EE2330 : ADVANCED DSP

HOMEWORK : 06

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A WRITEUP FOR QUES NO. 2

THE ESSENCE OF
LAPLACE TRANSFORM
AND HOW IT DIFFERS FROM
FOURIER TRANSFORM

IIT HYDERABAD

1 Why should we understand frequency domain in signal processing ?

Many of us have been habituated of viewing a signal in time domain and feel as if it's the only format we could ever represent a signal. But there is also the frequency domain where we can represent the signal and sometimes the frequency domain signal representation assists us to get a better picture about the signal.

Well for all the people who find it difficult to feel the frequency domain . Here's a smart but simple analogy for that :

"We see signals through time domain,
while hear them in frequency domain !!!"

Apart from that , a few operations over signals becomes much more simple when we deal with frequency domain. For eg; Convolution

I personally feel that time and frequency are two different languages which we use to interpret our signal , however the interpretation of the signal can be different but the meaning is not altered !

2 What is Fourier Transform ?

The Fourier Transform is a mathematical technique that transforms a function of time, x(t), to a function of frequency, X(ω).

$$X(\omega) = \int\limits_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

NOTE : Existence of the Fourier Transform requires that the x(t) be absolutely integrable i.e;

$$\int\limits_{-\infty}^{+\infty} |x(t)|\, dt < \infty$$

3 What is Laplace Transform ?

The Laplace transform of a function f(t), defined for all real numbers t 0, is the function F(s), which is a unilateral transform defined by

$$F(s) = \int_0^{\infty} f(t)e^{-st}\, dt$$

s is a complex number frequency parameter

$$s = \sigma + i\omega \quad \text{,with real numbers } \sigma \text{ and } \omega$$

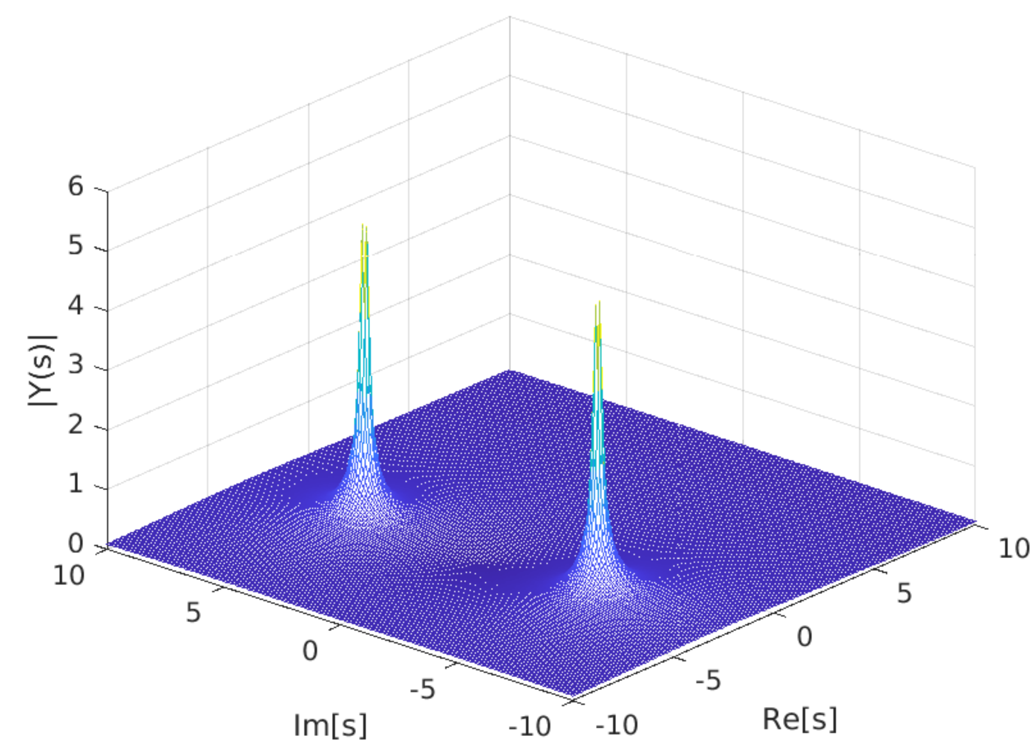
NOTE : The reason for the limits not extending till $t = -\infty$ is that as $'t' --> -\infty$ in signal analysis we often try to find the transient response where we consider the signal $f(t)$ to be causal i.e; starting from zero. Hence it makes no sense to integrate it from ∞.

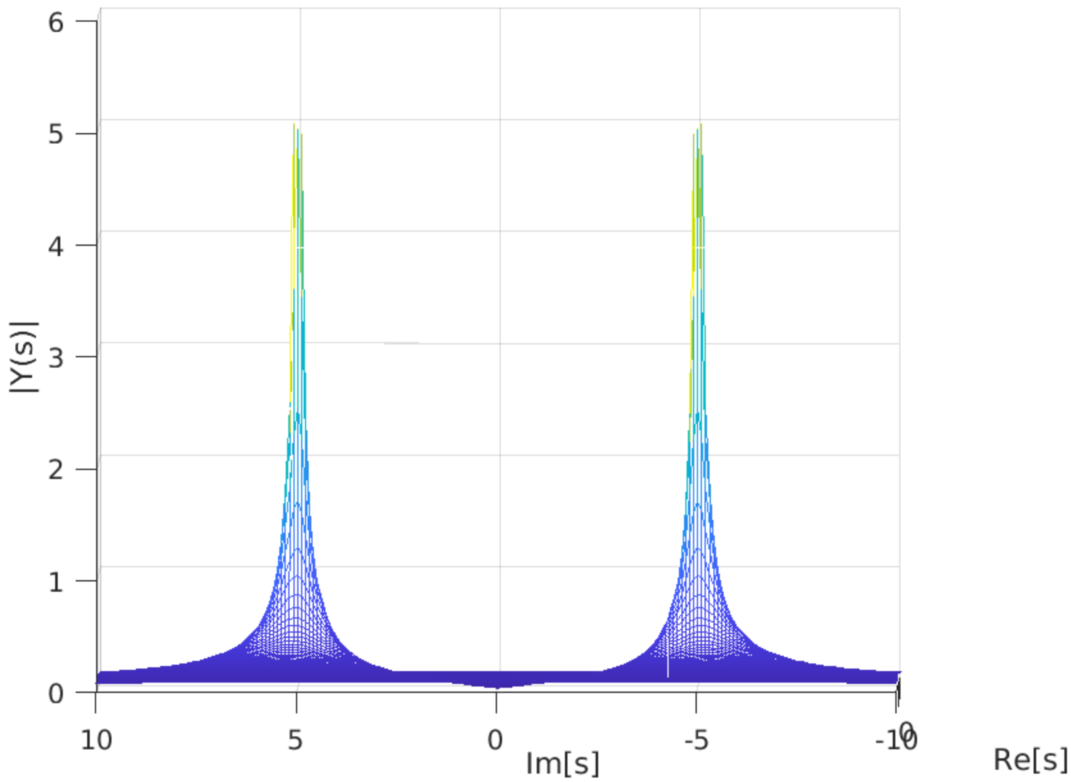
4 Why should we really care aboutROC a.k.a Region of Convergence ?

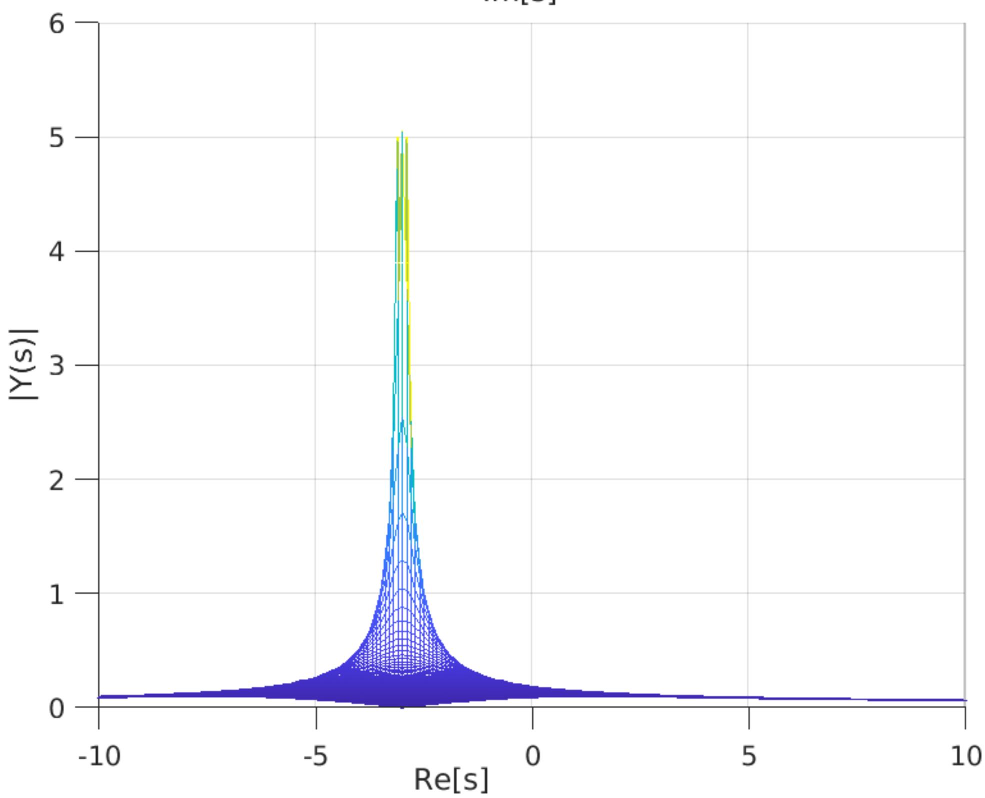
One of the main things to deal in Laplace domain is having the pole-zero plot. The below is an example for that ...

Lets consider the signal $y(t) = e^{-3t}cos(5t)$
whose laplace transform is as follows : $Y(s) = \frac{s+3}{(s+3)^2+25}$

Its magnitude plot will be as follows :







- We can see that the magnitude plot blows up at $s = -3 + 5i$ and $s = -3 - 5i$
- These are called the poles . And the ROC is the area to the right of the poles i.e region holding $Re(s) > -3$

- Reason : Here the laplace transform will be in this form :

$$Y(s) = \int_0^{\infty} e^{-(3+\sigma)t} cos(5t) e^{-j\omega}\, dt$$

- Therefore when $\sigma < -3$, The exponential term has a positive power; and when integration is done over $(0, \infty)$ the integration becomes infinite instead. Thus its generally defined to find the LT in the ROC which is $Re(s) > -3$ so that we can obtain a finite answer.

5 How are these transforms different ?

- Definition wise :
  - The Fourier transforms tells us about the frequency of sinusoids are present in the given signal.
  - While the Laplace tells the frequencies of sinusoids as well as the exponentials present in the signal .
  - This makes the Fourier Transform a subset of Laplace transform.

  Mathematically speaking , when the real part of s in Laplace Transform is set to zero the resultant will be the Fourier transform.

- Application wise :
  - The Fourier Transform deals with Steady state signal analysis , while the Laplace is good at transient analysis.
  - This is because the $s = \sigma + j\omega$ in the Laplace transform has a real part to it which resembles a exponential decaying function. Which dies with time and thus contributes to the transient part of the system.
  - Due to the absence of real part in Fourier domain , we can only deal with sinusoids which will last longer and thus does not contribute to the transient.
  - This is why we will come across Laplace transforms much often in control systems since its good at analysing both the steady and transient analysis.

6 Other uses of Laplace

- Well the real essence of this Transform is seen in solving complex Differential equation . By just converting the equation into s domain, we transform this into a algebraic problem.

- Hence we can see the use of s domain in circuit analysis where the relation between voltages and currents in seen as differential equation.

- Thus by applying Laplace Transform we can turn this into a algebraic relation , and we evaluate quantities individually and find its time domain format just by inverse Laplace Transform.

— ***THE END*** —