

QUES 4)

UNDERSTANDING THE PRACTICAL USE OF Z-TRANSFORMS.

- The practical use of z transforms is that we can make a different type of system by just varying its poles and zeros in the z domain.
- These changes affect the relationship between the input and output in the discrete-time domain and this can be easily computed in a computer. Thus proving the essence of DSP.
- For example, I implemented a low pass filter, which lowers the amplitude of high-frequency waveforms while barely affecting the low-frequency components.
- I came up with the filter's frequency response in the z domain just by knowing its zeros and poles.
- Here if we consider the normalized frequency which ranges between $[0, \pi]$ in the circular domain.
- Here 0 refers to the low-frequency components while π referring to high-frequency ones.
- Let z be the point moving along the unit circle $|z| = 1$, where frequency refer to the angle of a point .

- So we need to lower the strength of the components as we approach π i.e $z = -1$. Therefore $z = -1$ is the zero of the impulse response of the filter.
- While $z = 0$ can be the pole of it, since it does not contribute to the amplitude of the transfer function as the distance between the moving point and origin is always unity.
- Therefore the impulse response is $H(z) = (z + 1) \div 2z$
- This gives us the relation of $y[n] = (x[n] + x[n-1])/2$
Which is nothing but a moving average operation.
The amplitude of the impulse is nothing but the euclidean distance between z on the unit circle and the poles and the zeros.

❖ AN INTUITIVE EXPLANATION OF WHY DOES A MOVING AVERAGE SYSTEM WOULD WORK?

ANS : let us take $x[n] = [1, 100, 1, 100, \dots]$

This can be decomposed into $[50, 50, 50, 50, \dots]$ (**low freq component**) + $[-49, 50, -49, 50, \dots]$ (**high freq component**) since its changing more frequently.

If we apply a moving average operation to it.

We see low freq = $[50, 50, 50, 50]$

But high freq = [0.5,0.5,0.5,0.5] components are reduced in magnitude and now are barely changing .

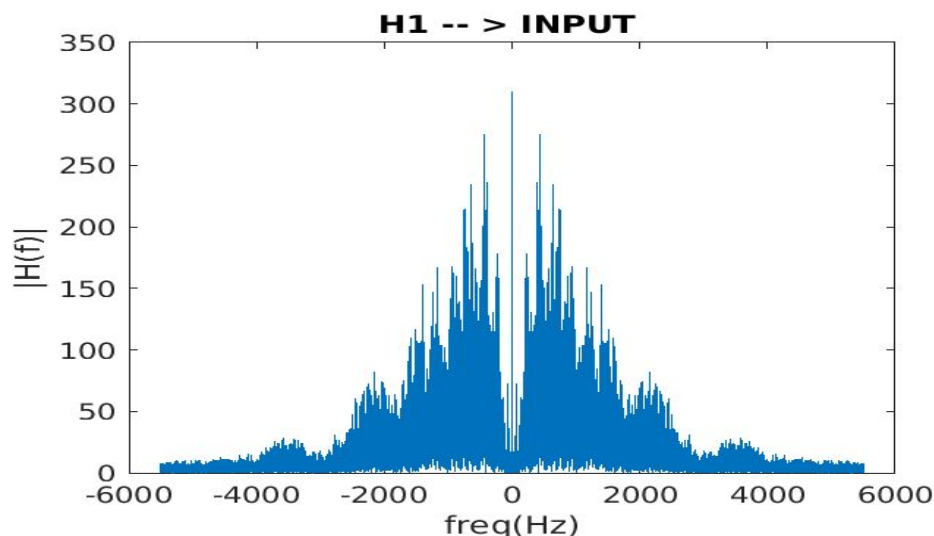
Thus we can see that the moving average operation lowers the magnitude of the high-frequency components.

❖ SIMULATION RESULTS :

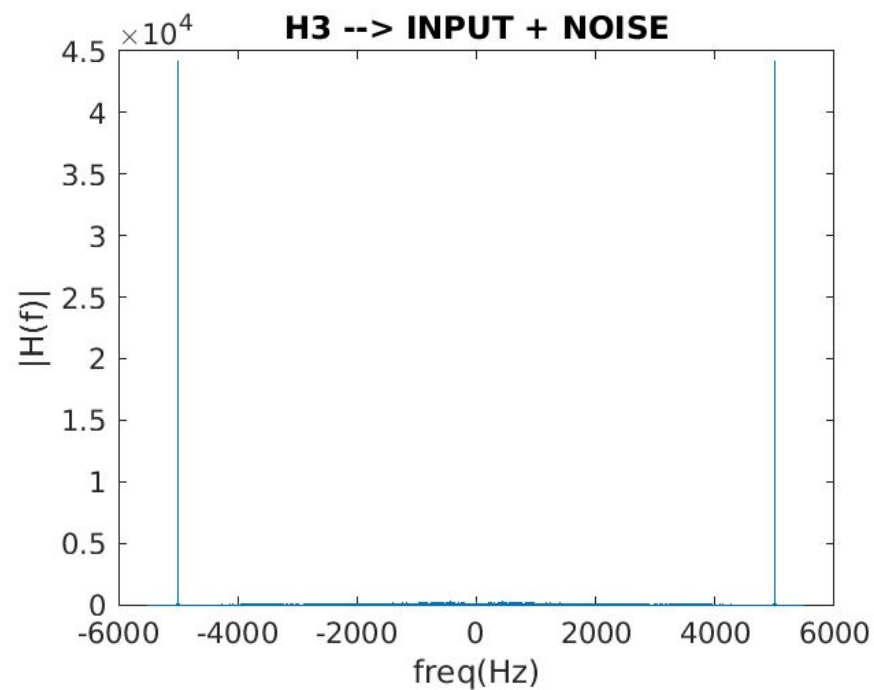
I took a .wav file (sampled at $F_s = 11025$ Hz) from the internet, added a sinusoid noise $\cos(2 * \pi * 5000 * t)$ and tried to pass it through the moving average system and the notch filter. The below are the results .

➤ For notch filter :

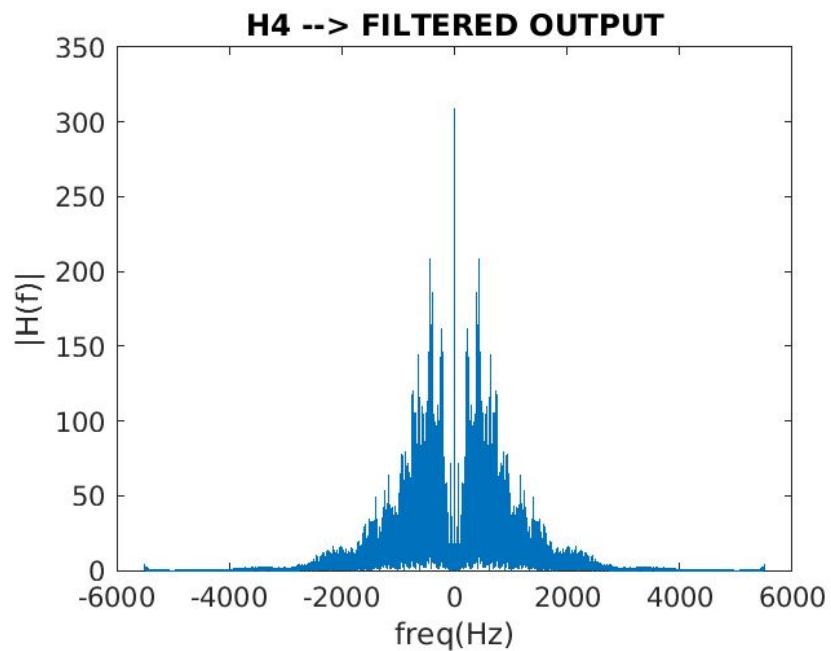
1) Input frequency spectrum.



2) Input + noise frequency spectrum



Filtered outputs frequency spectrum.

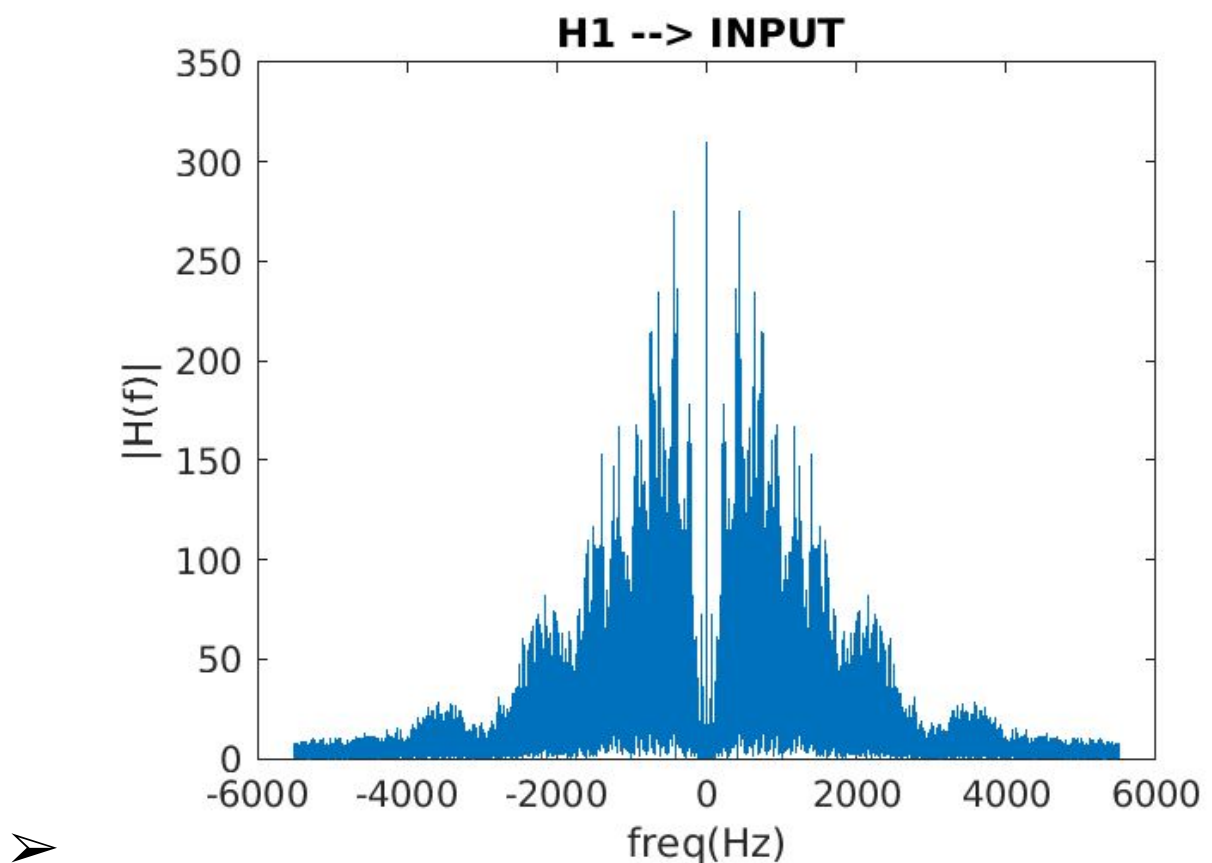


OBSERVATIONS :

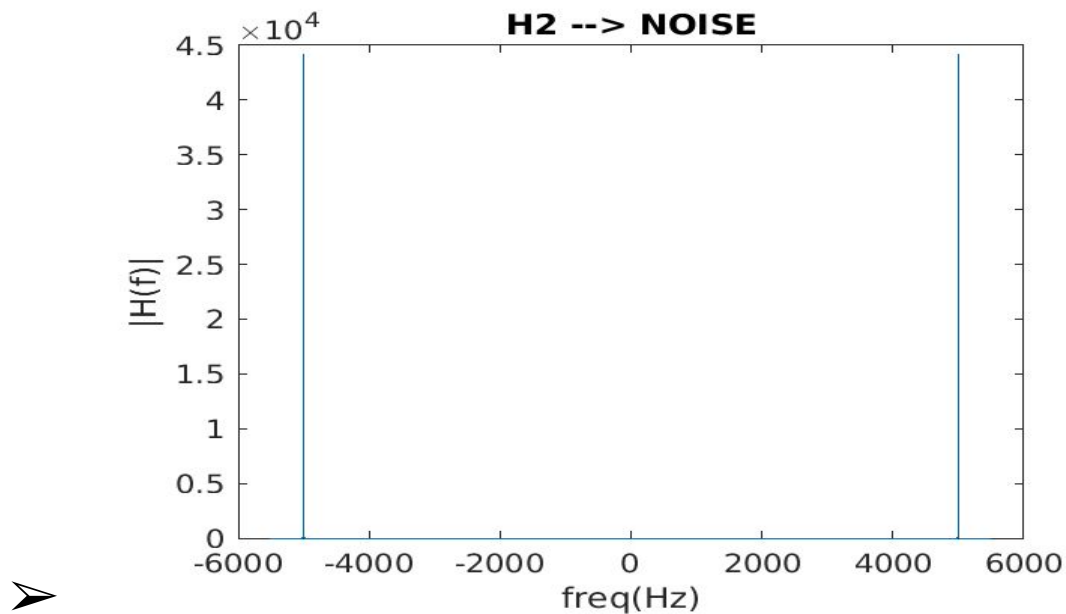
We can see that when we use the notch filter we could remove the added noise at 5000Hz completely. We can observe the spike at 5000Hz in the input + noise fft completely vanish when passed through the filter.

➤ For Moving Average System :

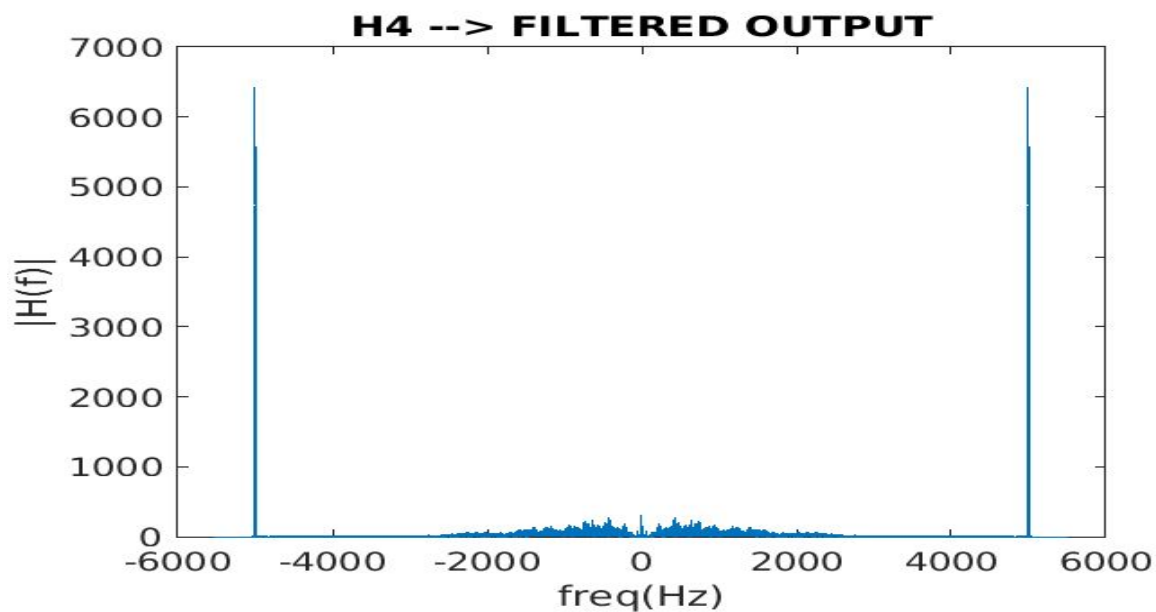
1) Input frequency spectrum.



2) With input + noise (The input spectrum is not visible due to high magnitude of sinusoids fft)



➤ 3) Output after applying the moving average operation over discrete time domain.



OBSERVATIONS :

Although the operation did not completely wipe out the noise, it lowered the amplitude of the noise from 450000 to 7000 which is great filtering in terms of frequency.