

Time Complexity and Data Structure Performance in Delivery Orders Management

1. Array Implementation

- Operations Performed:
 - Added a new delivery order at the end.
 - Displayed the entire list of orders.
 - Searched for a specific order by Order ID (linear search).
- Time Complexity:
 - Add new delivery order at the end: $O(1)$

The array has a fixed size in the example, and adding at the end is done in constant time if there is space.

 - Display entire list of orders: $O(n)$
Traversing and printing each order in the array takes linear time.
 - Search for specific order (linear search): $O(n)$
Searching for an order by ID requires checking each element one by one.

2. Singly Linked List Implementation

- Operations Performed:
 - Convert the array to a singly linked list.
 - Added a new delivery order at the end.
 - Traversed and printed all orders in the list.
 - Searched for an order by Order ID.
- Time Complexity:
 - Add new delivery order at the end: $O(n)$
In the singly linked list, adding at the end requires traversing the list first to find the last node.
 - Traverse and print orders: $O(n)$
To print the list, we must traverse each node in the list.
 - Search for an order: $O(n)$
Searching for an order by ID requires a linear search, checking each node in the list.

3. Doubly Linked List Implementation

- Operations Performed:
 - Convert the singly linked list to a doubly linked list for bi-directional traversal.
 - Added a new delivery order at the end.
 - Removed an order from the list.
 - Traversed the list in reverse order.
 - Updated the priority of an existing order.
- Time Complexity:
 - Add new delivery order at the end: $O(1)$
By maintaining references to both the head and the tail of the list, adding a new node at the end can be done in constant time.

- Remove an order: $O(n)$
Removing an order requires finding the specific node first, which takes linear time.
- Traverse in reverse order: $O(n)$
Traversing the list backward still requires checking each node, so it's $O(n)$.
- Update priority of an existing order: $O(n)$
To update the priority, we must search for the order first, which requires linear time.

4. Skip List Implementation

- Operations Performed:
 - Implemented a skip list for fast searching and ordered insertion of delivery orders.
 - Inserted a new delivery order.
 - Searched for a specific delivery order.
 - Deleted an order from the list.
- Time Complexity:
 - Insert new delivery order: $O(\log n)$
In the skip list, insertion is optimized by utilizing multiple layers, allowing for faster insertion in logarithmic time.
 - Search for an order by Order ID: $O(\log n)$
The skip list allows for efficient search using the multi-level structure, reducing search time to logarithmic complexity.
 - Delete an order: $O(\log n)$
Like searching, deletion is performed efficiently in logarithmic time due to the skip list's structure.

Insights on Data Structure Performance

1. Array

- Best for static, fixed-size data where frequent insertions and deletions are not needed.
- Searching is slower (linear search), but adding at the end is efficient if there's enough space.
- When to use: Best for applications where data is accessed sequentially and doesn't change often.

2. Singly Linked List

- Great for dynamic data where new items are added frequently.
- Insertion is slower at the end compared to arrays ($O(n)$), and searching is linear.
- When to use: Best for dynamic, ever-changing data where insertions and deletions are more common than searches.

3. Doubly Linked List

- Provides bi-directional traversal, making it useful when data might need to be accessed in both directions.
- Insertion at both ends can be $O(1)$, but searching and removal are still $O(n)$.

- When to use: When frequent insertions and deletions occur, and you need efficient traversal in both directions.

4. Skip List

- Optimized for fast searching, insertion, and deletion with logarithmic time complexity.
- Ideal for large datasets where you need fast lookups.
- When to use: Best for applications with many search, insert, and delete operations, especially when ordering by key (e.g., Order ID).

Conclusion

- Array is optimal when data size is fixed, and there is minimal modification required.
- Singly Linked List performs well for dynamic data with frequent insertions but suffers from slower searches.
- Doubly Linked List provides more flexibility for bi-directional traversal and deletion but still performs similarly to a singly linked list for most operations.
- Skip List is the best choice for applications that require efficient searching, inserting, and deleting in large datasets.