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Parameter Estimation Method using an Extended Kalman Filter

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Abstract. *Fast parameter estimation is a non-trivial task, and it is critical when the system parameters evolve with time, as demanded in real-time control applications. In this study, a new computational approach for parameter identification is proposed based on the application of polynomial chaos theory. The polynomial chaos approach has been shown to be considerably more efficient than Monte Carlo in the simulation of systems with a small number of uncertain parameters. In the framework of this new approach, a (suboptimal) Extended Kalman Filter (EKF) is used to recalculate the polynomial chaos expansions for the uncertain states and the uncertain parameters. As a case study, the proposed parameter estimation method is applied to a four degree-of-freedom roll plane model of a vehicle for which the vertical stiffnesses of the tires are estimated from periodic observations of the displacements and velocities across the suspensions. The results obtained with this approach are close to the actual values of the parameters. In addition, the EKF approach gives more information about the parameters of interest than a simple estimated value: the estimation comes in the form of a probability density function. The approach presented in this paper has shown great promise for an improvement in the computational efficiency of current parameter estimation methods. Possible applications of this theory to the field of off-road vehicle simulations include the estimation of various vehicle parameters of interest, as well as the estimation of parameters related to the tire-terrain contact.*

Keywords. Parameter Estimation, Polynomial Chaos, Collocation, Extended Kalman Filter (EKF), Hammersley Algorithm, Off-road Vehicle Dynamics.

1 Introduction

The polynomial chaos theory has been shown to be consistently more efficient than Monte Carlo simulations in assessing uncertainties in mechanical systems [7, 8]. This paper extends the polynomial chaos theory to the problem of parameter estimation, and applies it to a four degree of freedom (DOF) roll plane model of a ground vehicle. Parameter estimation is an important problem, because many parameters simply cannot be measured physically with good precision, especially in real time applications.

Parameter estimation is a very difficult problem, especially for large systems, and a lot of effort devoted to it would be needed. Estimating a large number of parameters often proved to be computationally too expensive. This has led to the development of techniques determining which parameters affect the system's dynamics the most, in order to choose the parameters that are important to estimate [10]. Sohns, et al. [10] proposed the use of activity analysis as an alternative to sensitivity-based and principal component-based techniques. Their approach combines the advantages of the sensitivity-based techniques (i.e., being efficient for large models) and the sensitivity-based techniques (i.e., keeping parameters that can be physically interpreted).

2 Problem Formulation

Optimal parameter estimation combines information from three different sources: the physical of evolution (encapsulated in the model), the reality (as captured by the observations), and the current best estimate of the parameters (all with associated errors). Consider a dynamic model which advances the state in time,

$$y^k = M\left(t^{k-1}, y^{k-1}, \theta\right), \quad y^0 = y\left(t^0\right), \quad k = 1, 2, \ldots, N \tag{1}$$

The state of the model y^k at time moment t^k depends implicitly on the set of parameters $\theta \in \Re^p$, possibly uncertain (the model has *n* states and *p* parameters). M is the discrete model solution operator which integrates the model equations forward in time (starting from state y^{k-1} at time t^{k-1} to state y^k at time t^k).

Using polynomial chaoses the uncertain parameters can be modeled explicitly as functions of a set of random variables $\xi \in \Omega \subset \Re^p$ with a joint probability density function $\rho(\xi)$. The explicit dependency is in the form of an expansion in terms of orthogonal polynomial basis functions,

$$\theta(\xi) = \sum_{i=1}^{S} \theta^i \phi^i(\xi), \quad y^k(\xi) = \sum_{i=1}^{S} \left(y^k\right)^i \phi^i(\xi) \tag{2}$$

The time evolution of the uncertain model state can be obtained from (1) and (2) via a Galerkin or collocation approach [7, 8].

For parameter estimation it is convenient to formally extend the model state to include the model parameters and extend the model with equations for parameters (such that parameters do not change during the model evolution):

$$\begin{bmatrix} y^k \\ \theta^k \end{bmatrix} = \begin{bmatrix} M\left(t^{k-1}, y^{k-1}, \theta^{k-1}\right) \\ \theta^{k-1} \end{bmatrix} \tag{3}$$

The optimal estimation of the uncertain parameters is thus reduced to the problem of optimal state estimation. Observations of quantities that depend on system state are available at discrete times t^k

$$y^k_{obs} = h\left(y^k\right) + \varepsilon^k_{obs} \approx H_k\, y^k + \varepsilon^k_{obs}\,, \quad \left\langle \varepsilon^k_{obs} \right\rangle = 0\,, \quad \left\langle \left(\varepsilon^k_{obs}\right)\left(\varepsilon^k_{obs}\right)^T \right\rangle = R_k \tag{4}$$

where $y^k_{obs} \in \Re^m$ is the observation vector at t^k, h is the (model equivalent) observation operator and H_k is the linearization of h about the solution y^k. Note that there are *m* observations for the *n*-dimensional state vector, and that typically *m* < *n*. Each observation is corrupted by observational (measurement and representativeness) errors [1]. We denote by $\langle\cdot\rangle$ the ensemble average over the observation uncertainty $\varepsilon^k_{obs} \in \Re^m$ space. The observational error is the experimental uncertainty associated with the measurements and is usually considered to have a Gaussian distribution with zero mean and a known covariance matrix R_k.

The aim of data assimilation is to find $P[y(t^k) \,|\, y^k_{obs} \ldots y^0_{obs}]$, the PDF of the true state at time t^k conditioned by all previous observations (including the most recent one).

3 Parameter Estimation Using the Extended Kalman Filter

The Kalman filter [2, 3, 4, 6] assumes that the model (1) is linear, and the model state at previous time t^{k-1} is normally distributed with mean y^{k-1}_a and covariance matrix P^{k-1}_a. The Extended Kalman Filter (EKF) allows for nonlinear models and observations by assuming the error propagation is linear. In the EKF approach, the nonlinear observation operators are linearized, $y^k_{obs} = H_k y^k + \varepsilon^k_{obs}$.

The state is propagated from t^{k-1} to t^k using model equations, and the covariance matrix is explicitly propagated using the tangent linear operator and its adjoint,

$$y^k_f = M\left(t^{k-1}, y^{k-1}_a, p\right), \qquad P^k_f = M' P^{k-1}_a M'^* + Q \tag{5}$$

where the subscripts *f* and *a* stand for forecast and analysis, respectively. M is the model (1) propagator (from t^{k-1} to t^k), M' is the corresponding tangent linear propagator and M'^* is its adjoint. Q represents the covariance of the model errors.

Under linear, Gaussian assumptions, the PDFs of the forecast and assimilated fields are also Gaussian, and completely described by the mean state and the covariance matrix. The assimilated field y^k_a and its covariance matrix P^k_a are computed from the model forecast y^k_f, the current observations y^k_{obs}, and from their covariances using:

$$\begin{aligned} y^k_a &= y^k_f + P^k_f H^T_k \left(R_k + H_k P^k_f H^T_k\right)^{-1} \left(y^k_{obs} - H_k y^k_f\right), \\ P^k_a &= P^k_f - P^k_f H^T_k \left(R_k + H_k P^k_f H^T_k\right)^{-1} H_k P^k_f\,. \end{aligned} \tag{6}$$

One step of the data assimilation with the extended Kalman filter can be represented as:

$$\begin{array}{l} y^{k-1}_a \ \text{and}\ P^{k-1}_a \xrightarrow{\text{Model \& Tangent Linear Model}} y^k_f \ \text{and}\ P^k_f \xrightarrow{\text{Filter}} y^k_a \ \text{and}\ P^k_a \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\; y^k_{obs} \ \text{and}\ R_k \end{array} \tag{7}$$

For parameter estimation extend the model state to formally include the model parameters:

$$\begin{bmatrix} y_f^k \\ \theta_f^k \end{bmatrix} = \begin{bmatrix} M\left(t^{k-1}, y_a^{k-1}, \theta_a^{k-1}\right) \\ \theta_a^{k-1} \end{bmatrix} \tag{8}$$

The covariance matrix of the extended state vector can be estimated from the polynomial chaos expansions of $y(\xi)$ and $\theta(\xi)$.

$$P_f^k = \operatorname{cov}\left(\begin{bmatrix} y_f^k(\xi) \\ \theta_f^k(\xi) \end{bmatrix}\right) = \begin{bmatrix} \operatorname{cov}\left(y_f^k\right) & \operatorname{cov}\left(\left[y_f^k, \theta_f^k\right]\right) \\ \operatorname{cov}\left(\left[\theta_f^k, y_f^k\right]\right) & \operatorname{cov}\left(\theta_f^k\right) \end{bmatrix} \tag{9}$$

Using this covariance matrix compute the Kalman gain matrix using the formula:

$$K_k = P_f^k H_k^T \left(R_k + H_k P_f^k H_k^T\right)^{-1} \tag{10}$$

The Kalman filter formula computes the assimilated state and parameter vector as:

$$\begin{bmatrix} y_a^k \\ \theta_a^k \end{bmatrix} = \begin{bmatrix} y_f^k \\ \theta_f^k \end{bmatrix} + K_k \left(y_{obs}^k - H_k \begin{bmatrix} y_f^k \\ \theta_f^k \end{bmatrix}\right) = \left(I - K_k H_k\right)\begin{bmatrix} y_f^k \\ \theta_f^k \end{bmatrix} + K_k y_{obs}^k \tag{11}$$

Assuming that no direct observations are made on the parameters, and only the state is observed, we obtain:

$$\begin{bmatrix} y_a^k \\ \theta_a^k \end{bmatrix} = \begin{bmatrix} y_f^k \\ \theta_f^k \end{bmatrix} + K_k \left(y_{obs}^k - H_k y_f^k\right) = \left(I - K_k H_k\right) y_f^k + K_k y_{obs}^k \tag{12}$$

Using the polynomial chaos expansions of the forecast state and the parameters:

$$\begin{bmatrix} y_f^k(\xi) \\ \theta_f^k(\xi) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{S} \left(y_f^k\right)^i \phi^i(\xi) \\ \sum_{i=1}^{S} \left(\theta_f^k\right)^i \phi^i(\xi) \end{bmatrix} \tag{13}$$

the Kalman filter formula is used to determine the polynomial chaos expansion of the assimilated model and parameters. For this, first insert the polynomial chaos expansions into the filter formula:

$$\begin{bmatrix} \sum_{i=1}^{S} \left(y_a^k\right)^i \phi^i(\xi) \\ \sum_{i=1}^{S} \left(\theta_a^k\right)^i \phi^i(\xi) \end{bmatrix} = \left(I - K_k H_k\right) \begin{bmatrix} \sum_{i=1}^{S} \left(y_f^k\right)^i \phi^i(\xi) \\ \sum_{i=1}^{S} \left(\theta_f^k\right)^i \phi^i(\xi) \end{bmatrix} + K_k y_{obs}^k \phi^1(\xi) \tag{14}$$

Note that the term with the observations does not depend on the random variables and is therefore associated with only the first (constant) basis function. By a Galerkin projection we see that the polynomial chaos coefficients of the assimilated state and parameters are:

$$\begin{bmatrix} \left(y_a^k\right)^i \\ \left(\theta_a^k\right)^i \end{bmatrix} = \left(I - K_k H_k\right) \begin{bmatrix} \left(y_f^k\right)^i \\ \left(\theta_f^k\right)^i \end{bmatrix} + K_k y_{obs}^k \delta_{i-1}, \quad i = 1, \cdots, S \tag{15}$$

If all the observations are made only on the state of the system we have that:

$$\begin{bmatrix} \left(y_a^k\right)^i \\ \left(\theta_a^k\right)^i \end{bmatrix} = \left(I - K_k H_k\right)\left(y_f^k\right)^i + K_k y_{obs}^k \,\delta_{i-1}\,, \quad i = 1,\cdots,S \tag{16}$$

The covariance of the extended state vector is:

$$P = \mathrm{cov}\left(\begin{bmatrix} y \\ \theta \end{bmatrix}\right) = \begin{bmatrix} \mathrm{cov}(y) & \mathrm{cov}([y,\theta]) \\ \mathrm{cov}([\theta, y]) & \mathrm{cov}(\theta) \end{bmatrix} = \begin{bmatrix} P_{yy} & P_{y\theta} \\ P_{\theta y} & P_{\theta\theta} \end{bmatrix}, \quad \begin{bmatrix} H & 0 \end{bmatrix} P \begin{bmatrix} H^T \\ 0 \end{bmatrix} = HP_{yy}H^T \tag{17}$$

The Kalman gain reads:

$$K = P\begin{bmatrix} H^T \\ 0 \end{bmatrix}\left(R + HP_{yy}H^T\right)^{-1} = \begin{bmatrix} P_{yy}H^T \\ P_{\theta y}H^T \end{bmatrix}\left(R + HP_{yy}H^T\right)^{-1} \tag{18}$$

The parameter estimate is then:

$$\theta_a = \theta + P_{\theta y}H^T\left(R + HP_{yy}H^T\right)^{-1}\left(y_{obs} - Hy\right) \tag{19}$$

In the polynomial chaos framework the covariance matrices P_{yy} and $P_{\theta y}$ can be estimated from the polynomial chaos expansion of the solution and the parameters. Then the polynomial chaos coefficients of the parameters are adjusted as:

$$\theta_a^i = \theta^i + P_{\theta y}H^T\left(R + HP_{yy}H^T\right)^{-1}\left(y_{obs}\cdot\delta_{i-1} - Hy^i\right), \; i = 1,\cdots,S \tag{20}$$

Let’s note that the Kalman filter formula is optimal for the linear Gaussian case. For non-Gaussian uncertainties the Kalman filter formula is sub-optimal, but is still expected to work.

4 Application to a Mechanical System

4.1 Roll Plane Modeling Vehicle

The model used as a case study for the theory presented in this article is based on the four degree of freedom roll plane model of a vehicle used in [9], which is shown in Figure 1. The difference is that the suspension dampers and the suspension springs used in this study are nonlinear and the tire vertical stiffnesses k_{t1} and k_{t2} are assumed to be uncertain.

If x is the relative displacement across the suspension spring with a stiffness k_i (*i* = 1, 2), the force across the suspension spring is given by:

$$F_{K_i}(x) = k_i x + k_{i,3}\, x^3, \quad i = 1, 2 \tag{21}$$

If v is the relative velocity across the damper with a damping coefficient c_i (*i* = 1, 2), the force across the damper is given by:

$$F_{C_i}(v) = c_i\left(0.2\,\tanh(10v)\right) \tag{22}$$

It is assumed that the probability density functions of the values of k_{t1} and k_{t2} can be represented with a Beta (1, 1) distribution, with an uncertainty of +/- 20%.

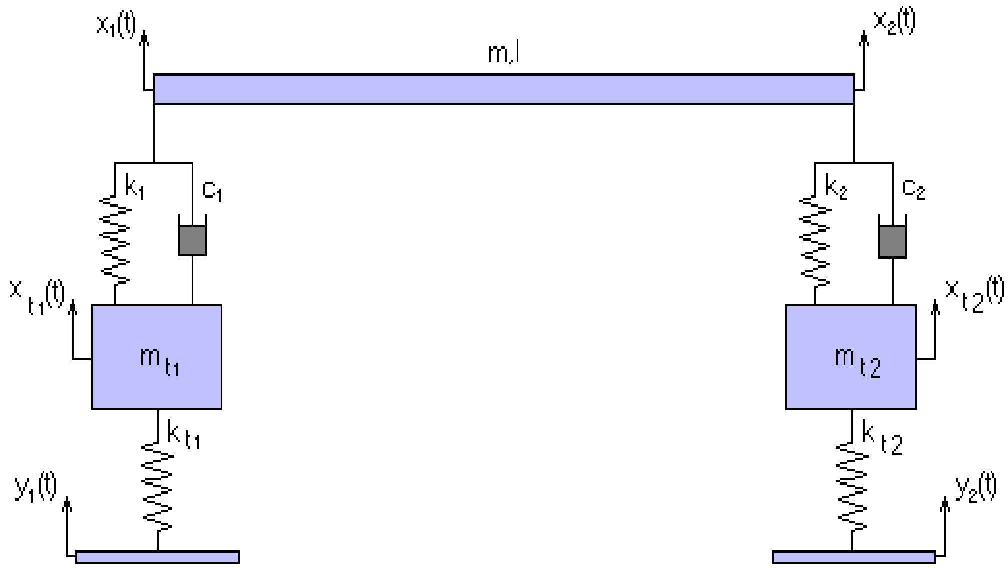


Figure 1. Four Degree of Freedom Roll Plane Model [8].

The body of the vehicle is represented as a bar of mass m (sprung mass), length l, and moment of inertia I. The unsprung masses, i.e., the mass of each tire/axle combination, are represented by m_{t1} and m_{t2}.

The motion variables x_1 and x_2 correspond to the vertical position of each side of the vehicle body, while the motion variables x_{t1} and x_{t2} correspond to the vertical position of the tires.

The inputs to this system are y_1 and y_2, which represent the road profile under each wheel. The equations of motion of the system are described by Equations (23) – (26):

$$\begin{aligned}\frac{m}{2}(\ddot{x}_2+\ddot{x}_1)+F_{K_1}(x_1-x_{t1})+F_{K_2}(x_2-x_{t2})\\ =F_{C_1}(\dot{x}_{t1}-\dot{x}_1)+F_{C_2}(\dot{x}_{t2}-\dot{x}_2)\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{I}{l}(\ddot{x}_2-\ddot{x}_1)+\frac{l}{2}F_{K_2}(x_2-x_{t2})-\frac{l}{2}F_{K_1}(x_1-x_{t1})\\ =\frac{l}{2}F_{C_1}(\dot{x}_1-\dot{x}_{t1})-\frac{l}{2}F_{C_2}(\dot{x}_2-\dot{x}_{t2})\end{aligned} \tag{24}$$

$$m_{t1}\ddot{x}_{t1}+F_{K_1}(x_{t1}-x_1)+F_{C_1}(\dot{x}_{t1}-\dot{x}_1)=k_{t1}(y_1-x_{t1}) \tag{25}$$

$$m_{t2}\ddot{x}_{t2}+F_{K2}(x_{t2}-x_2)+F_{C_2}(\dot{x}_{t2}-\dot{x}_2)=k_{t2}(y_2-x_{t2}) \tag{26}$$

where $F_{K_1}, F_{K_2}, F_{C_1}, \text{and } F_{C_2}$ are defined in Equations (21) and (22).

The parameters used in this study and their values are shown in Table 1. They are the same parameters used by [9], with the addition of nonlinearities and uncertainties.

Table 1. Parameters used for the Parameter Estimation Case Study

Parameter	Description	Value
m	Mass of the Roll Bar	580 kg
m_{t1}, m_{t2}	Mass of the tire/axle	36.26 kg
c_1, c_2	Damping coefficients	710.70 N s /m
k_1, k_2	Spring constants – linear component	19,357.2 N/m
$k_{1,3}$, $k_{2,3}$	Spring constants – cubic component	15,000 N/m^3
l	Length of the Roll Bar	1.524 m
I	Inertia of the Roll Bar	63.3316 kg m^2
k_{t1}, k_{t2}	Tires vertical stiffnesses	96,319.76 N/m+/-20%, with Beta (1, 1) distribution

The uncertainties of 20% on the values of k_{t1} and k_{t2} can be represented as:

$$k_{t1} = k_{t1\,nom}(1+0.20\,\xi_1), \quad \xi_1 \in [-1,\ 1] \tag{27}$$

$$k_{t2} = k_{t2\,nom}(1+0.20\,\xi_2), \quad \xi_2 \in [-1,\ 1] \tag{28}$$

where $k_{t1\,nom}$ and $k_{t2\,nom}$ are the nominal values of the vertical stiffnesses of the tires ($k_{t1\,nom} = 96,319.76$ N/m and $k_{t2\,nom} = 96,319.76$ N/m).

The distributions of the uncertainties related to the values of k_{t1} and k_{t2}, defined on the interval $[-1,\ 1]$, are represented in Figure 2. They have the following Probability Density Functions (PDF):

$$w(\xi_i) = \frac{3}{4}\left(1-\xi_i^{\ 2}\right), \quad i = 1, 2 \tag{29}$$

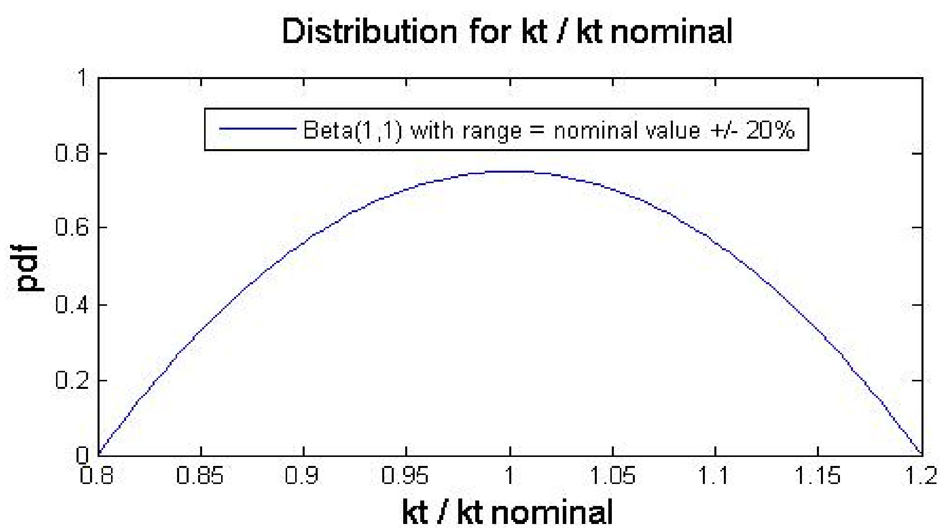


Figure 2. Beta (1, 1) Distribution

4.2 Collocation Points

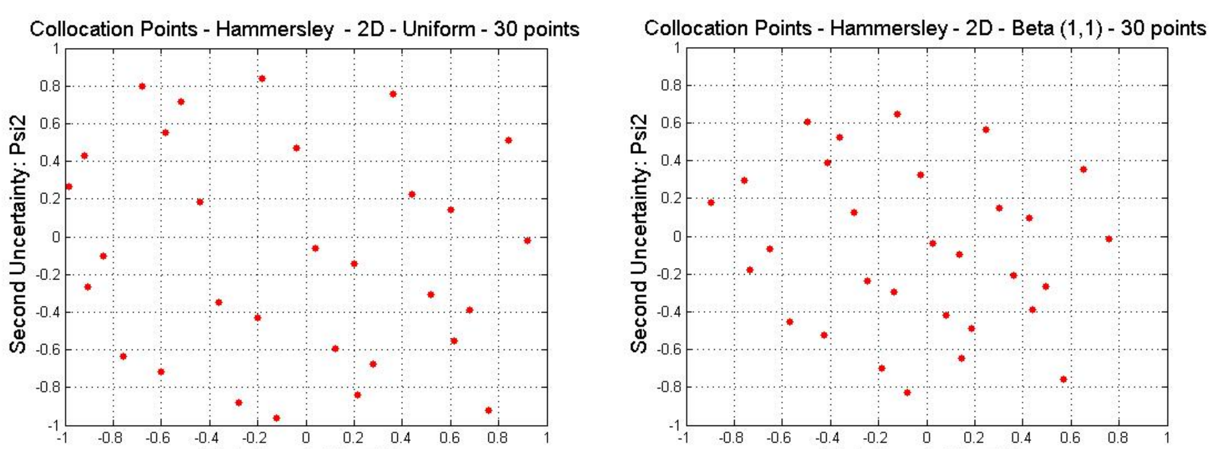
The generalized polynomial chaos theory is explained in [7], in which direct stochastic collocation is proposed as a less expensive alternative to the traditional Galerkin approach for propagating the uncertainties through the system of equations of motion. The collocation approach consists of imposing that the system of equations holds at a given set of collocation points. If the polynomial chaos expansion for the uncertain parameter(s) considered contains 15 terms for instance, then at least 15 collocation points are needed in order to have at least 15 equations for 15 unknown polynomial chaos coefficients. It is desirable to have more collocation points than polynomial coefficients to solve for. In that case a least-squares algorithm is used to solve the system with more equations than unknowns.

Unless otherwise specified, the polynomial chaos expansions of k_{t1} and k_{t2} will use 15 terms. All the other variables affected by the uncertainties on k_{t1} and k_{t2} will be modeled by a polynomial chaos expansion using 15 terms as well. The collocation approach is the one used in this study.

Unless otherwise specified, 30 collocation points will be used to derive the coefficients associated to each of the 15 terms of the different polynomial chaos expansions. The collocation points used in this study are obtained using the Hammersley algorithm [5]. These collocation points for a uniform distribution are shown in Figure 3 (a).

One of the advantages of the Hammersley points used in this study is that when the number of points is increased, the new set of points still contains all the old points. We therefore know that more points should result in a better approximation. The collocation points for a Beta (1, 1) distribution, which is used in this study, are shown in Figure 3 (b).

The transformation from the collocation points for a uniform distribution to the points for a Beta (1, 1) distribution is achieved by applying the inverse Cumulative Distribution Function of the Beta (1, 1) distribution. Let's note that there is no collocation point at the boundary, i.e., no point associated with an uncertainty equal to -1 or 1, which is needed in order to avoid having a cost function equal to infinity.



(a) **(b)**

Figure 3. Hammersley Collocation Points for Uniform Distribution (2-Dimensions, 30 Points)

(a) Uniform Distribution, (b) Beta (1, 1) Distribution

4.3 Experimental Setting

In order to assess the efficiency of the polynomial chaos theory for parameter estimation, k_{t1} and k_{t2} will be estimated using a plot of four motion variables obtained for a given road input: the displacements across the suspensions ($x_1 - x_{t1}$ and $x_2 - x_{t2}$), and their corresponding velocities ($\dot{x}_1 - \dot{x}_{t1}$ and $\dot{x}_2 - \dot{x}_{t2}$). The road profile is shown in Figure 4, and the road input is obtained assuming the vehicle has a constant speed of 16 km/h (10 mph). The road profile can be seen as a long speed bump. The first tire is subjected to a ramp at *t* = 0, and reaches a height of 15 cm (6”) for a vertical displacement of 1 m, then stays at the same height for 1 m, and goes back down to its initial height. The second tire is subjected to the same kind of input, but with a time delay of 20% and it reaches a maximum height of only 12 cm.

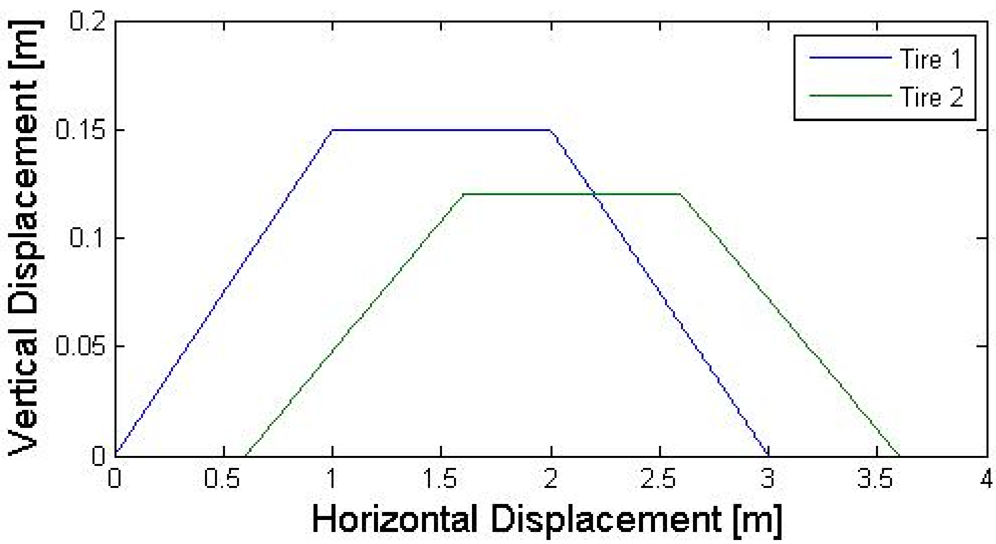


Figure 4. Road Profile

The four motion variables are plotted from *t* = 0 to *t* = 3 seconds using $k_{t1\ ref} = 100{,}800$ N/m and $k_{t2\ ref} = 88{,}855$ N/m (i.e., $\xi_{1\ ref} = 0.2326$ and $\xi_{2\ ref} = -0.3875$) and assuming these values can only be measured with a sampling rate of 0.3 seconds, which means that 10 measurement points are available in order to estimate the vertical stiffnesses of the tires. However, for the proof of concept of the parameter estimation method presented in this paper, we pretend that the values of k_{t1} and k_{t2} are unknown, the objective being to estimate those values based on the plot of the four motion variables shown in Figure 5. Let’s note that 3 seconds of data correspond to a horizontal displacement of 13.33 meters. The end of the speed bump occurs at *t* = 0.675 seconds.

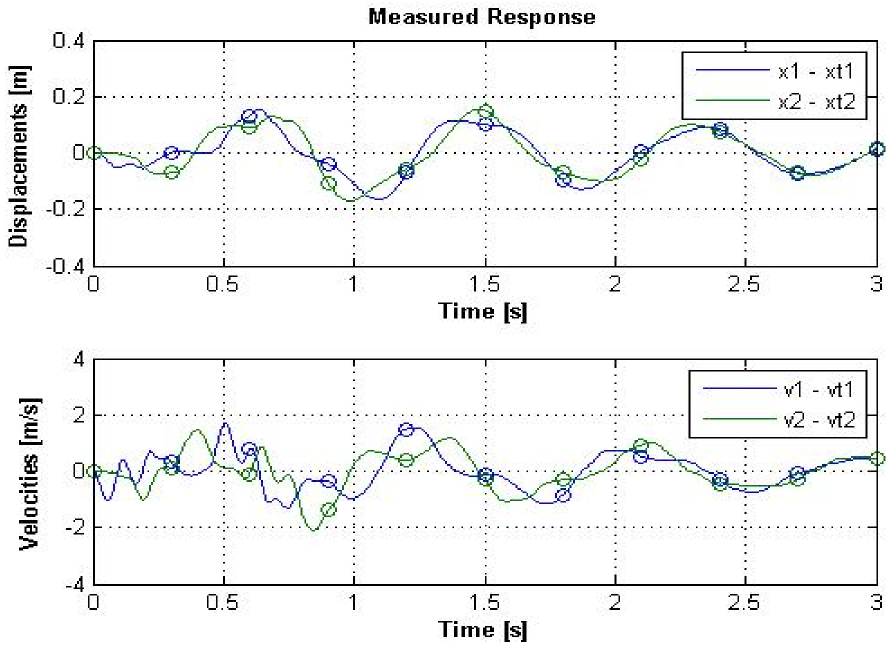
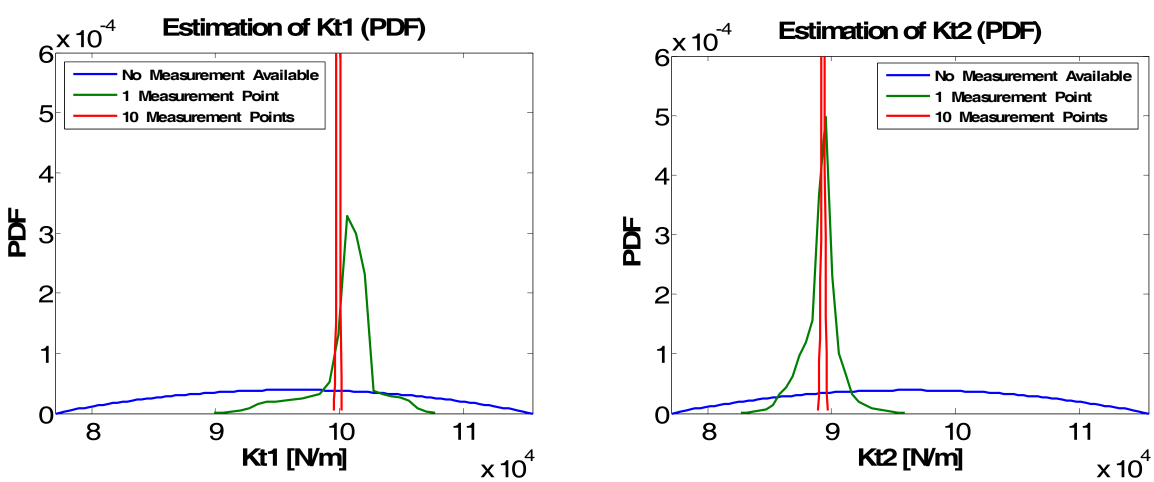


Figure 5. Observed States - Displacements and Velocities

Parameters estimation is performed using the EKF approach. A Gaussian measurement noise with zero mean and 1% variance is added to the measurements. Parameter estimation is performed using the noisy signal.

4.4 Results Using the EKF Approach

The EKF estimations come in the form of PDFs, as shown in Figure 6. .



(a) **(b)**

Figure 6. Estimations of the Tires Vertical Stiffnesses - (a) Tire 1, (b) Tire 2

The estimated value of a parameter is given by the first term of its polynomial chaos expression. The estimated values of ξ_1 and ξ_2 obtained using the EKF approach are $\xi_{1\ est} = 0.1863$ and $\xi_{2\ est} = -0.3612$, i.e., $k_{t1\ est} = 99{,}909$ N/m and $k_{t2\ est} = 89{,}362$ N/m. The actual values were $\xi_{1\ ref} = 0.2326$ and $\xi_{2\ ref} = -0.3875$, i.e., $k_{t1\ ref} = 100{,}800$ N/m and $k_{t2\ ref} = 88{,}855$ N/m. It seems to be a rather good estimation considering that only 10 measurement points were used and that there is noise associated to the measurements. In a pending article for the ASME 2007 International Design Engineering Technical Conferences (IDETC) to be held in Las Vegas, Nevada, in September 2007, we considered the problem of parameter estimation using a Bayesian approach that is also based on the polynomial chaos theory, in which the maximum likelihood estimates are obtained by minimizing a cost function derived from the Bayesian theorem. The estimated values of ξ_1 and ξ_2 obtained using the Bayesian approach were $\xi_{1\ est} = 0.2460$ and $\xi_{2\ est} = -0.3783$, i.e., $k_{t1\ est} = 101{,}059$ N/m and $k_{t2\ est} = 89{,}032$ N/m. For this specific test case, the quality of the estimations using the Bayesian approach is better than the estimation using the EKF approach. The EKF and the Bayesian approach can therefore both work with noisy measurements. Let's note that the vertical tire stiffness has very little effect on the displacement of the suspensions, which means that a noise of 1% for a specific time measurement is equivalent to having quite a different vertical stiffness for that given time. The EKF and the Bayesian approaches therefore seem to be quite robust. With a Gaussian measurement noise with zero mean and 0.01% variance, the EKF would give very good estimations: $\xi_{1\ est} = 0.2302$ and $\xi_{2\ est} = -0.3837$, i.e. $k_{t1\ est} = 100{,}753$ N/m and $k_{t2\ est} = 88{,}929$ N/m.

4.4 Effect of the Frequency for Harmonic Inputs

For the specific test case studied in 4.3., the Bayesian approach worked better than the EKF approach. In order to compare the two approaches in more general way, the estimations can be performed for different harmonic inputs. Figure 7 shows the harmonic inputs that will be used at two different frequencies: 0.33 Hz and 1 Hz.

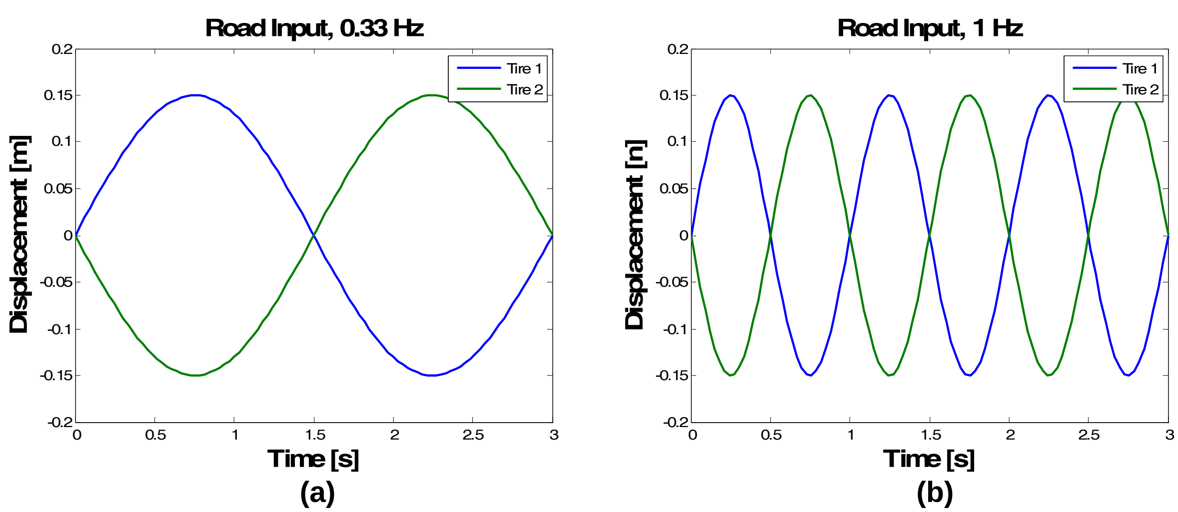


Figure 7: Road Inputs at Different Frequencies: (a) At 0.33 Hz, (b) At 1 Hz

The harmonic inputs used for this study have amplitudes of 15 cm, and the inputs at each of the two tires are 180 degrees out of phase. Each of the tires is subjected to the input for 3 seconds. It is still assumed that measurements can only be obtained at a sampling rate of 0.3 s. Therefore, 10 measurement points are available in order to estimate the vertical stiffnesses of the tires. Although an amplitude of 15 cm might not be very realistic at high frequencies, it will give an insight of when the EKF seems to be a better option than the Bayesian approach.

The results obtained for frequencies ranging from 0.33 Hz to 100 Hz with the EKF approach are shown in Figure 8 (a). The results obtained with the Bayesian approach are shown in Figure 8 (b) for comparison. Let us recall that the actual values of the parameters ξ_1 and ξ_2 are $\xi_{1\ ref} = 0.2326$ and $\xi_{2\ ref} = -0.3875$.

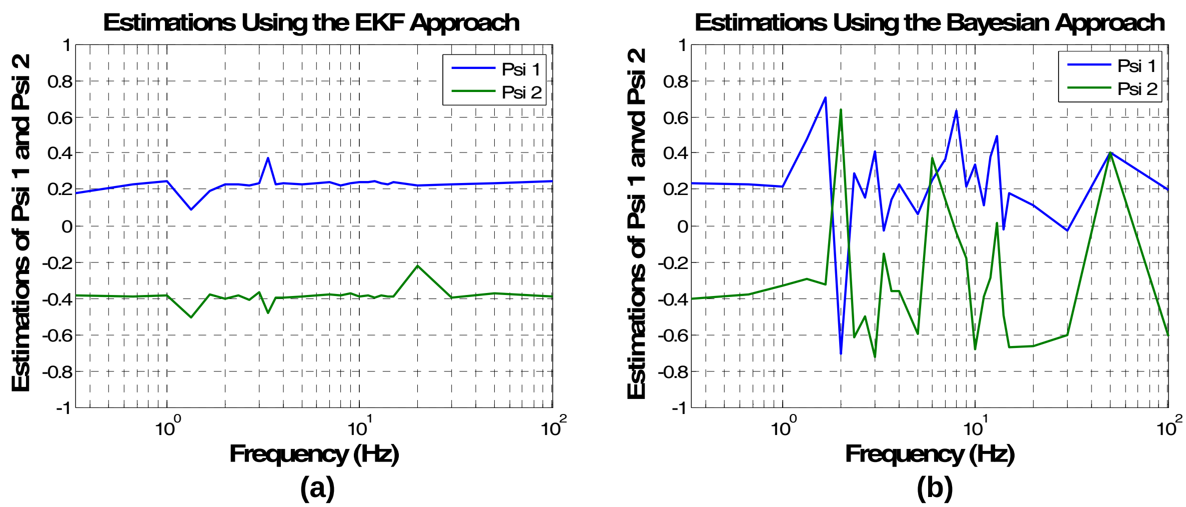


Figure 8. Estimations of the Tires Vertical Stiffnesses at Different Frequencies

(a) EKF Approach, (b) Bayesian Approach

The EKF approach clearly gives better results than the Bayesian approach for frequencies higher than 1 Hz. The reason why the Bayesian approach gave a better estimate of the vertical stiffnesses of the tires for the road profile shown in Figure 4 is that the profile was very smooth and did not have any small irregularity.

Let's note that the estimations with the EKF approach are not very good at 1.33 Hz. The 1.33 Hz frequency is close to the natural frequency of the sprung mass of the corresponding linearized system. A transfer function can easily be derived if we linearize the system and make it a quarter-car since both sides of the half-car are the same. For that equivalent linear quarter-car, the natural frequency of the sprung mass is 1.22 Hz. There are several possible explanations for the lack of precision of the estimation around the natural frequency of the sprung mass. The deflections and velocities across the suspensions have large values, which makes the system reach very nonlinear regimes, and makes good estimation harder to obtain. Another thing is that a noise of 1% in the measurement corresponds to higher errors in absolute values. It doesn't seem that there is a straightforward explanation for the few other frequencies resulting in large estimation errors, especially for the estimation of the vertical stiffness of the second tire at 20 Hz.

Figure 9 (a) shows the results obtained with the EKF approach when the Gaussian measurement noise has a 0.01% variance instead of a 1%. Figure 9 (b) shows the same estimations with a Bayesian approach.

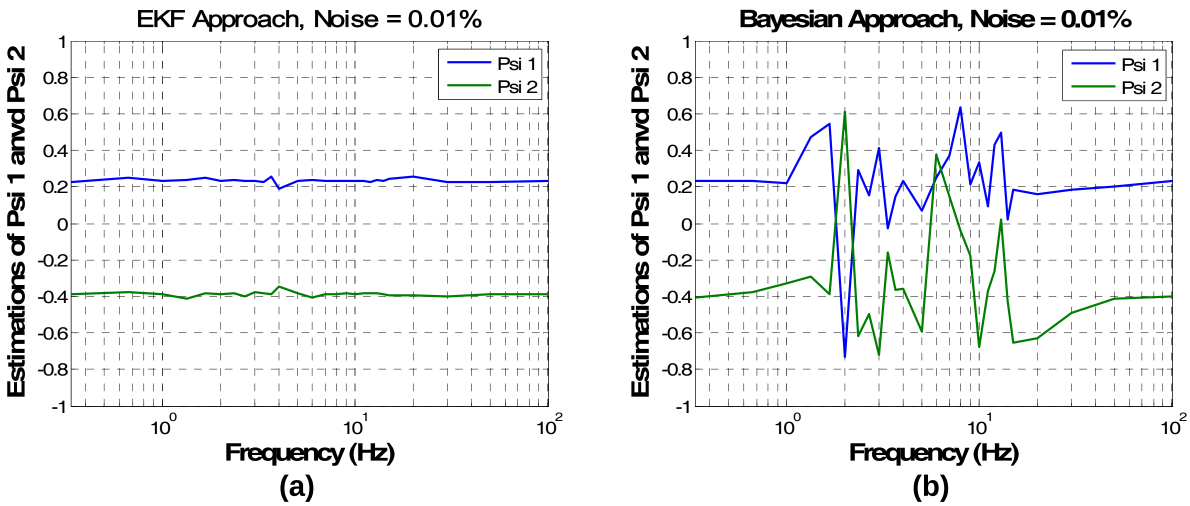


Figure 9. Estimations of the Vertical Stiffnesses of the Tires With Gaussian Measurement Noise With Zero Mean and 0.01% Variance: (a) EKF Approach, (b) Bayesian Approach

With a very low noise level, the EKF approach gives very good estimations for every frequency, whereas the Bayesian approach still gives poor estimations for many frequencies, despite a clear improvement at 50 Hz to 100 Hz.

5 Conclusion

This paper applies the polynomial chaos theory to the problem of parameter estimation, using direct stochastic collocation and an approach based on the Extended Kalman Filter (EKF). In this approach, an updated polynomial chaos representation of the state accounts for all the observations, given the polynomial chaos representation of the current state. Parameter estimation was performed on a four degree of freedom roll plane model of a vehicle, where uncertainties on the values of the vertical stiffnesses of the tires are assumed to have a Beta (1, 1) distribution. The vertical stiffnesses of the tires were estimated from periodic observations of the displacements and velocities across the suspensions. The proposed EKF approach resulted in a good estimation of the vertical stiffnesses of the tires.

This approach was compared to a Bayesian approach that is also based on the polynomial chaos theory, in which the maximum likelihood estimates are obtained by minimizing a cost function derived from the Bayesian theorem. Estimations performed for different harmonic inputs showed that the EKF approach clearly gave better estimations than the Bayesian approach for frequencies higher that 1 Hz. With noise associated to the measurement of the displacements and velocities across the suspensions, the EKF approach gave very good results for most frequencies and no incoherent results at any frequency. Given the fact that the vertical stiffnesses of the tires have very little effect on the displacements and velocities across the suspensions, the fact that the EKF estimation procedure can work with noisy measurements tends to show that the EKF approach is quite robust.

Estimating the vertical stiffnesses of the tires based on the behavior of the suspensions is not a very realistic task since the other parameters of the system would have to be perfectly known due to their greater effect on the displacements of the suspensions. However, the case study that was presented in this article was chosen as a proof of concept and shows that the EKF approach works well. The proposed method has several advantages. Simulations using Polynomial Chaos methods are much faster than Monte Carlo simulations. In addition, the EKF approach gives more information about the parameters of interest than a simple estimated value: the estimation comes in the form of a probability density function. However, a few issues exist with this approach. It is sub-optimal and, unlike the Bayesian approach, it cannot treat non-Gaussian uncertainties. Finally, it is shown that measurement noise can have a negative impact on the quality of the estimations. These issues will be investigated in the future. The proposed estimation procedures may benefit from regularization techniques.

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