Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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5.1. Consider the state space realization :

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -9 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 45 \end{pmatrix} u(t) \quad (5.1.1)$$

with initial conditions :

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5.1.2)$$

, where $u(t)$ denotes unit step function. Find the value of $\lim_{t\to\infty} \left| \sqrt{x_1^2(t) + x_2^2(t)} \right|$

5.2. **Solution:** The state space model is given in the form :

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (5.2.1)$$

X(s) can be directly determined using the below formula :

$$X(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}U(s) + (s\mathbf{I} - \mathbf{A})^{-1}x(0) \quad (5.2.2)$$

Where :

$$A = \begin{pmatrix} 0 & 0 \\ 0 & -9 \end{pmatrix} \tag{5.2.3}$$

$$B = \begin{pmatrix} 0 \\ 45 \end{pmatrix} \tag{5.2.4}$$

$$x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5.2.5}$$

$$\begin{aligned} u(t) &= unit\ step\ function \\ \implies U(s) &= \frac{1}{s} \end{aligned} \tag{5.2.6}$$

$$\begin{aligned} X(s) = &\left(\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -9 \end{pmatrix}\right)^{-1} \begin{pmatrix} 0 \\ 45 \end{pmatrix} \frac{1}{s} \\ &+ \left(\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -9 \end{pmatrix}\right)^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{aligned} \tag{5.2.7}$$

Solving X(s) :

$$X(s) = \begin{pmatrix} s & 0 \\ 0 & s+9 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 45 \end{pmatrix} \frac{1}{s} \tag{5.2.8}$$

$$= \begin{pmatrix} 0 \\ \frac{45}{s(s+9)} \end{pmatrix} \tag{5.2.9}$$

Hence :

$$X(s) = \begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{45}{s(s+9)} \end{pmatrix} \tag{5.2.10}$$

By comparing elements in the matrices :

$$X_1(s) = 0 \implies x_1(t) = 0 \tag{5.2.11}$$

Using this result we can simplify the required expression as follows :

$$\lim_{t\to\infty} \left| \sqrt{x_1^2(t) + x_2^2(t)} \right| = \left| \lim_{t\to\infty} x_2(t) \right| \tag{5.2.12}$$

Using the final value theorem :

$$\begin{aligned} \left| \lim_{t\to\infty} x_2(t) \right| &= \left| \lim_{s\to 0} sX_2(s) \right| \\ &= \left| \lim_{s\to 0} \left(s \frac{45}{s(s+9)} \right) \right| \\ &= \left| \lim_{s\to 0} \left(\frac{45}{s+9} \right) \right| \\ &= \left| \frac{45}{9} \right| \\ &= |5| \\ &= 5 \end{aligned} \tag{5.2.13}$$

Hence $\lim_{t\to\infty} \left| \sqrt{x_1^2(t) + x_2^2(t)} \right| = 5$

5.3. verify the answer with python code https://github.com/Surya291/EE2227-Control-systems/tree/master/Codes

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