EE18BTECH11026_A4

February 17, 2022

1 Assignment 04

2 KOIDALA SURYA PRAKASH

2.1 EE18BTECH11026

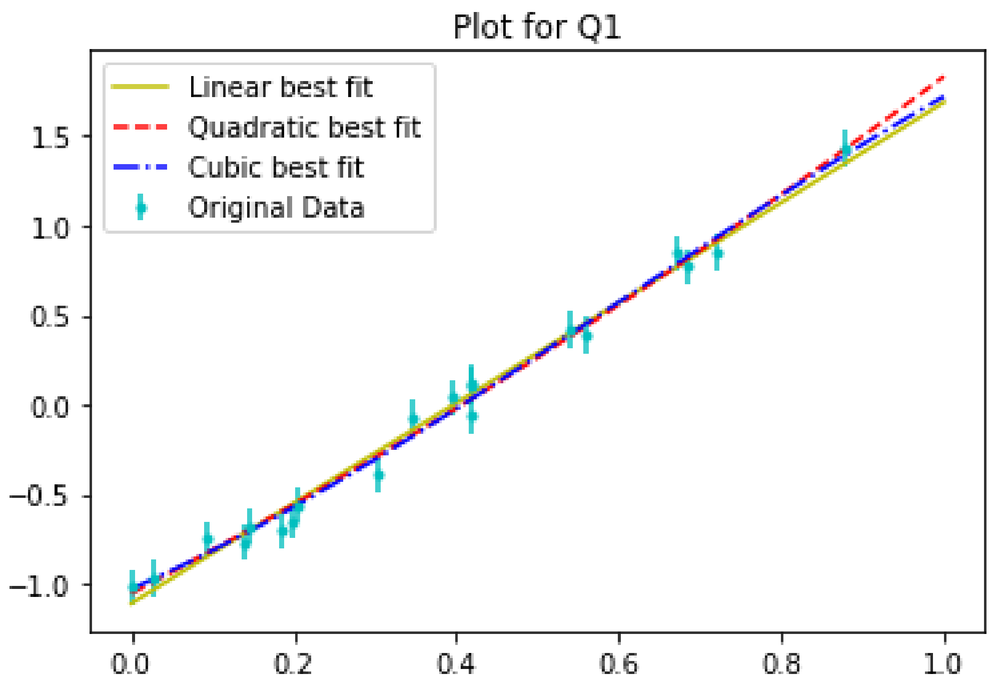
[10]:
```
from scipy import stats as stats_scipy
import numpy as np
import matplotlib.pyplot as plt
from astropy import stats as stats_astropy
from astroML import stats as stats_astroML
from scipy.optimize import curve_fit
```

3 Q1

[15]:
```
### HELPER FUNCS

## function calls for models
def linear(x,m,b) :
    return m*x + b

def quad(x, a,b ,c):
    return a*(x**2) + b*x + c

def cubic(x, a,b,c,d):
    return a*(x**3) + b*(x**2) + c*x + d

def likelihood(param, model):
    if model=='linear':
        y = linear(X, *param)
    elif model=='quadratic':
        y = quad(X, *param)
    elif model=='cubic':
        y = cubic(X, *param)
```

```
    return sum(stats_scipy.norm.logpdf(*args)
               for args in zip(Y, y, Sigma))

m_lin = 2
m_quad = 3
m_cubic = 4
```

[26]:
```
data = np.loadtxt('q1_data.dat')

X = data[:,0]
Y = data[:,1]
Sigma = data[:,2]

n = X.shape[0]



#Best fit parameters for linear  quad and cubic
optparam_lin, cov_lin = curve_fit(linear, X, Y, sigma=Sigma,␣
 ↪absolute_sigma=True)
optparam_quad, cov_quad = curve_fit(quad, X, Y, sigma=Sigma,␣
 ↪absolute_sigma=True)
optparam_cubic, cov_cubicicic = curve_fit(cubic, X, Y, sigma=Sigma,␣
 ↪absolute_sigma=True)



#plotting
x = np.linspace(0,1,1000)
#Plot original data, lin best fit , quad best fit , cubic best fit

plt.errorbar(X, Y, yerr=Sigma, fmt=".", color = 'c', label='Original Data')
plt.plot(x, linear(x, *optparam_lin),'y-',label='Linear best fit')
plt.plot(x, quad(x, *optparam_quad),'r--', label='Quadratic best fit')
plt.plot(x, cubic(x, *optparam_cubic), 'b-.', label='Cubic best fit')
plt.title('Plot for Q1')
plt.legend()
plt.show()
```



[27]:
```
#Frequentist comparision

chisq_lin = np.sum(((Y - linear(X, *optparam_lin))/Sigma)**2)
chisq_quad = np.sum(((Y - quad(X, *optparam_quad))/Sigma)**2)
chisq_cubic = np.sum(((Y - cubic(X, *optparam_cubic))/Sigma)**2)

#p-value when linear model is null hypothesis
pval_quad_lin = 1-stats_scipy.chi2(m_quad - m_lin).cdf(chisq_lin - chisq_quad)
pval_cubic_lin = 1-stats_scipy.chi2(m_cubic - m_lin).cdf(chisq_lin -␣
↪chisq_cubic)

#AIC

#Considered dataset is small => take AIC values
AIC_lin = -2*likelihood(optparam_lin, 'linear') + (2.0*m_lin*n)/(n-m_lin-1)
AIC_quad = -2*likelihood(optparam_quad, 'quadratic') + (2.0*m_quad*n)/
↪(n-m_quad-1)
AIC_cubic = -2*likelihood(optparam_cubic, 'cubic') + (2.0*m_cubic*n)/
↪(n-m_cubic-1)
AIC_min = min([AIC_lin, AIC_quad, AIC_cubic])
delta_AIC_lin = AIC_lin - AIC_min
delta_AIC_quad = AIC_quad - AIC_min
delta_AIC_cubic = AIC_cubic - AIC_min
```

```
#BIC
BIC_lin = m_lin*np.log(n) - 2*likelihood(optparam_lin, 'linear')
BIC_quad =  m_quad*np.log(n)  -2*likelihood(optparam_quad, 'quadratic')
BIC_cubic = m_cubic*np.log(n) -2*likelihood(optparam_cubic, 'cubic')
BIC_min = min([BIC_lin, BIC_quad, BIC_cubic])
delta_BIC_lin = BIC_lin - BIC_min
delta_BIC_quad = BIC_quad - BIC_min
delta_BIC_cubic = BIC_cubic - BIC_min


print("p-value for preferred model for Quadratic & null hypothesis ie, Linear =␣
 ↪", pval_quad_lin)
print("p-value for preferred model for Cubic & null hypothesis ie, Linear = ",␣
 ↪pval_cubic_lin)
print('\n')

print("AIC value for linear model = ", AIC_lin)
print("AIC value for quadratic model = ", AIC_quad)
print("AIC value fot cuboc model = ", AIC_cubic)
print('\n')
print("Delta AIC value for linear model = ", delta_AIC_lin)
print("Delta AIC value for quadratic model = ", delta_AIC_quad)
print("Delta AIC value fot cubic model = ", delta_AIC_cubic)
print('\n')
print("BIC value for linear model = ", BIC_lin)
print("BIC value for quadratic model = ", BIC_quad)
print("BIC value fot cuboc model = ", BIC_cubic)
print('\n')
print("Delta BIC value for linear model = ", delta_BIC_lin)
print("Delta BIC value for quadratic model = ", delta_BIC_quad)
print("Delta BIC value fot cuboc model = ", delta_BIC_cubic)
```

```
p-value for preferred model for Quadratic & null hypothesis ie, Linear =
0.17813275695316733
p-value for preferred model for Cubic & null hypothesis ie, Linear =
0.32887884419522884


AIC value for linear model =  -39.33080446313153
AIC value for quadratic model =  -38.34982062400561
AIC value fot cuboc model =  -35.594151850935894


Delta AIC value for linear model =  0.0
Delta AIC value for quadratic model =  0.9809838391259191
Delta AIC value fot cubic model =  3.736652612195634
```

```
BIC value for linear model =  -38.04522226896472
BIC value for quadratic model =  -36.86262380334364
BIC value fot cuboc model =  -34.2778894233866


Delta BIC value for linear model =  0.0
Delta BIC value for quadratic model =  1.1825984656210835
Delta BIC value fot cuboc model =  3.7673328455781245
```

3.1 Comments for Q1

3.2 1. Comparing models with linear

While considering Linear model as the null hypothesis, and comparing them with other two models : the p-vals are as follows

Comparing with Quadratic 0.17813275695316733

Comparing with Cubic 0.32887884419522884

The p-values are quite higher (greater than 0.05) and thus we cannot reject the null hypothesis.

3.3 2. DELTA AIC vals

Delta AIC value for linear model = 0.0

Delta AIC value for quadratic model = 0.9809838391259191

Delta AIC value fot cubic model = 3.736652612195634

delta AIC for quad model lies in (0,2), thus the model has subtantial support delta AIC for cubic model lies in (2,4), thus the model has less support

3.4 3. DELTA BIC vals

Delta BIC value for linear model = 0.0

Delta BIC value for quadratic model = 1.1825984656210835

Delta BIC value fot cuboc model = 3.7673328455781245

delta BIC for quad model lies in (0,2), thus the no evidence against the model

delta BIC for cubic model lies in (2,4), thus positive evidence against the model

4 Q2

[6]:
```
data = np.array([[ 0.42,  0.72,  0.  ,  0.3 ,  0.15,
                    0.09,  0.19,  0.35,  0.4 ,  0.54,
                    0.42,  0.69,  0.2 ,  0.88,  0.03,
                    0.67,  0.42,  0.56,  0.14,  0.2  ],
                  [ 0.33,  0.41, -0.22,  0.01, -0.05,
                   -0.05, -0.12,  0.26,  0.29,  0.39,
                    0.31,  0.42, -0.01,  0.58, -0.2 ,
                    0.52,  0.15,  0.32, -0.13, -0.09 ],
                  [ 0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                    0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                    0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                    0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1  ]])
X, Y, Sigma = data
```

[7]:
```
# free paramemter count for each model
m_lin , m_quad, m_cubic= 2,3,4
#find best parameters

#Best fit parameters for linear, quadratic & cubic
optparam_lin, cov_lin = curve_fit(linear, X, Y, sigma=Sigma,␣
 ↪absolute_sigma=True)
optparam_quad, cov_quad = curve_fit(quadratic, X, Y, sigma=Sigma,␣
 ↪absolute_sigma=True)
n = X.shape[0]


def likelihood(param, model):
    if model=='linear':
        y = linear(X, *param)
    elif model=='quadratic':
        y = quadratic(X, *param)
    elif model=='cubic':
        y = cubic(X, *param)
    return sum(stats_scipy.norm.logpdf(*args)
               for args in zip(Y, y, Sigma))

#AIC
#Considered dataset is small => take AIC values
AIC_lin = -2*likelihood(optparam_lin, 'linear') + (2.0*m_lin*n)/(n-m_lin-1)
AIC_quad = -2*likelihood(optparam_quad, 'quadratic') + (2.0*m_quad*n)/
 ↪(n-m_quad-1)
AIC_min = min([AIC_lin, AIC_quad])
delta_AIC_lin = AIC_lin - AIC_min
delta_AIC_quad = AIC_quad - AIC_min
```

```python
#BIC
BIC_lin = m_lin*np.log(n) - 2*likelihood(optparam_lin, 'linear')
BIC_quad =  m_quad*np.log(n)  -2*likelihood(optparam_quad, 'quadratic')
BIC_min = min([BIC_lin, BIC_quad])
delta_BIC_lin = BIC_lin - BIC_min
delta_BIC_quad = BIC_quad - BIC_min



print("AIC value for linear model : ", AIC_lin)
print("AIC value for quadratic model : ", AIC_quad)
print('\n')
print("Delta AIC value for linear model : ", delta_AIC_lin)
print("Delta AIC value for quadratic model : ", delta_AIC_quad)
print('\n')
print("BIC value for linear model : ", BIC_lin)
print("BIC value for quadratic model : ", BIC_quad)
print('\n')
print("Delta BIC value for linear model : ", delta_BIC_lin)
print("Delta BIC value for quadratic model : ", delta_BIC_quad)
```

```
AIC value for linear model :  -39.31585166028409
AIC value for quadratic model :  -38.38302717300821


Delta AIC value for linear model :  0.0
Delta AIC value for quadratic model :  0.9328244872758802


BIC value for linear model :  -38.03026946611728
BIC value for quadratic model :  -36.89583035234624


Delta BIC value for linear model :  0.0
Delta BIC value for quadratic model :  1.1344391137710446
```

4.1 Comments

Since delta AIC for quadratic model is in between (0,2) , we can conclude that quadratic model has a substatial support

While delta BIC for quadratic model is in between (0,2) , we can conclude that quadratic model has no evidence against itself.

Thus we can conclude that both metrics infer the same and thus , linear model cannot be rejected !!

5 Q3

I read through the paper ”Remarks on generating realistic synthetic meteoroid orbits” authored by T.J. Jopek. link : https://arxiv.org/pdf/2101.02175.pdf

The paper talks about 5 ways of generating synthetic meteroid objects, while compared with orbits of observed ones. It talks about how K-S test has been used in this case. The author considered both 1d and 2d has been used while the latter should not be used as 2d ks test has no unique way to order the points for computing EDFs. Mainly , the KS test is not used to derive a model , rather to to compare it with synthetic data.

6 Q4

[28]:

```
## finding significance

## ATLAS DISCOVERY : 1.7*10 POW -9
sign_1 = stats_scipy.norm.isf(1.7*10**-9)
## LIGO DISCOVERY P VAL : 2 *  10 POW -7
sign_2= stats_scipy.norm.isf(2*10**-7)
## GOF OF CHI SQ
p_val = stats_scipy.chi2(67).sf(65.2)

print('Significance of Higgs boson disc {}'.format(sign_1))
print('Significance of LIGO disc {}'.format(sign_2))
print('GOF of chisquare in best fit oscillation {}'.format(p_val))
```

```
Significance of Higgs boson disc 5.911017938341624
Significance of LIGO disc 5.068957749717791
GOF of chisquare in best fit oscillation 0.5394901931099036
```

7 The End