(3f)

$$p(x_1, x_2) = x_1^2 + 2x_2^2 - x_1x_2 - x_1$$
$$+ \lambda_1(x_1 - 2x_2 - u_1)$$
$$+ \lambda_2(x_1 + 4x_2 - u_2)$$
$$+ \lambda_3(5x_1 - 76x_2 - 1)$$

$\frac{\partial p}{\partial x_1} =$ Objective can be rewritten as

$$Q = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 2 \end{pmatrix}, \quad c = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad A = \begin{pmatrix} 1 & -2 \\ 1 & 4 \\ 5 & -76 \end{pmatrix} \quad b = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$$

$$p(\underline{x}) = x^TQx + c^Tx + \lambda(Ax - b)$$

$\frac{\partial \mathcal{L}(x, \lambda)}{\partial x} = 2Qx + c + A^T\lambda$

Equating $\frac{\partial \mathcal{L}(x, \lambda)}{\partial x} = 0$ to find primal & optimal dual variable

$$\Rightarrow 2\begin{pmatrix} 1 & -0.5 \\ -0.5 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 5 \\ -2 & 4 & -76 \end{pmatrix}\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 2x_1 - x_2 - 1 + \lambda_1 + \lambda_2 + 5\lambda_3 = 0 \rightarrow (1)$

$\Rightarrow -x_1 + 4x_2 - 2\lambda_1 + 4\lambda_2 - 76\lambda_3 = 0 \ (2)$

* Values obtained using cvxpy

$\lambda_1^* = 2.8645$

$x_1^* = -2.33$

$\lambda_2^* = 2.2980$

$x_2^* = -0.1667$

$\lambda_3^* = 0.0675$

To verify the relationship

Consider (1):

$$2x_1 - x_2 + 1 = -\lambda_1 - \lambda_2 - 5\lambda_3$$

* Computing LHS:

$$2x_1^* - x_2^* + 1 = -4.66 + 0.167 + 1$$

$$= -5.49$$

* Computing RHS:

$$-\lambda_1^* - \lambda_2^* - 5\lambda_3^* = -5.4975$$

∴ We can see than LHS = RHS

Hence the values are correct

---x---