

$$= \frac{T}{2X} \pm \sqrt{\frac{T^2}{4X^2} - \frac{3600D}{X}}$$

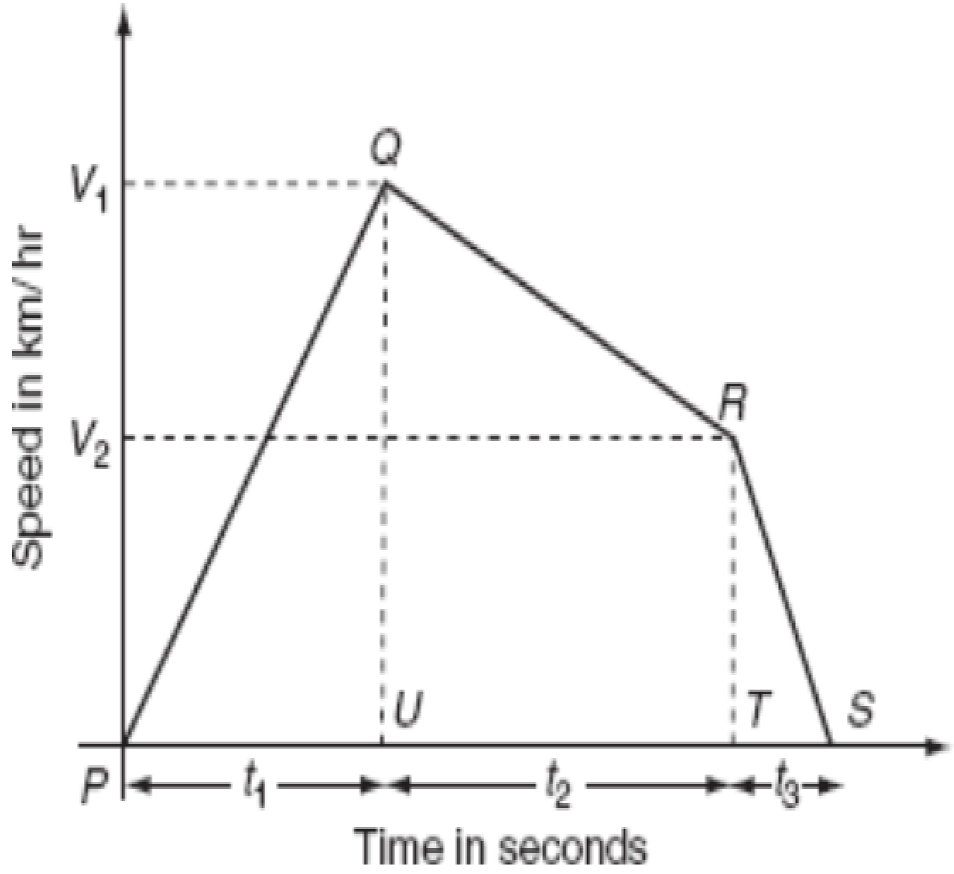
- By considering positive sign, we will get high values of crest speed, which is practically not possible, so negative sign should be considered:

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3600D}{X}}$$

$$V_m = \frac{\alpha\beta}{\alpha+\beta}T - \sqrt{\left(\frac{\alpha\beta}{\alpha+\beta}\right)^2 T^2 - 7200\left(\frac{\alpha\beta}{\alpha+\beta}\right)D}$$

Analysis of quadrilateral speed–time curve

- Quadrilateral speed–time curve for urban and suburban services for which the distance between two stops is less.
- The assumption for simplified quadrilateral speed–time curve is the initial acceleration and coasting retardation periods are extended, and there is no free-running period.
- Simplified quadrilateral speed–time curve is shown in Figure.



Quadrilateral speed–time curve

ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE

- Let V_1 be the **speed at the end of accelerating period** in km/h, V_2 be the **speed at the end of coasting retardation period** in km/h, and β_c **be the coasting retardation** in km/h/sec.

$$Time\ for\ acceleration, t_1 = \frac{V_1 - 0}{\alpha} = \frac{V_1}{\alpha}$$

$$Time\ for\ \text{coasting period}, t_2 = \frac{V_2 - V_1}{\beta}$$

$$Time\ period\ for\ braking\ retardation\ \text{period}, t_3 = \frac{V_2 - 0}{\beta} = \frac{V_2}{\beta}$$

Total distance travelled during the running period D: = **the area of triangle PQU + the area of rectangle UQRS + the area of triangle TRS**

= the distance travelled during acceleration + the distance travelled during coasting retardation + the distance travelled during braking retardation

ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE

But, the distance travelled during acceleration = average speed × time for Acceleration

$$= \frac{0 + V_1}{2} x\, t_1\, k\,m/h\, x \sec$$

$$= \frac{V_1}{2}\, x \frac{t_1}{3600}\; km$$

The distance travelled during coasting retardation

$$= \frac{V_2 + V_1}{2}\; x\; t_2\; k\,m/h\, x \sec$$

$$= \frac{V_2 + V_1}{2}\; x \frac{t_2}{3600}\; km$$

The distance travelled during braking retardation = average speed × time for braking retardation

$$= \frac{0 + V_2}{2} x\, t_3\, k\,m/h\, x \sec$$

ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE

$$= \frac{V_2}{2}\ x\ \frac{t_3}{3600}\ km$$

The total distance travelled:

$$D = \frac{V_1}{2} x \frac{t_1}{3600} + \frac{(V_2 + V_1)}{2} \frac{(t_2)}{3600} + \frac{V_2}{2} x \frac{t_3}{3600}$$

$$= \frac{V_1 t_1}{7200} + \frac{(V_2 + V_1) t_2}{7200} + \frac{V_2 t_3}{7200}$$

$$= \frac{V_1}{7200}(t_1 + t_2) + \frac{V_2}{7200}(t_2 + t_3)$$

$$= \frac{V_1}{7200}(T - t_3) + \frac{V_2}{7200}(T - t_1)$$

$$= \frac{(V_1 + V_2)T}{7200} - \frac{V_2 t_3}{7200} - \frac{V_2 t_1}{7200}$$

ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE

$$= \frac{(V_1 + V_2)T}{7200} - \frac{V_1 V_2}{7200\beta} - \frac{V_1 V_2}{7200\alpha}$$

$$= \frac{T}{7200}(V_1 + V_2) - \frac{V_1 V_2}{7200}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$7200\,D = (V_1 + V_2)T - V_1 V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

The time of coasting is given by,

$$t_2 = \frac{V_1 - V_2}{\beta_c}$$

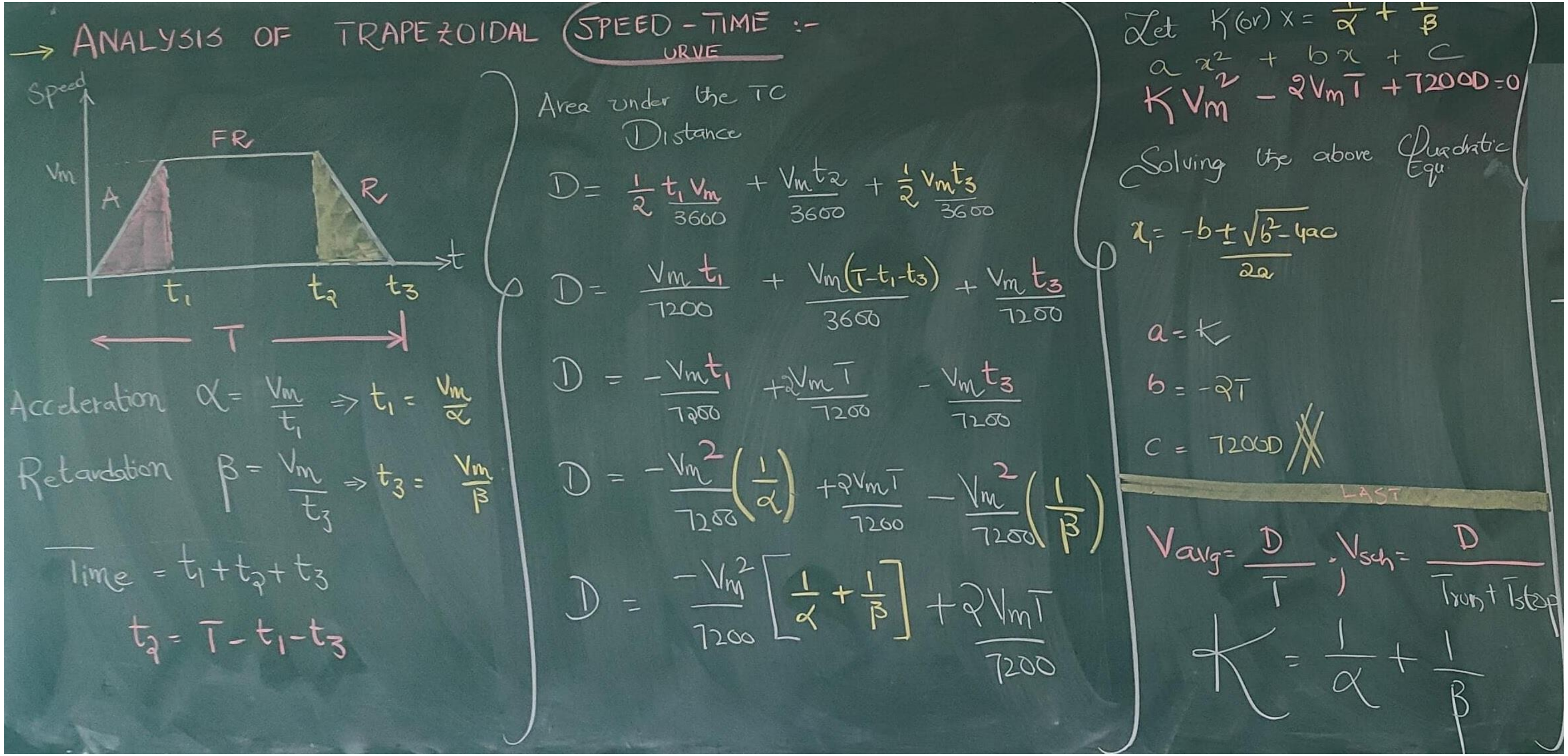
$$V_2 = V_1 - \beta_c t_2$$

$$\Rightarrow V_2 = V_1 - \beta_c(T - t_1 - t_3) = V_1 - \beta_c\left(T - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right)$$

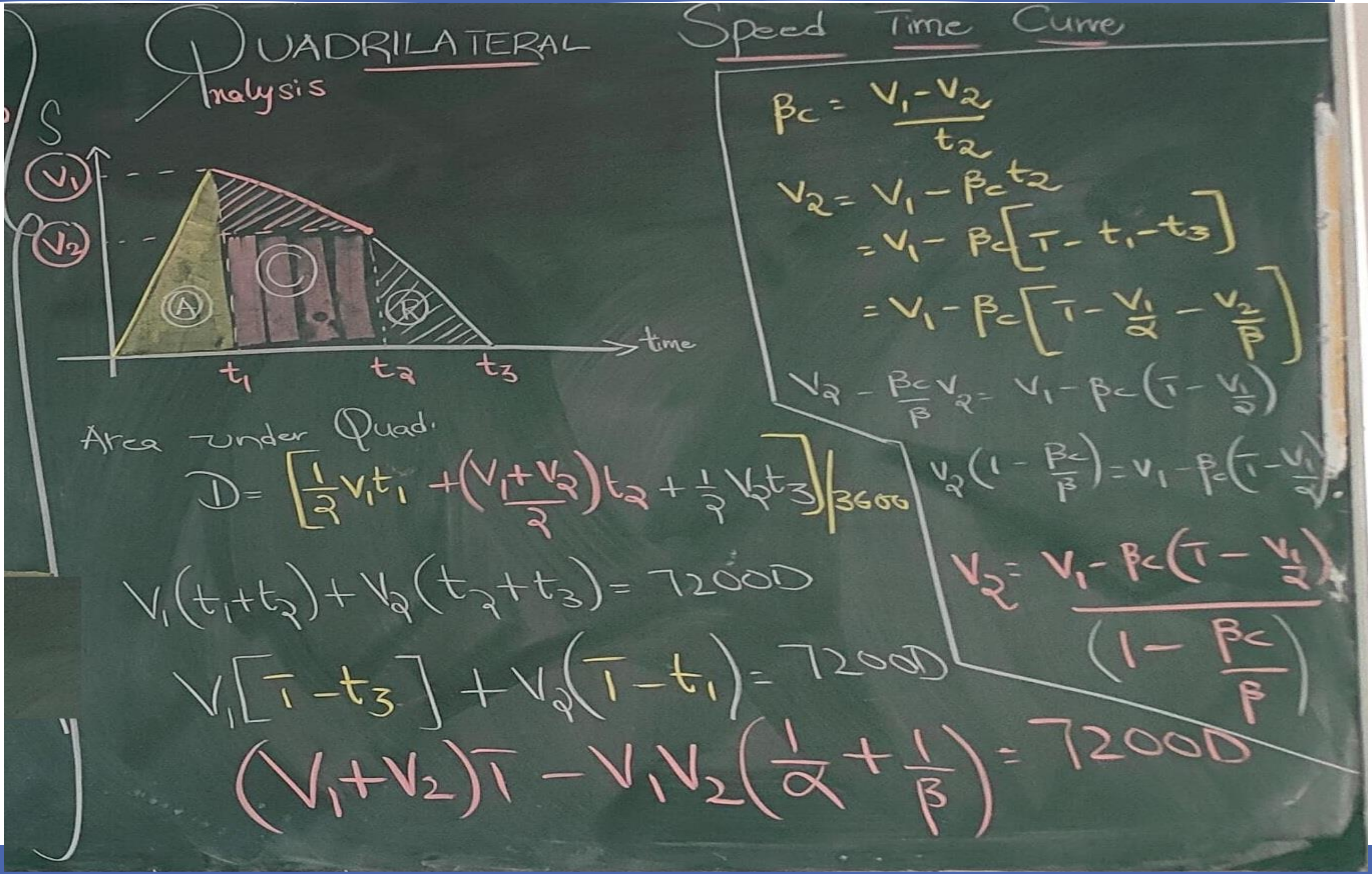
$$\Rightarrow V_2 - \frac{\beta_c V_2}{\beta} = V_1 - \beta_c\left(T - \frac{V_1}{\alpha}\right)$$

$$\therefore V_2 = \frac{V_1 - \beta_c T + \frac{\beta_c}{\alpha} V_1}{1 - \frac{\beta_c}{\beta}} \quad \cdots (3)$$

ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE



ANALYSIS OF QUADRILATERAL SPEED–TIME CURVE



Tractive effort required for propulsion of train

The tractive effort required for train propulsion is:

$$\mathbf{F_t = F_a + F_g + F_r}$$

where F_a is the force required for linear and angular acceleration, F_g is the force required to overcome the gravity, and F_r is the force required to overcome the resistance to the motion.

Force required for linear and angular acceleration (F_a)

According to the fundamental law of acceleration, the force required to accelerate the motion of the body is given by:

$$\text{Force} = \text{Mass} \times \text{acceleration}$$

(1) An electric train scheduled at a speed of 45 kmph (including a station stop of 20 sec.) has a maximum speed of 70 kmph. If the train accelerates at 1.5 kmph/sec, compute the value of retardation when the distance between stops is 4 km.

Sol:

$$V_{schedule} = \frac{D}{T_{run} + T_{stop}}$$

$$T_{run} + T_{stop} = \frac{D}{V_{schedule}} \times 3600 \text{ sec}$$

$$T_{run} + T_{stop} = \frac{4}{45} \times 3600 \text{ sec}$$

let $T = T_{run}$

$$T = \frac{4}{45} \times 3600 - T_{stop}$$

$$= \frac{4}{45} \times 3600 - 25$$

$$T = 295 \text{ sec.}$$

$$\boxed{V_m^2 \left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - V_m T + 3600 D = 0} \rightarrow (1)$$

D = Distance in km

V_m = maximum speed in kmph.

α = acceleration kmph/sec

β = Retardation kmph/sec.

T = Time of run in sec

$\alpha = 1.5$ kmph/s

$\beta = ?$

$V_m = 70$ kmph

$D = 4$ km

substitute in eq ①

$$V_m^2\left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - V_m T + 3600 D = 0$$

$$(70)^2\left(\frac{1}{3} + \frac{1}{2\beta}\right) - 70 \times 295 + 3600 \times 4 = 0$$

$$\frac{(70)^2}{3} + \frac{(70)^2}{2\beta} - 20650 + 14400 = 0$$

$\beta = 0.53$ kmph/sec

→ A train is required to run between two stations 1.5 kms apart at an average speed of 50 kmph. The run is to be made to a simplified quadrilateral speed-time curve. If the maximum speed is to be limited to 60 kmph, acceleration to 2 kmphps and coasting and braking retardations to 0.15 kmphps and 3 kmphps ~~respectively~~ respectively and the speed of the train before applying the brakes is 40 kmph. Determine the duration of acceleration, coasting and braking.

Sol Given

Distance of run $D = 1.5$ km

Average speed, $V_a = 50$ kmph

maximum speed, $v_1 = 60$ kmph

speed at the end of coasting, $V_2 = 48$ kmph

Acceleration, $\alpha = 2$ kmphps

coasting retardation, $\beta_c = 0.15$ kmphps

Braking retardation, $\beta = 3$ kmphps

$\therefore$ duration of acceleration is

$$t_1 = \frac{v_1}{\alpha} = \frac{60}{2} = 30 \text{ seconds}$$

duration of coasting is

$$t_2 = \frac{V_1 - V_2}{\beta_c} = \frac{60 - 4.8}{0.15} = 80 \text{ seconds}$$

duration of braking is

$$t_3 = \frac{V_3}{\beta} = \frac{48}{3} = 16 \text{ seconds.}$$

(2) A train has schedule speed of 25 kmph over a level track distance between the ~~sit~~ stations being 0.8 km. station stopping speed time is 25 sec. Assuming breaking retardation of 3 kmphs and maximum speed 20% greater than average speed, calculate acceleration required to run the service if the speed time curve is approximated by a trapezoidal curve.

2

sol given $V_{schedule} = 25$ kmph

$D = 0.8$, $T_{stop} = 25$ sec, $\beta = 3$ kmph

$$T_{run} + T_{stop} = \frac{D}{V_{schedule}} \times 3600 \text{ sec}$$

$$T + 25 = \frac{0.8}{25} \times 3600$$

$$T = 90.2 \text{ sec}$$

$$T = \underline{\underline{90.2}} \quad \underline{\underline{\text{sec}}}$$

$$V_{avg} = \frac{D}{T}$$

$$= \frac{0.8}{90.2} \times 3600 = 32 \text{ kmph}$$

$$V_m = 1.2\, V_{avg}$$
$$= 1.2 \times 32$$
$$= 38.4 \text{ kmph}$$

$$V_m^2\left[\frac{1}{2\alpha}+\frac{1}{2\beta}\right] - V_m T + D = 0$$

$$(38.4)^2\left[\frac{1}{2\alpha}+\frac{1}{6}\right] - V_m T + \cancel{D}3600D = 0$$

$$(38.4)^2\left[\frac{1}{2\alpha}+\frac{1}{6}\right] - 38.4 \times 90.2 + 3600 \times D = 0$$

$$1474.56\left[\frac{1}{2\alpha}\right] + \frac{1474.56}{6} - 3456 + 3600 \times 0.8 = 0$$

$$2\alpha = 1.98$$

$$\underline{\alpha = 0.99} \text{ kmphs.}$$

(3) A train has an average speed of 50 kmph between stops which are 1.5 km apart (i) Duration of stops as 30 sec (ii) Acceleration as 1.7 kmphps (iii) Retardation as 3.3 kmphs. Draw the speed-time curve for the run.

sol Avg speed, $V_a = 50$ kmph

Acceleration . $\alpha = 1.7$ kmphs

Retardation $\beta = 3.3$ kmphs

$D = 1.5$ km

$\cancel{T = \frac{V_{avg}}{D}}$ $T = \frac{D}{V_{avg}}$

$T = \frac{1.5}{V_{avg}} \times 3600 = \frac{1.5}{50} \times 3600$

Total running Time ; $T = 108$ sec

$$V_m^2\left[\frac{1}{2\alpha}+\frac{1}{2\beta}\right]-V_m T+3600\times D=0$$

$$V_m^2\left[\frac{1}{2\times 1.7}+\frac{1}{2\times 3.3}\right]-V_m\times 108+3600\times 1.5=0$$

$$V_m=70.56 \text{ kmph}$$

$$t_1=\frac{V_m}{\alpha}=\frac{70.56}{1.7}$$

$$t_1=41 \text{ sec}$$

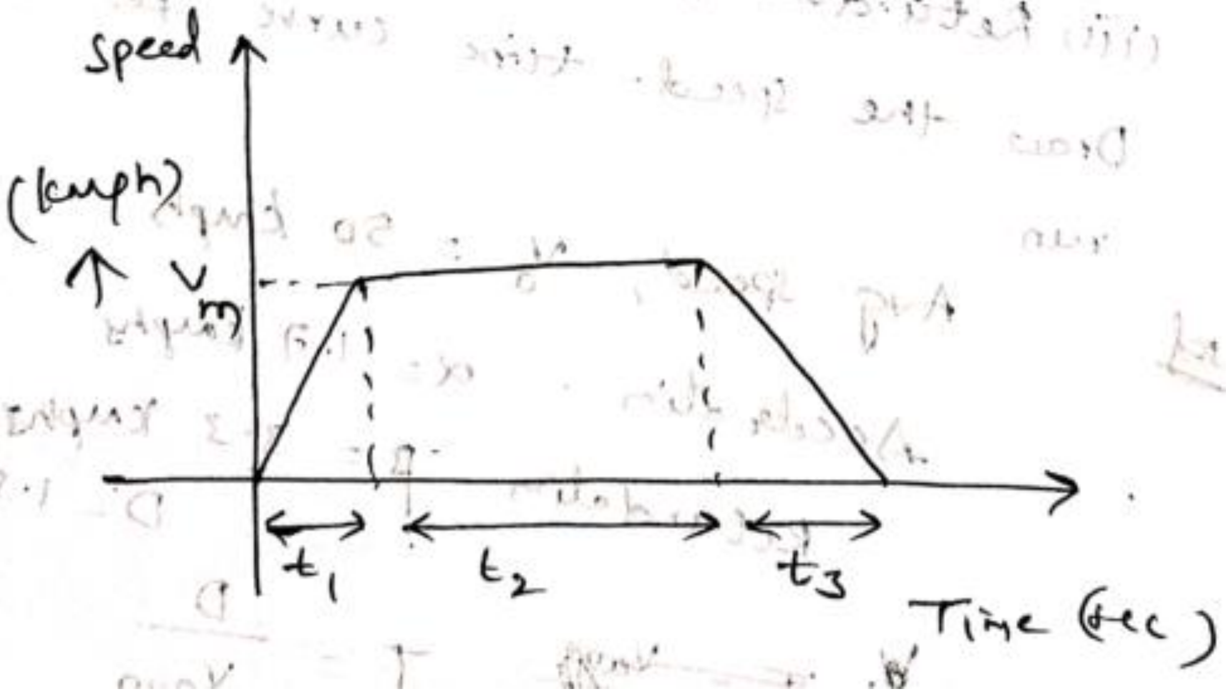
$$t_3=\frac{V_m}{\beta}=\frac{70.56}{3.3}$$

$$t_3=21.38 \text{ sec}$$

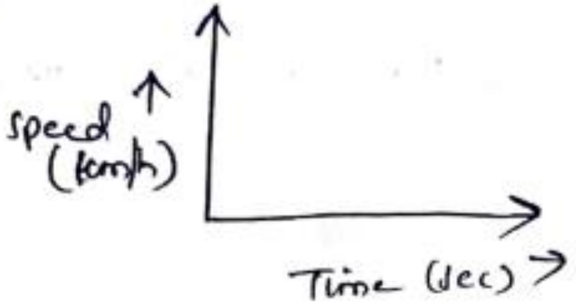
$$t_2 = T - (t_1 + t_3)$$

$$= 108 - (41 + 21.38)$$

$$t_2 = 45.62 \text{ sec}$$



→ The distance b/w two stations is 5 km and the average speed of the train is 50 kmph. The acceleration is 3 kmph, retardation during coasting is 0.2 kmph and the braking is 4 kmph, respectively. Taking quadrilateral approximation of speed-time curve, determine the duration of the accelerating, coasting and braking periods and distance covered during these period.



sol

$$T = \frac{5 \times 3600}{50} = 360 \text{ sec}.$$

from ų: $7200D = (v_1 + v_2)T - v_1 v_2 \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$v_1 T + v_2\left[T - v_1\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)\right] = 7200D$$

$$v_2 = \frac{v_1 - \beta_c\left(T - \frac{v_1}{\alpha}\right)}{\left(1 - \frac{\beta_c}{\beta}\right)}$$

$$v_2 = \frac{\left[v_1 - 0.2\left(360 - \frac{v_1}{3}\right)\right]}{\left(1 - \frac{0.2}{4}\right)} = \frac{v_1 - 0.2\left(360 - \frac{v_1}{3}\right)}{\frac{(3.8)}{4}}$$

$$v_2 = v_1 - \left(72 - \frac{v_1}{15}\right)\frac{4}{3.8}$$

$$v_2 = 1.0526\, v_1 - 75.79 + \frac{4v_1}{57}$$

$$v_2 = 1.12277 v_1 - 75.8$$

① A suburban electric train has a maximum speed of 65 kmph. It has a scheduled speed of 43.5 kmph with a stop of 30 sec. The acceleration of the train is 1.3 kmph. Calculate the retardation. The average distance between stops is 3 km.

Sol $V_{max} = 65$ kmph, $\alpha = 1.3$ kmph,

$D = 3$ km.

$$\text{schedule speed} = \frac{\text{Distance b/w stations}}{\text{Actual time of run} + \text{Time for stops}}$$

$$\frac{43.5}{3600} = \frac{3}{T+30}$$

$$(T+30) = \frac{3 \times 3600}{43.5} = \frac{10800}{43.5}$$

$$= 248 \cdot 27 \text{ sec}$$

$$\therefore T = 248 \cdot 27 \text{ sec.}$$

Average speed $\Rightarrow V_a = \frac{3 \times 3600}{218 \cdot 27} = 49.48 \text{ kmph.}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7200\,D}{V_m^2}\left[\frac{V_m}{V_a} - 1\right]$$

$$\frac{1}{1 \cdot 3} + \frac{1}{\beta} = \frac{7200 \times 3}{(65)^2}\left[\frac{65}{49.48} - 1\right]$$

$$\frac{1}{\beta} = \frac{7200 \times 3}{65 \times 65}\left[1 \cdot 3136 - 1\right] - \frac{1}{1 \cdot 3}$$

$$= \frac{21600}{65 \times 65}\left[0 \cdot 3136\right]$$

$$= 1 \cdot 6032 - 0 \cdot 7692 = \underline{\underline{0 \cdot 834}}$$

$\therefore$ Retardation $\beta = \underline{\underline{1 \cdot 199}}$ kmph

Example: A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.

Assume a simplified speed/time curve.

Solution