EE2227: CONTROL SYSTEM

Presentation 1
GATE: 2017 ECE Q17

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Question

. Consider the state space realization :

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

with the initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

, where u(t) denotes unit step function

The value of $\lim_{t\to\infty} |\sqrt{x_1^2(t) + x_2^2(t)}|$ **is ?**

State Space Reprensentation :
A state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.
For Linear systems :

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t)$$

where $\dot{\mathbf{x}}(t) := \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t)$

$$L\{x(t)\} = X(s)$$

$$L\{x(\dot{t})\} = sX(s) - x(0)$$

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

By applying Laplace transform on both sides, we get

$$sX_1(s) - x_1(0) = 0$$

$X_1(s) = \frac{x_1(0)}{s} = 0 \qquad \because x_1(0) = 0$
So, $\quad x_1(t) = 0$
and $\quad sX_2(s) - x_2(0) = -9X_1(s) + \frac{45}{s}$

Solution

$$\begin{aligned} \text{Required value} &= \lim_{t\to\infty} \left|\sqrt{x_1^2(t) + x_2^2(t)}\right| \\ &= \left|\lim_{t\to\infty} x_2(t)\right| \end{aligned}$$

$\lim_{t\to\infty} x_2(t) = \lim_{s\to 0} sX_2(s) = \frac{45}{9} = 5$

So Required value $= |5| = 5$

Verification