Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

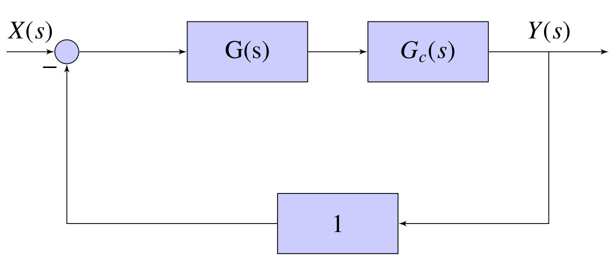
1 Compensators

1.1 Phase Lead

1.1. Given the unity feedback system of Fig. **??** , with

$$G(s) = \frac{K}{s(s+5)(s+20)} \tag{1.1.1}$$

The uncompensated system has about 55% peak overshoot and a peak time if 0.5 seconds when $K_v = 10$. Use frequency response technique to design a lead compensator to reduce the percent overshoot to 10% , while keeping the peak time and steady state error about the same or less. Consider second order approximations.



.

Fig. 1.1

1.2. **Solution:**

$$K_v = \lim_{s\to 0} sG(s) = 10 \tag{1.2.1}$$

$$\implies K = 1000 \tag{1.2.2}$$

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The bode plot for G(s)is as follows :

$$G(s) = \frac{1000}{s(s+5)(s+20)} \tag{1.2.3}$$

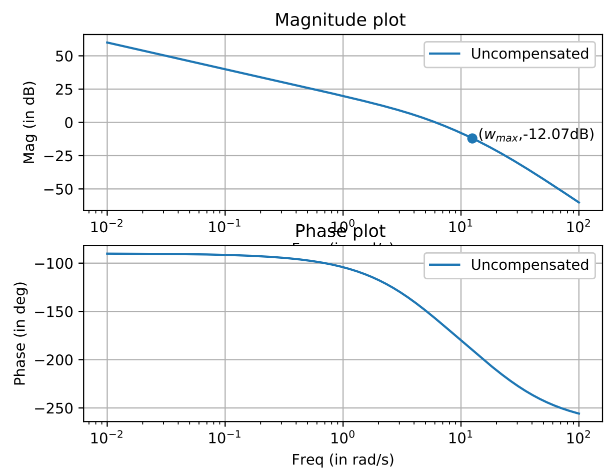

Fig. 1.2: G(s) Bode Plot

$$\zeta = \frac{-\ln\left(\frac{OS\%}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\frac{OS\%}{100}\right)\right)^2}} \tag{1.2.4}$$

$$PhaseMargin = \phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}\right) \tag{1.2.5}$$

The following code computes the above quantities.

```
codes/ee18btech11026/ee18btech11026_1.py
```

The required additional phase contribution by the compensator will be:

$$\phi_{max} = 58.9 - 21.16 + correctionfactor \tag{1.2.6}$$

$$CorrectionFactor = 25^\circ \tag{1.2.7}$$

Specifications	Actual	Expected
OS%	55%	10%
ζ	0.186	0.591
ϕ_m	21.16°	58.59°
T_p	0.5	<= 0.5
K_v	10	<= 10

TABLE 1.2: Table of Specifications

$$\phi_{max} = 62^\circ \tag{1.2.8}$$

Note : Since we know that the lead network will also increase the phase-margin frequency, we add a correction factor to compensate for the lower uncompensated systems phase angle.Choosing the correction factor is a trail and error procedure so as to reach our expected specifications.
The gain compensator's T.F will be of the form:

$$G_c(s) = \frac{1}{\beta}\left(\frac{s + \frac{1}{T}}{s + \frac{1}{T\beta}}\right) \tag{1.2.9}$$

This form of T.F does not influence the steady state error.
Important Relations to find T and β:

$$\phi_{max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} \tag{1.2.10}$$

The Compensator's magnitude at the phase margin frequency ω_{max}

$$|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}} \tag{1.2.11}$$

$$T = \frac{1}{\omega_{max}\sqrt{\beta}} \tag{1.2.12}$$

Using the above formulae :

$$\beta = 0.062 \tag{1.2.13}$$

$$|G_c(j\omega_{max})| = 12.07dB \tag{1.2.14}$$

If we select ω_{max} to be the new phase-margin frequency, the uncompensated systems magnitude at this frequency must be -12.07 dB to yield a 0 dB crossover at ω_{max} for the compensated system.
From the bode plot of the un-compensated system, find ω_{max} where the magnitude is -12.07 dB. This becomes our new phase-margin frequency.

$$\omega_{max} = 12.5rad/sec \tag{1.2.15}$$

$$T = 0.321 \tag{1.2.16}$$

The Compensator's T.F is as follows :

$$G_c(s) = 16.13\left(\frac{s + 3.115}{s + 50.25}\right) \tag{1.2.17}$$

The open loop T.F for the compensated system is :

$$G(s).G_c(s) = 16130\left(\frac{(s+3.115)}{s(s+50.25)(s+5)(s+20)}\right) \tag{1.2.18}$$

1.3. **Verification :** We could observe the affect of the lead-phase compensator from the phase plots.

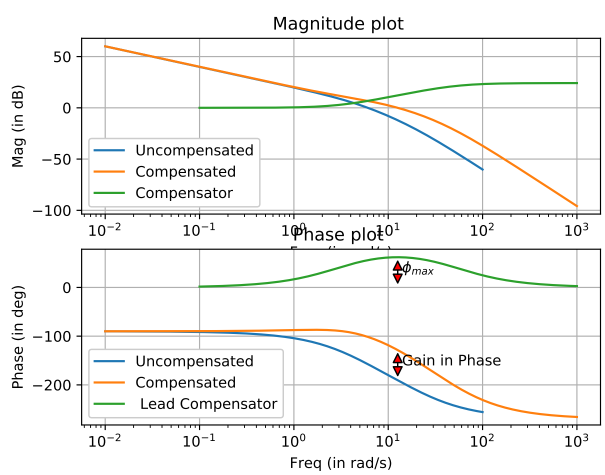

Fig. 1.3: Combined Bode Plots

The time responses for a unit step input in a unity feedback system with and without a compensator are as follows :

These plots are generated using the below code:

```
codes/ee18btech11026/ee18btech11026_2.py
```

1.4. **Result :** The below is the summary for the designed lead-compensator

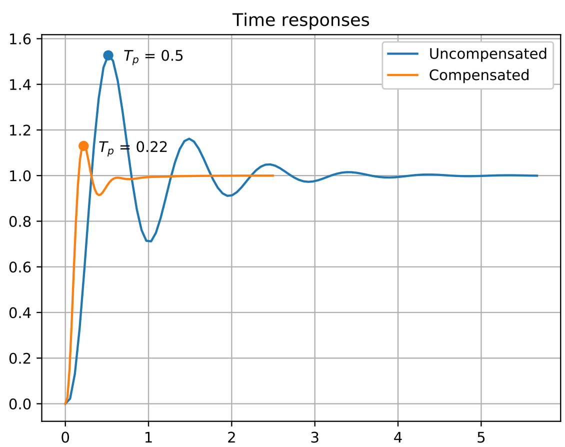

Fig. 1.3: Time response for a unit step input

Specifications	Expected	Proposed
OS%	10%	11%
T_p	<= 0.5	0.22
K_v	<= 10	10

TABLE 1.4: Comparing the desired and obtained results