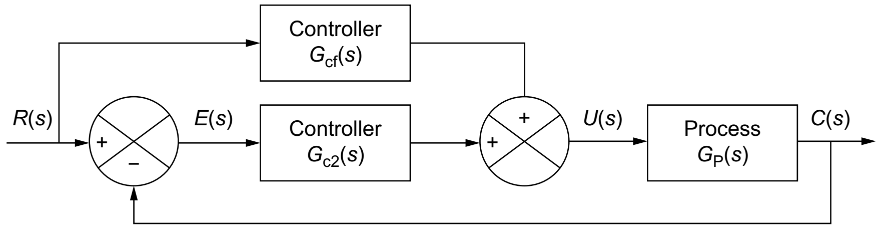


Fig. 11.1(f) | Feed-forward compensation

The compensated systems shown in Figs. 11.1(a), 11.1(b) and 11.1(d) have one degree of freedom which intimates that the system has single controller. The disadvantage of one degree of freedom controller is that the performance criteria realized using these compensation techniques are limited.

In simple words, the compensators introduce additional poles/zeros to an existing system so that the desired specification is achieved.

11.2.7 Effects of Addition of Poles

The following are the effects of addition of poles to an existing system:

(i) The root locus of a compensated system will be shifted towards the right-hand side of the *s*-plane.
(ii) Stability of a system gets lowered.
(iii) Settling time of a system increases.
(iv) Accuracy of a system is improved by the reduction of steady-state error.

11.2.8 Effects of Addition of Zeros

The following are the effects of addition of zeros to an existing system:

(i) The root locus of a compensated system will be shifted towards the left-hand side of the *s*-plane.
(ii) Stability of a system gets increased.
(iii) Settling time of a system decreases.
(iv) Accuracy of a system is lowered as steady-state error of the system increases.

In this chapter, three types of compensators used in the electrical systems are discussed. They are:

(i) Lag compensators
(ii) Lead compensators
(iii) Lag–lead compensators

11.2.9 Choice of Compensators

The choice of compensators from the different categories discussed in the previous sections is based on the following factors:

(i) Nature of signal to the system
(ii) Available components
(iii) Experience of the designer
(iv) Cost
(v) Power levels at different points and so on

11.3 Lag Compensator

The lag compensator is one that has a simple pole and a simple zero in the left half of the s-plane with the pole nearer to the origin. The term *lag* in the lag compensator means that the output voltage lags the input voltage and the phase angle of the denominator of the transfer function is greater than that of numerator. The general transfer function of the lag compensator is given by

$$G_{\mathrm{la}}(s) = K_{\mathrm{c}} \frac{s+\dfrac{1}{T}}{s+\dfrac{1}{\beta T}}, \quad \beta > 1 \tag{11.1}$$

where β and T are the constants and $K_{\mathrm{c}} = \dfrac{1}{\beta}$.

The pole-zero configuration of the lag compensator is shown in Fig. 11.2.

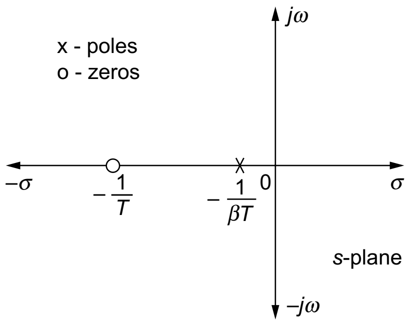


Fig. 11.2 | Pole-zero configuration of $G_{\mathrm{la}}(s)$

The corner frequencies present in the lag compensator, whose transfer function is given by Eqn. (11.1) are at $\omega = \dfrac{1}{T}$ and $\omega = \dfrac{1}{\beta T}$. The Bode plot and polar plot of the lag compensator are shown in Figs. 11.3(a) and 11.3(b) respectively.

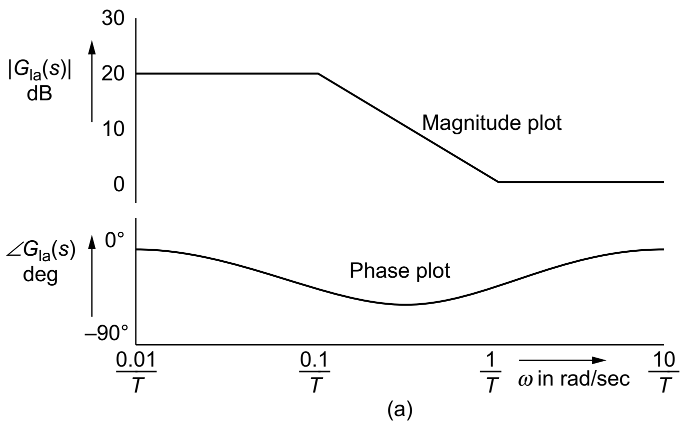
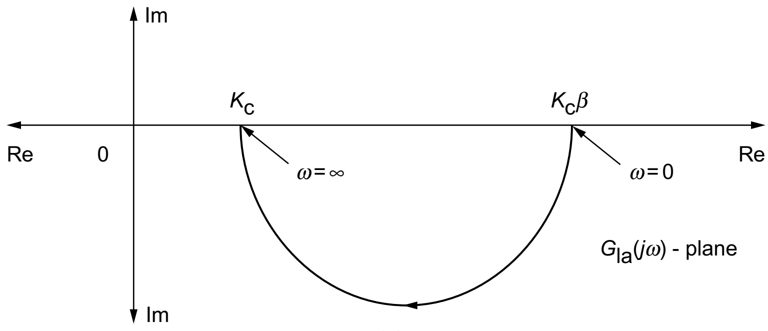


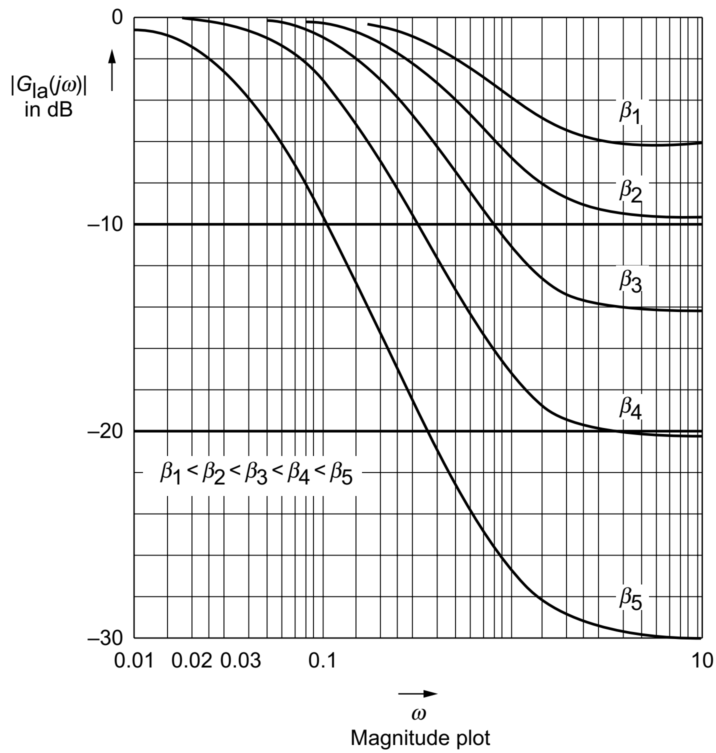
Fig. 11.3 | Plots of lag compensator



(b)

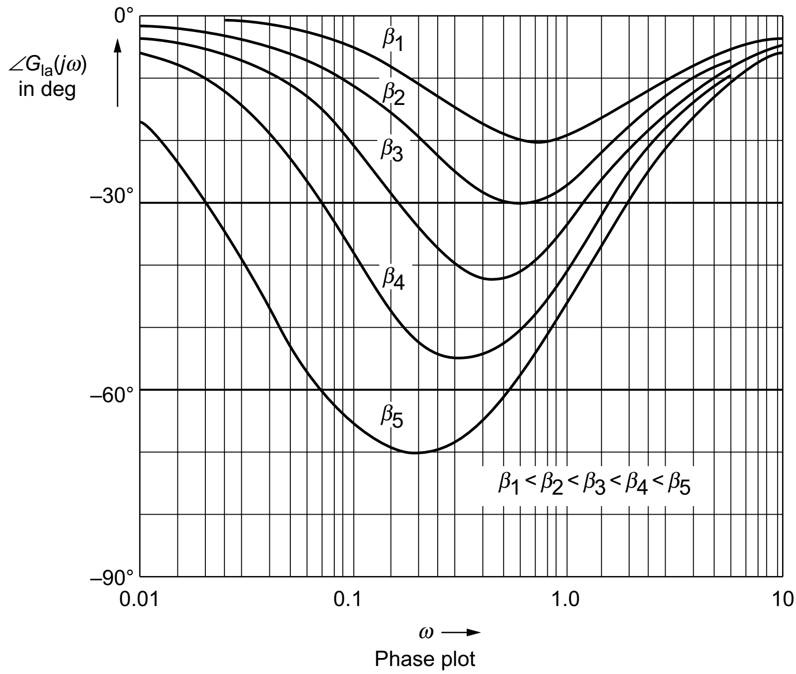
Fig. 11.3 | (Continued)

The Bode plot shown in Fig. 11.3(a) is plotted with $K_c = 0.1$ and $\beta = 10$. It is inferred that (i) magnitude of lag compensator is high at low frequencies and (ii) magnitude of lag compensator is zero at high frequencies. Hence, from the above conclusions, it is clear that the lag compensator behaves like a low-pass filter. The value of β is chosen between 3 and 10. The magnitude plot and phase plot of the compensator with different values of β are shown in Figs. 11.4(a) and 11.4(b) respectively.



(a)

Fig. 11.4 | Bode plot for different values of β (a) magnitude plot and (b) phase plot



(b)

Fig. 11.4 | (Continued)

11.3.1 Determination of Maximum Phase Angle ϕ_m

The modified transfer function of the lag compensator is

$$G_{la}(s) = \left(\frac{1+sT}{1+s\beta T}\right) \tag{11.2}$$

The magnitude and phase angle of the lag compensator are

$$M = \frac{\sqrt{1+\omega^2 T^2}}{\sqrt{1+\omega^2 \beta^2 T^2}} \tag{11.3}$$

and $$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\omega \beta T) \tag{11.4}$$

The maximum phase angle ϕ_m occurs at

$$\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_m} = 0$$

Differentiating Eqn. (11.4) with respect to ω, we obtain

$$\frac{T}{1+\omega_{\mathrm{m}}^2 T^2} - \frac{\beta T}{1+\beta^2\omega_{\mathrm{m}}^2 T^2} = 0$$

$$\frac{1}{1+\omega_{\mathrm{m}}^2 T^2} = \frac{\beta}{1+\beta^2\omega_{\mathrm{m}}^2 T^2}$$

$$\omega_{\mathrm{m}}^2\left(\beta T^2 - \beta^2 T^2\right) = 1-\beta$$

Solving the above equation, we obtain

$$\omega_{\mathrm{m}} = \frac{1}{T\sqrt{\beta}} = \left(\frac{1}{\sqrt{T}}\right)\left(\frac{1}{\sqrt{\beta T}}\right) \tag{11.5}$$

Substituting the above equation in Eqn. (11.4), we obtain

$$\tan\phi_{\mathrm{m}} = \frac{1-\beta}{2\beta} \tag{11.6}$$

Thus, Eqn. (11.5) gives the frequency at which the phase angle of the system is maximum and Eqn. (11.6) gives the maximum phase angle of the lag compensator.

11.3.2 Electrical Representation of the Lag Compensator

A simple lag compensator using resistor and capacitor is shown in Fig. 11.5.

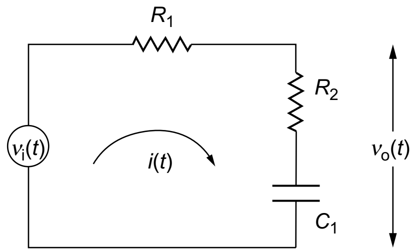


Fig. 11.5 | A simple lag compensator

Applying Kirchoff's voltage law to the above circuit, we obtain

$$v_i(t) = (R_1 + R_2)i(t) + \frac{1}{C_1}\int i(t) \tag{11.7}$$

and

$$v_0(t) = R_2 i(t) + \frac{1}{C_1}\int i(t) \tag{11.8}$$

Taking Laplace transform on both sides of Eqs. (11.7) and (11.8), we obtain

$$V_i(s) = \left(R_1 + R_2 + \frac{1}{sC_1}\right) I(s) \tag{11.9}$$

and

$$V_0(s) = \left(R_2 + \frac{1}{sC_1}\right) I(s) \tag{11.10}$$

Substituting $I(s)$ from Eqs. (11.10) to (11.9), we obtain

$$V_i(s) = \left(R_1 + R_2 + \frac{1}{sC_1}\right) \frac{V_0(s)}{\left(R_2 + \frac{1}{sC_1}\right)}$$

Therefore, the transfer function of the above circuit is

$$\frac{V_0(s)}{V_i(s)} = \frac{\left(R_2 + \frac{1}{sC_1}\right)}{\left(R_1 + R_2 + \frac{1}{sC_1}\right)}$$

Rearranging the above equation, we obtain

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\left(\frac{R_1 + R_2}{R_2}\right)} \left(\frac{s + \frac{1}{R_2C_1}}{s + \frac{1}{\left(\frac{R_1+R_2}{R_2}\right)R_2C_1}} \right) \tag{11.11}$$

Comparing Eqn. (11.1) and Eqn. (11.11), we obtain

$$T = R_2C_1,\ \beta = \frac{R_1 + R_2}{R_2} \text{ and } K_c = \frac{R_2}{R_1 + R_2} = \frac{1}{\beta}.$$

11.3.3 Effects of Lag Compensator

The following are the effects of adding lag compensator to a given system are:

(i) The lag compensator attenuates the high-frequency noise signals in the control loop.
(ii) It increases the steady-state error constants of a system.

(iii) Gain crossover frequency of a compensated system gets lowered.
(iv) Bandwidth of the compensated system decreases.
(v) Maximum peak overshoot, rise time and settling time of the system increases.
(vi) The system becomes more sensitive to the parameter variations.
(vii) As it acts like a proportional integral controller, it makes the system less stable.
(viii) Transient response of the compensated system becomes slower.

11.3.4 Design of Lag Compensator

The objective of designing the lag compensator is to determine the values of β and T for an uncompensated system $G(s)$ based on the desired system requirements. The design of lag compensator is based on the frequency domain specifications or time-domain specifications. The Bode plot is used for designing the lag compensator based on the frequency domain specifications, whereas the root locus technique is used for designing the lag compensator based on the time-domain specifications. Once the values of β and T are determined, the transfer function of the lag compensator $G_{\mathrm{la}}(s)$ can be obtained. The transfer function of the compensated system is $G_{\mathrm{c}}(s) = G_{\mathrm{la}}(s)G(s)$. If the Bode plot or root locus technique is plotted for the compensated system, it will satisfy the desired system requirements.

11.3.5 Design of Lag Compensator Using Bode Plot

Consider the open-loop transfer function of the uncompensated system $G(s)$. The objective is to design a lag compensator $G_{\mathrm{la}}(s)$ for $G(s)$ so that the compensated system $G_{\mathrm{c}}(s)$ will satisfy the desired system requirements. The steps for determining the transfer function of the compensated system $G_{\mathrm{la}}(s)$ using Bode plot are explained below:

Step 1: If the open-loop transfer function of the system $G(s)$ has a variable K, then $G_1(s) = G(s)$ or $G_1(s) = KG(s)$.

Step 2: Depending on the input and TYPE of the system, the variable K present in the transfer function of the uncompensated system $G_1(s)$ is determined based on either the steady-state error or the static error constant of the system.
The static error constants of the system are:

Position error constant, $K_{\mathrm{p}} = \underset{s \to 0}{Lt}\, G_1(s)$

Velocity error constant, $K_{\mathrm{v}} = \underset{s \to 0}{Lt}\, sG_1(s)$

Acceleration error constant, $K_{\mathrm{a}} = \underset{s \to 0}{Lt}\, s^2 G_1(s)$

The relation between the steady-state error and static error constant based on the TYPE of the system and input applied to the system can be referred to Table 5.5 of Chapter 5.

Step 3: Construct the Bode plot for $G_1(s)$ with gain K obtained in the previous step and determine the frequency domain specifications of the system (i.e., phase margin, gain margin, phase crossover frequency and gain crossover frequency).

Step 4: Let the desired phase margin of the system be $(p_{\mathrm{m}})_{\mathrm{d}}$. With a tolerance ε, determine $\left((p_{\mathrm{m}})_{\mathrm{d}}\right)_{\mathrm{new}}$ as

$$\left((p_{\mathrm{m}})_{\mathrm{d}}\right)_{\mathrm{new}} = (p_{\mathrm{m}})_{\mathrm{d}} + \varepsilon$$

where $\varepsilon = 5$ to 10.

Step 5: Determine the new gain crossover frequency of the system for the phase margin $\left((p_{\mathrm{m}})_{\mathrm{d}}\right)_{\mathrm{new}}$. Let it be $(\omega_{\mathrm{gc}})_{\mathrm{new}}$.

Step 6: Determine the magnitude A of the system in dB from the magnitude plot corresponding to $(\omega_{\mathrm{gc}})_{\mathrm{new}}$.

Step 7: As the magnitude of system at the gain crossover frequency must be zero, the Bode plot must be either increased or decreased by A dB.

Step 8: The value of β in the transfer function of the lag compensator will be determined as

$$-20 \log \beta = \pm A$$

Here $+A$ is when the magnitude plot is to be increased and $-A$ is when the magnitude plot is to be decreased.

Step 9: The value of T in the transfer function of the lag compensator will be determined by using the equation, $\dfrac{1}{T} = \dfrac{(\omega_{\mathrm{gc}})_{\mathrm{new}}}{10}$.

Step 10: Thus, the transfer function of the lag compensator will be determined as

$$G_{\mathrm{la}}(s) = K_{\mathrm{c}} \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\beta T}}$$

Step 11: The transfer function of the compensated system will be $G_{\mathrm{c}}(s) = G_1(s) G_{\mathrm{la}}(s)$. If the Bode plot for the compensated system is drawn, it will satisfy the desired system requirements.

The flow chart for determining the parameters present in the transfer function of the lag compensator using Bode plot is shown in Fig. 11.6.

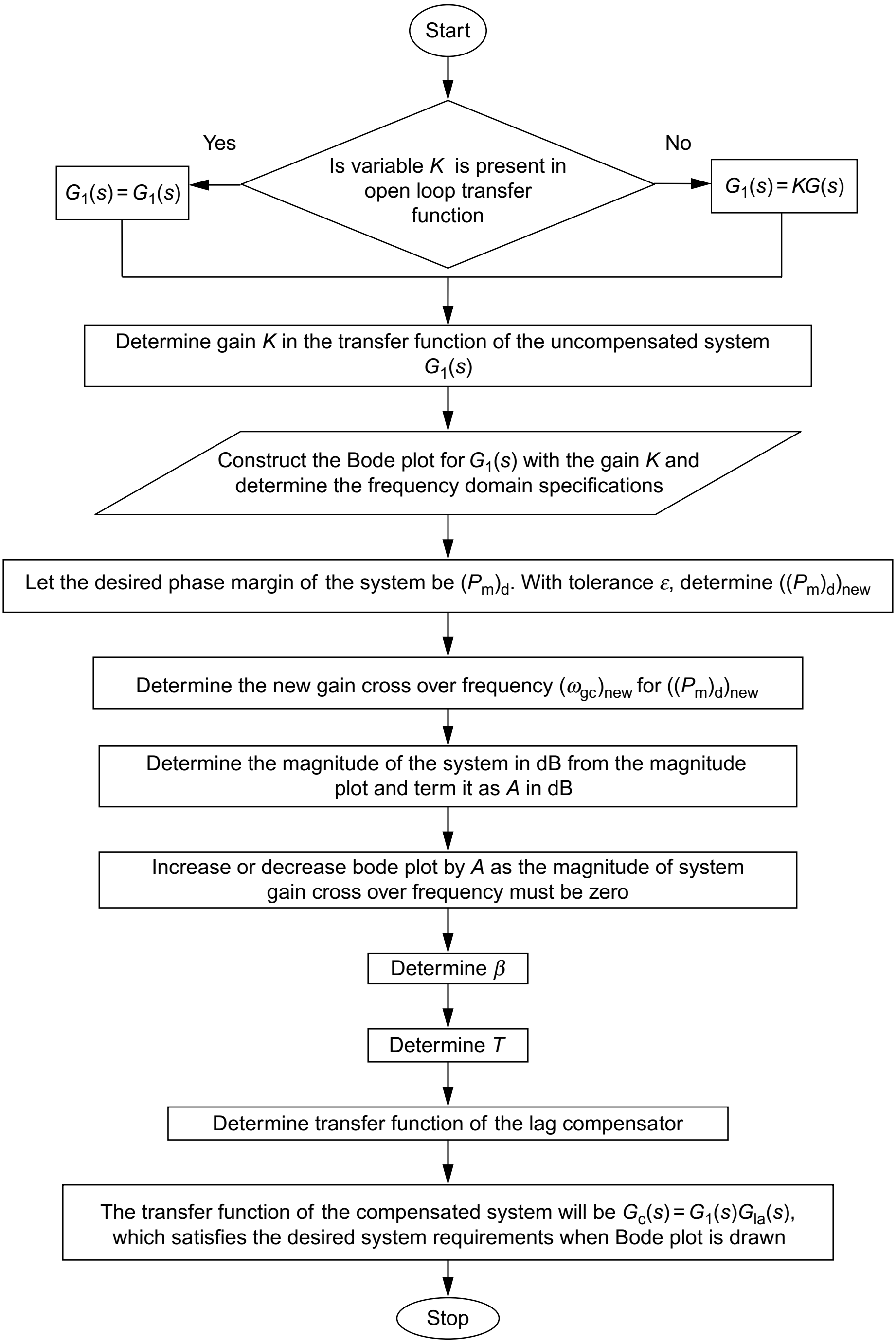


Fig. 11.6 | Flow chart for designing the lag compensator using Bode plot

Example 11.1 **Consider a unity feedback uncompensated system with the open-loop transfer function as $G(s) = \dfrac{5}{s(s+2)}$. Design a lag compensator for the system such that the compensated system has static velocity error constant K_v = 20 sec^{-1}, phase margin p_m = 55° and gain margin g_m = 12 dB.**

Solution

1. Let $G_1(s) = KG(s) = \dfrac{5K}{s(s+2)}$ and desired phase margin $(p_m)_d = 55^\circ$.
2. The value of K is determined by using $K_v = \underset{s \to 0}{Lt}\, sG_1(s)$ as

$$20 = \underset{s \to 0}{Lt}\, s\frac{5K}{s(s+2)}$$

Solving the above equation, we obtain $K = 8$.

3. The Bode plot for $G_1(s) = \dfrac{40}{s(s+2)}$ $H(s) = 1$ is drawn.

(a) Given $G_1(s)H(s) = \dfrac{40}{s(s+2)}$

(b) Substituting $s = j\omega$ and $K = 1$ in the above equation, we obtain

$$G_1(j\omega)H(j\omega) = \frac{20}{j\omega(1 + j0.5\omega)}$$

(c) The corner frequency existing in the given system is

$$\omega_{c1} = \frac{1}{0.5} = 2 \text{ rad/sec}$$

To sketch the magnitude plot:

(d) The changes in slope at different corner frequencies are given in Table E11.1(a).

Table E11.1(a) | Determination of change in slope at different corner frequencies

Term	Corner frequency ω rad/sec	Slope of the term in dB/decade	Change in slope in dB/decade
$\dfrac{1}{j\omega}$	–	−20	–
$\dfrac{1}{(1 + j0.5\omega)}$	$\omega_{c1} = 2$	−20	−20 − 20 = −40

(e) Assume the lower frequency as $\omega_l = 0.1$ rad/sec and higher frequency as $\omega_h = 20$ rad/sec.
(f) The values of gain at different frequencies are determined and given in Table E11.1(b).

Table E11.1(b) | Gain at different frequencies

Term	Frequency	Change in slope in dB	Gain A_i
$\frac{20}{(j\omega)}$	$\omega_1 = \omega_l = 0.1$	–	$A_1 = 20\log\left(\frac{20}{\omega_1}\right) = 46.02$ dB
$\frac{20}{(j\omega)}$	$\omega_2 = \omega_{c1} = 2$	–	$A_2 = 20\log\left(\frac{20}{\omega_2}\right) = 20$ dB
$\frac{1}{(1+j0.5\omega)}$	$\omega_3 = \omega_h = 20$	–40	$A_3 = \left[-40 \times \log\left(\frac{\omega_3}{\omega_2}\right)\right] + A_2 = -20$ dB

(g) The magnitude plot of the given system is plotted using Table E11.1(b) and is shown in Fig. E11.1.

To sketch the phase plot:

(h) The phase angle of the given loop transfer function as a function of frequency is obtained as

$$\phi = -90 - \tan^{-1}(0.5\omega)$$

(i) The phase angle at different frequencies is obtained using the above equation and the values are tabulated as shown in Table E11.1(c).

Table E11.1(c) | Phase angle of a system for different frequencies

Frequency ω rad/sec	$-\tan^{-1}(0.5\omega)$	Phase angle ϕ (in degree)
0.2	–5.71°	–95.71
2	–45°	–135
4	–63.53°	–153.43
10	–78.69°	–168.69
∞	–90°	–180

(j) The phase plot of a given system is plotted with the help of Table E11.1(c) and is shown in Fig. E11.1(a).

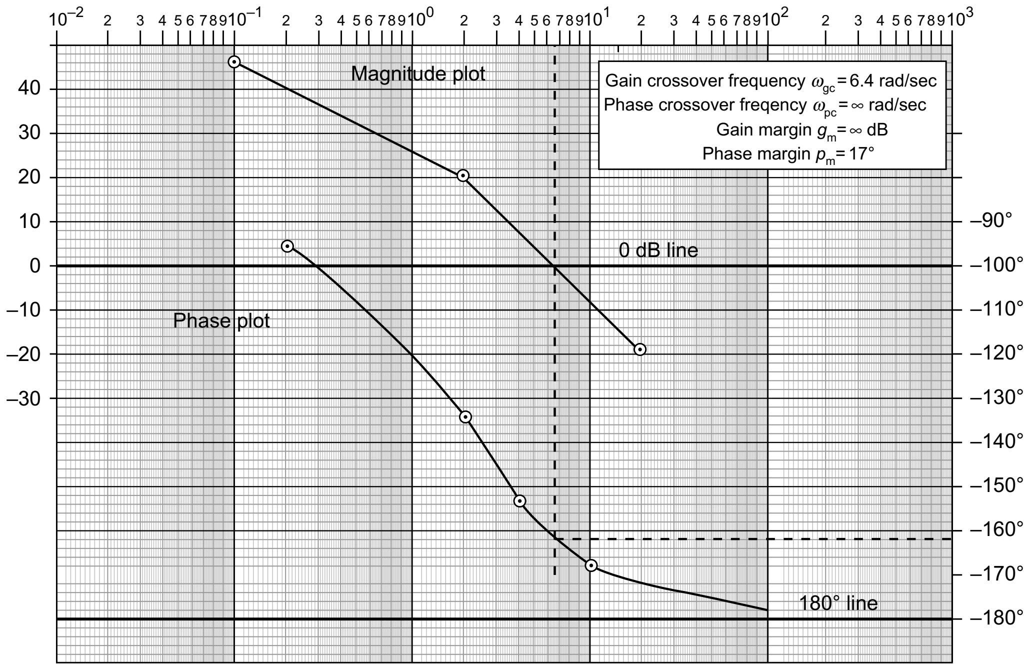


Fig. E11.1(a)

(k) The frequency domain specifications of the given system are:

Gain crossover frequency, $\omega_{gc} = 6.4$ rad/sec.

Phase crossover frequency, $\omega_{pc} = \infty$ rad/sec.

Gain margin, $g_m = \infty$ dB

Phase margin, $p_m = 17°$

It can be noted that the uncompensated system is stable, but the phase margin of the system is less than the desired phase margin which is 55°.

4. Let $\left((p_m)_d\right)_{new} = (p_m)_d + \varepsilon = 55° + 5° = 60°$.

5. The new gain crossover frequency for $\left((p_m)_d\right)_{new}$ is $(\omega_{gc})_{new} = 0.92$ rad/sec.

6. The magnitude of the system corresponding to $(\omega_{gc})_{new}$ is $A = 27$ dB.

7. The magnitude plot of the uncompensated system is decreased by 27 dB so that the gain at $(\omega_{gc})_{new}$ is zero dB.

8. The value of β in the lag compensator will be determined as

$$-20\log\beta = -27$$

i. e., $\beta = 22.38$

9. The value of T in the transfer function of the lag compensator is determined as

$$\frac{1}{T} = \frac{(\omega_{gc})_{new}}{10}$$

i. e., $T = 10.86$

10. Thus, the transfer function of the lag compensator is

$$G_{la}(s) = \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\beta T}\right)} = \frac{(s + 0.092)}{(s + 0.0041)}$$

11. Thus, the transfer function of the compensated system is $G_c(s) = \dfrac{40(s + 0.092)}{s(s+2)(s + 0.0041)}$

11.3.6 Design of Lag Compensator Using Root Locus Technique

When a system is desired to meet the static error constant alongwith other time-domain specifications such as peak overshoot, rise time, settling time, damping ratio of the system and undamped natural frequency of oscillation, then lag compensator will be designed using root locus technique. The step-by-step procedure for designing the lag compensator using root locus technique is discussed below:

Step 1: The root locus of an uncompensated system with the loop transfer function $G(s)H(s)$ is constructed.

Step 2: Determine θ using $\theta = \cos^{-1}(\xi)$.

Step 3: Draw a line from origin with an angle θ from the negative real axis and determine the point at which it cuts the root locus of the uncompensated system. Let that point be the dominant pole of the closed-loop system P.

Step 4: If the uncompensated system has a gain K, the gain K is determined by using the formula $\left|G(s)H(s)\right|_{s=P} = 1$, or else, we can proceed to the next step.

3. The gain K at dominant closed-loop poles is obtained using the magnitude condition,

$$\left|G(s)H(s)\right|_{s=-0.4+j0.6993} = 1$$

$$\frac{|K|}{|-0.4+j0.6993||0.6+j0.6993||3.6+j0.6993|} = 1$$

$$\frac{K}{0.8\times 0.9167\times 3.667} = 1$$

i. e., $$K = 2.6892.$$

Therefore, the transfer function of the compensated system is $G(s) = \dfrac{2.6892}{s(s+1)(s+4)}$.

4. The static error constant for the uncompensated system is

$$K_{\text{static}} = \underset{s\to 0}{Lt}\, sG(s) = \frac{2.6892}{1\times 4} = 0.6723.$$

5. The factor by which the static error constant to be increased is determined by using

$$\text{Factor} = \frac{K_{\text{desired}}}{K_{\text{static}}} = \frac{5}{0.6723} = 7.437$$

6. Let the zero of the compensator be at 0.1 and the pole of the compensator be at 0.01.
7. Therefore, the transfer function of the lag compensator is

$$G_{\text{la}}(s) = K_{\text{c}}\frac{(s+0.1)}{(s+0.01)}.$$

8. Thus, the transfer function of the compensated system is

$$G_c(s) = K_c\frac{(s+0.1)}{(s+0.01)}\times\frac{2.6892}{s(s+1)(s+4)} = \frac{K(s+0.1)}{s(s+1)(s+4)(s+0.01)}$$

where $K = 2.6892K_{\text{c}}$

11.4 Lead Compensator

The lead compensator is one that has a simple pole and a simple zero in the left half of the s-plane with the zero nearer to the origin. The term *lead* in the lead compensator refers that the output voltage leads the input voltage and the phase angle of the numerator of the transfer function is greater than that of denominator. The general transfer function of the lead compensator is given by

$$G_{le}(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, 0 < \alpha < 1 \tag{11.12}$$

where α, T and K_c are constants.

The pole-zero configuration of the lead compensator is shown in Fig. 11.8.

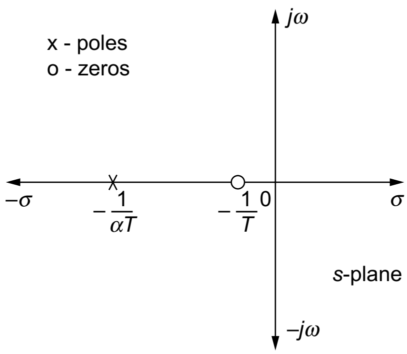
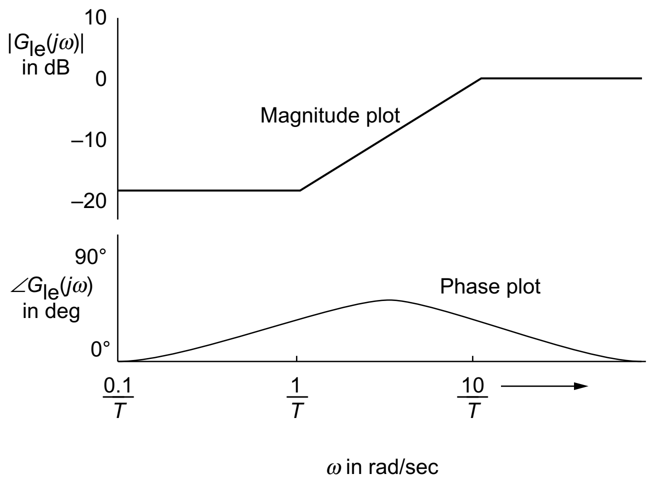


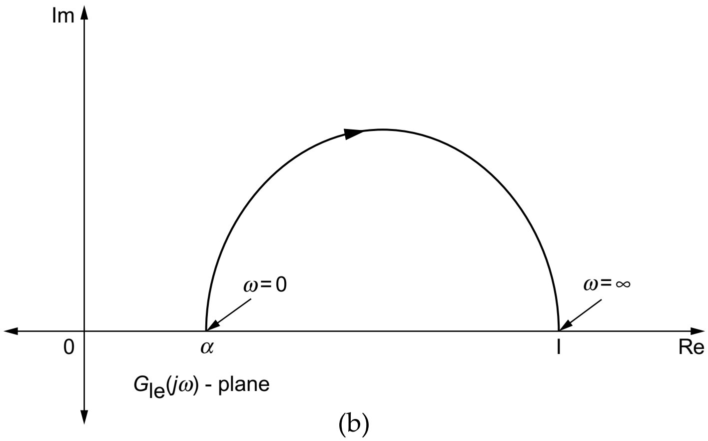
Fig. 11.8 | Pole-zero configuration of $G_{le}(s)$

The corner frequencies present in the lead compensator whose transfer function is given by Eqn. (11.12) are at $\omega = \frac{1}{T}$ and $\omega = \frac{1}{\alpha T}$. The Bode plot and polar plot of the lead compensator are shown in Figs. 11.9(a) and 11.9(b) respectively.



(a)

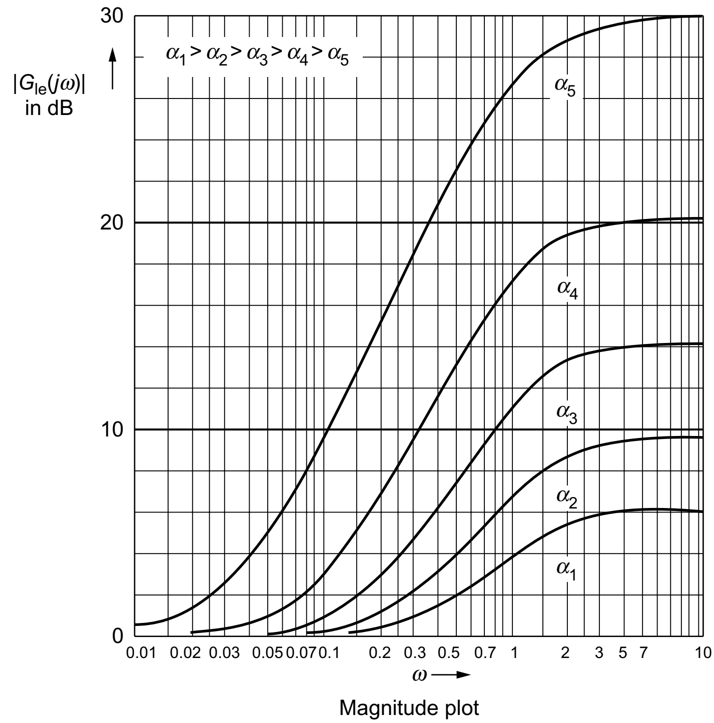
Fig. 11.9 | Plots of lead compensator



(b)

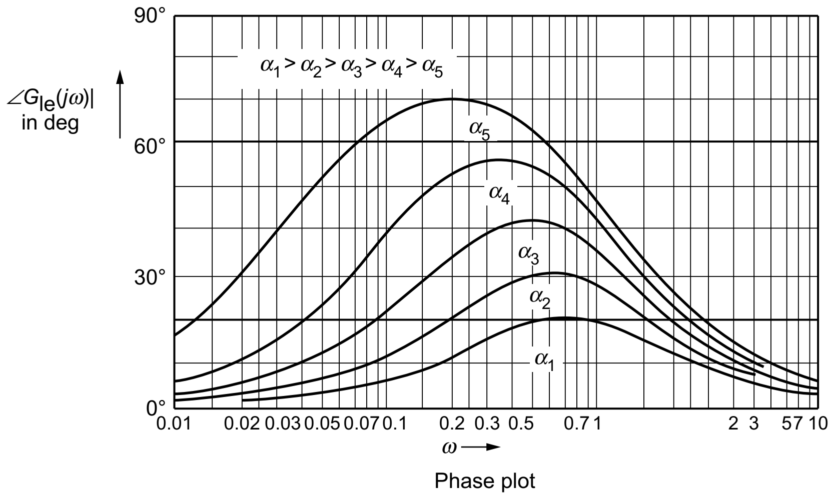
Fig. 11.9 | (Continued)

The Bode plot shown in Fig. 11.9(a) is plotted with $K_c = 1$ and $\alpha = 0.1$. It is inferred that (i) magnitude of lead compensator is low at low frequencies and (ii) magnitude of lead compensator is zero at high frequencies. Hence, the lead compensator behaves like a high-pass filter. The magnitude and phase plots for different values of α are shown in Figs. 11.10(a) and 11.10(b) respectively.



(a)

Fig. 11.10 | Bode plot for different values of α (a) magnitude plot and (b) phase plot



(b)

Fig. 11.10 | (Continued)

11.4.1 Determination of Maximum Phase Angle ϕ_m

The modified transfer function of the lead compensator is

$$G_{le}(s) = \alpha\left(\frac{1+sT}{1+s\alpha T}\right) \tag{11.13}$$

The magnitude and phase angle of the lead compensator are

$$M = \frac{\alpha\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\alpha^2T^2}} \tag{11.14}$$

and
$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T) \tag{11.15}$$

The maximum phase angle ϕ_m occurs at

$$\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_m} = 0$$

Differentiating Eqn. (11.15) with respect to ω, we obtain

$$\frac{T}{1+\omega_m^2T^2} - \frac{\alpha T}{1+\alpha^2\omega_m^2T^2} = 0$$

$$\frac{1}{1+\omega_m^2T^2} = \frac{\alpha}{1+\alpha^2\omega_m^2T^2}$$

$$\omega_m^2\left(\alpha T^2 - \alpha^2T^2\right) = 1-\alpha$$

Solving the above equation, we obtain

$$\omega_{\mathrm{m}} = \frac{1}{T\sqrt{\alpha}} = \left(\frac{1}{\sqrt{T}}\right)\left(\frac{1}{\sqrt{\alpha T}}\right) \tag{11.16}$$

Substituting the above equation in Eqn. (11.15), we obtain

$$\tan\phi_{\mathrm{m}} = \frac{1-\alpha}{2\alpha} \tag{11.17}$$

Thus, Eqn. (11.16) gives the frequency at which the phase angle of the system is maximum and Eqn. (11.17) gives the maximum phase angle of the lag compensator.

11.4.2 Electrical Representation of the Lead Compensator

A simple lead compensator using resistor and capacitor is shown in Fig. 11.11.

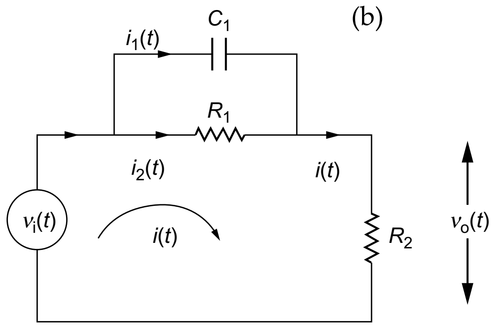


Fig. 11.11 | A simple lead compensator

Applying Kirchoff's current law to the above circuit, we obtain

$$i_1(t) + i_2(t) = i(t)$$

$$C_1 \frac{d\big(v_i(t) - v_0(t)\big)}{dt} + \frac{1}{R_1}\big(v_i(t) - v_0(t)\big) = \frac{1}{R_2} v_o(t) \tag{11.18}$$

Taking Laplace transform on both sides, we obtain

$$sC_1\big(V_i(s) - V_o(s)\big) + \frac{1}{R_1}\big(V_i(s) - V_0(s)\big) = \frac{1}{R_2} V_0(s) \tag{11.19}$$

$$sC_1 V_i(s) - sC_1 V_o(s) + \frac{1}{R_1} V_i(s) - \frac{1}{R_1} V_0(s) = \frac{1}{R_2} V_0(s)$$

$$\left(sC_1 + \frac{1}{R_1}\right) V_i(s) = \left(\frac{1}{R_2} + sC_1 + \frac{1}{R_1}\right) V_0(s)$$

Therefore, the transfer function of the above circuit is

$$\frac{V_0(s)}{V_i(s)} = \frac{\left(s + \dfrac{1}{R_1 C_1}\right)}{\left(s + \dfrac{1}{\left(\dfrac{R_2}{R_1 + R_2}\right) R_1 C_1}\right)} \tag{11.20}$$

Comparing Eqs. (11.12) and (11.20), we obtain

$$T = R_1 C_1 \text{ and } \alpha = \frac{R_2}{R_1 + R_2}$$

11.4.3 Effects of Lead Compensator

The effects of adding lead compensator to the given system are:

(i) Damping of the closed-loop system increases since a dominant zero is added to the system.
(ii) Peak overshoot of the system, rise time and settling time of the system decrease and as a result of which the transient response of the system gets improved.
(iii) Gain margin and phase margin of the system get increased.
(iv) Improves the relative stability of the system.
(v) Increases the bandwidth of the system that corresponds to the faster time response.

11.4.4 Limitations of Lead Compensator

The limitations of adding lead compensator to the given system are:

(i) Single-phase lead compensator can provide a maximum phase lead of 90°. If a phase lead of more than 90° is required, multistage compensator must be used.
(ii) There is always a possibility of reaching the conditionally stable condition even though the desired system requirements are achieved.

11.4.5 Design of Lead Compensator

The objective of designing the lead compensator is to determine the values of α and T for an uncompensated system $G_1(s)$ based on the desired system requirements. The design of lead compensator can be either based on the frequency domain specifications or time- domain specifications. The Bode plot is used for designing the lead compensator based on the frequency domain specifications and root locus technique is used for designing the lead compensator based on the time-domain specifications. Once α and T are determined, the transfer function of the lead compensator $G_{le}(s)$ can be obtained. The transfer function of the compensated system $G_c(s) = G_{le}(s) G_1(s)$. If the Bode plot or root locus technique is plotted for the compensated system, it will satisfy the desired system requirements.

11.4.6 Design of Lead Compensator Using Bode Plot

Let the open-loop transfer function of the uncompensated system be $G(s)$. The objective is to design a lead compensator $G_{le}(s)$ for $G(s)$ so that the compensated system $G_c(s)$ will satisfy the desired system requirements. The steps for determining the transfer function of the compensated system $G_{le}(s)$ using Bode plot are explained below:

Step 1: If the open-loop transfer function of the system $G(s)$ has a variable K, then $G_1(s) = G(s)$; otherwise $G_1(s) = KG(s)$.

Step 2: Depending on the input and TYPE of the system, the variable K present in the open-loop transfer function of the uncompensated system is determined based on either the steady-state error or the static error constant of the system.

The static error constants of the system are

$$\text{Position error constant, } K_p = \underset{s\to 0}{Lt}\, G_1(s)$$

$$\text{Velocity error constant, } K_v = \underset{s\to 0}{Lt}\, sG_1(s)$$

$$\text{Acceleration error constant, } K_a = \underset{s\to 0}{Lt}\, s^2G_1(s)$$

The relation between the steady-state error and static error constant based on the TYPE of the system and input applied to the system can be referred to Table 5.5 of Chapter 5.

Step 3: Construct the Bode plot for the uncompensated system with gain K obtained in the previous step and determine the frequency domain specifications of the system (i.e., phase margin, gain margin, phase crossover frequency and gain crossover frequency). Let the phase margin of the uncompensated system be p_m.

Step 4: Let the desired phase margin of the system be $(p_m)_d$. With a tolerance ε, determine $((p_m)_d)_{new}$ as

$$((p_m)_d)_{new} = (p_m)_d - p_m + \varepsilon$$

where $\varepsilon = 5$ to 10.

Step 5: Determine α using $\sin\left(\left(p_m\right)_d\right)_{new} = \dfrac{1-\alpha}{1+\alpha}$.

Step 6: Determine $-10\log\left(\dfrac{1}{\alpha}\right)$ in dB. Let it be A.

Step 7: Determine the frequency from the magnitude plot of the uncompensated system for the magnitude of AdB. Let this frequency be the new gain crossover frequency $(\omega_{gc})_{new}$.

Step 8: Determine T using $(\omega_{gc})_{new} = \dfrac{1}{T\sqrt{\alpha}}$.

Step 9: Determine K_c using $K = K_c\alpha$.

Step 10: Thus, the transfer function of the lead compensator will be determined as

$$G_{le}(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = K_c\alpha\left(\frac{1+sT}{1+s\alpha T}\right)$$

Step 11: The transfer function of the compensated system will be $G_c(s) = G(s)G_{le}(s)$ $= K_c\alpha\left(\frac{1+sT}{1+s\alpha T}\right)G_{le}(s)$. If the Bode plot for the compensated system is drawn, it will satisfy the desired system requirements.

Flow chart for designing the lead compensator using Bode plot

The flow chart for determining the parameters present in the transfer function of the lead compensator using Bode plot is shown in Fig. 11.12.

Example 11.3: Consider a unity feedback uncompensated system with the open-loop transfer function as $G(s) = \frac{K}{s(s+1)}$. Design a lead compensator for the system such that the compensated system has static velocity error constant K_v = 12sec^{-1} and phase margin $p_m = 40°$.

Solution:

1. Let $G_1(s) = G(s) = \frac{K}{s(s+1)}$.
2. The value of K is determined by using $K_v = \underset{s\to 0}{Lt}\, sG_1(s)$ as

$$12 = \underset{s\to 0}{Lt}\, s\frac{K}{s(s+1)}$$

Solving the above equation, we obtain $K = 12$.

3. The Bode plot for $G_1(s) = \frac{12}{s(s+1)}$ $H(s) = 1$ is drawn.

(a) Given $G_1(s)H(s) = \frac{12}{s(s+1)}$

(b) Substituting $s = j\omega$ in the above equation, we obtain

$$G_1(j\omega)H(j\omega) = \frac{12}{j\omega(1+j\omega)}$$

(c) The corner frequency existing in the given system is

$$\omega_{c1} = 1 \text{ rad/sec}$$

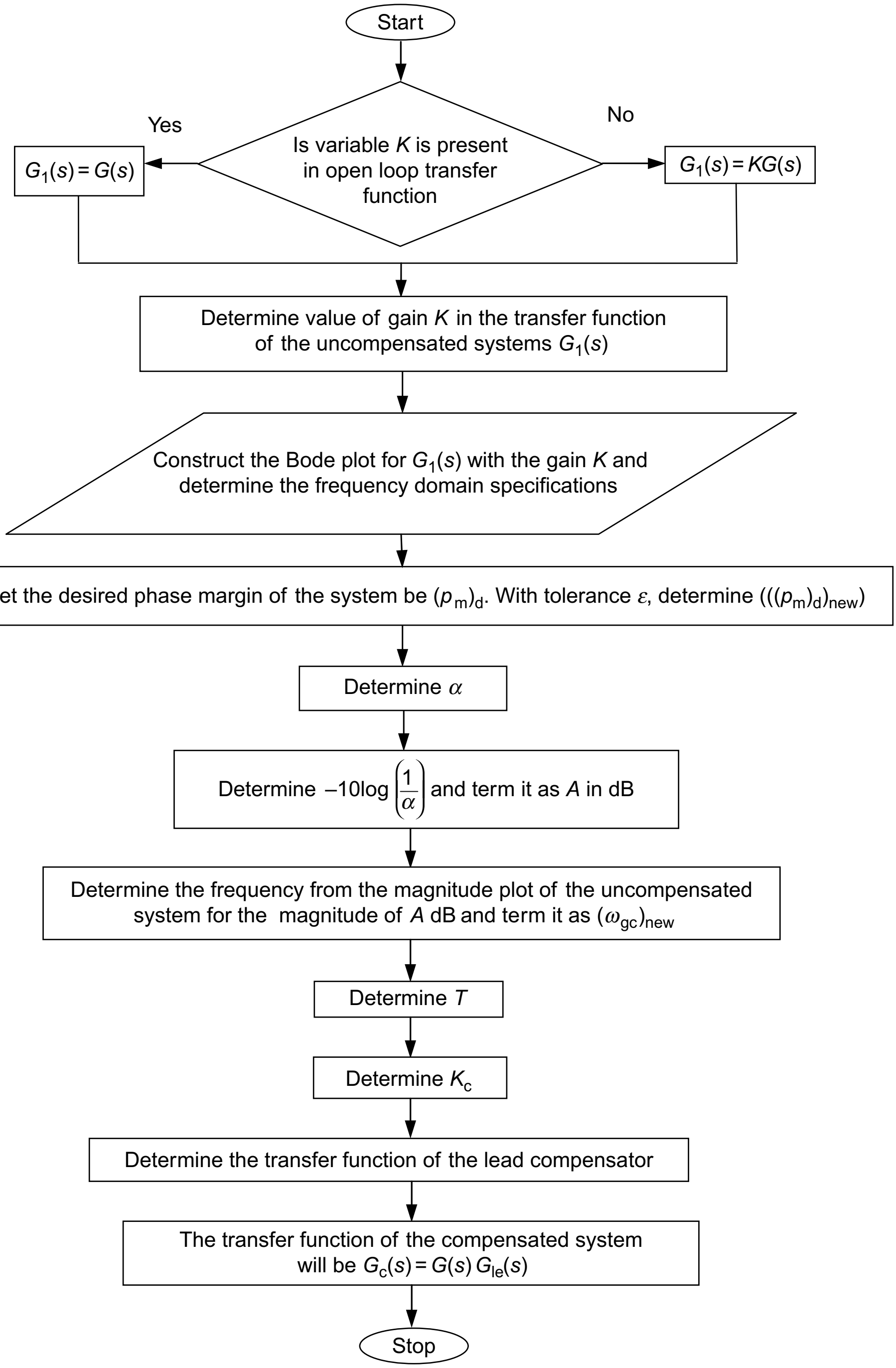


Fig. 11.12 | Flow chart for designing the lead compensator using Bode plot

To sketch the magnitude plot:

(d) The changes in slope at different corner frequencies are given in Table E11.3(a).

Table E11.3(a) | Determination of change in slope at different corner frequencies

Term	Corner frequency ω rad/sec	Slope of the term in dB/decade	Change in slope in dB/decade
$\frac{1}{j\omega}$	–	–20	–
$\frac{1}{(1+j\omega)}$	$\omega_{c1}=1$	–20	$-20-20=-40$

(e) Assume the lower frequency as ω_l = 0.1 rad/sec and higher frequency as ω_h = 20 rad/sec.

(f) The values of gain at different frequencies are determined and given in Table E11.3(b).

Table E11.3(b) | Gain at different frequencies

Term	Frequency	Change in slope in dB	Gain A_i
$\frac{12}{(j\omega)}$	$\omega_1=\omega_l=0.1$	–	$A_1=20\log\left(\frac{12}{\omega_1}\right)=41.58\ \text{dB}$
$\frac{12}{(j\omega)}$	$\omega_2=\omega_{c1}=1$	–	$A_2=20\log\left(\frac{12}{\omega_2}\right)=21.58\ \text{dB}$
$\frac{1}{(1+j\omega)}$	$\omega_3=\omega_h=20$	–40	$A_3=\left[-40\times\log\left(\frac{\omega_3}{\omega_2}\right)\right]+A_2=-30.46\ \text{dB}$

(g) The magnitude plot of the given system is plotted using Table E11.3(b) and is shown in Fig. E11.3.

To sketch the phase plot:

(h) The phase angle of the given loop transfer function as a function of frequency is obtained as

$$\phi=-90-\tan^{-1}(\omega)$$

(i) The phase angle at different frequencies is obtained using the above equation and the values are tabulated as shown in Table E11.3(c).

Table E11.3(c) | Phase angle of the system for different frequencies

Frequency ω rad/sec	$-\tan^{-1}(\omega)$	**Phase angle ϕ (in degree)**
0.2	-11.30°	–101.3
2	-63.43°	–153.43
5	-75.69°	-165.69°
10	-84.28°	–174.28
∞	-90°	–180

(j) The phase plot of the given system is plotted with the help of Table E11.3(c) and is shown in Fig. E11.3.

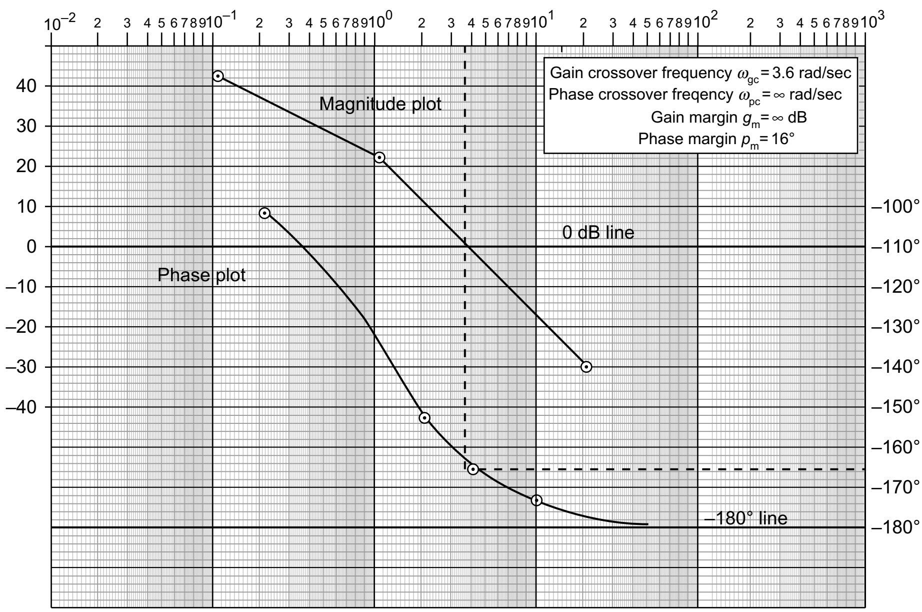


Fig. E11.3

(k) The frequency domain specifications of the given system are

Gain crossover frequency, $\omega_{gc} = 3.6$ rad/sec

Phase crossover frequency, $\omega_{pc} = \infty$ rad/sec

Gain margin, $g_m = \infty$ dB and

Phase margin, $p_m = 16^{\circ}$

It can be noted that the uncompensated system is stable, but the phase margin of the system is less than the desired phase margin.

4. With tolerance value $\varepsilon = 5^{\circ}$, $\left((p_{\mathrm{m}})_{\mathrm{d}}\right)_{\mathrm{new}}$ is calculated as

$$\text{i. e., } \left((p_{\mathrm{m}})_{\mathrm{d}}\right)_{\mathrm{new}} = 40^{\circ} - 16^{\circ} + 5^{\circ} = 29^{\circ}.$$

5. The value of α in the lead compensator is determined by using, $\sin(29^{\circ}) = \dfrac{1-\alpha}{1+\alpha}$.

$$\text{i.e., } 0.484(1+\alpha) = 1-\alpha$$

$$0.484 + 0.484\alpha = 1-\alpha$$

Solving the above equation, we obtain

$$\alpha = 0.3477$$

6. Let $B = -10\log\left(\dfrac{1}{\alpha}\right) = -4.58$ dB

7. The new gain crossover frequency $(\omega_{\mathrm{gc}})_{\mathrm{new}}$ corresponding to B from the magnitude plot of the uncompensated system is 5 rad/sec.

8. Using $(\omega_{\mathrm{gc}})_{\mathrm{new}} = \dfrac{1}{T\sqrt{\alpha}}$, determine T as $5 = \dfrac{1}{T\sqrt{0.3477}}$.

$$\text{i. e., } T = 0.339.$$

9. The value of K_{c} is determined as $12 = K_{\mathrm{c}}(0.3477)$ i.e., $K_{\mathrm{c}} = 34.51$.

10. The transfer function of the lead compensator is

$$G_{\mathrm{le}}(s) = K_{\mathrm{c}}\frac{\left(s+\dfrac{1}{T}\right)}{\left(s+\dfrac{1}{\alpha T}\right)} = 12\frac{(1+0.339s)}{(1+0.1178s)}$$

11. Thus, the transfer function of the compensated system is

$$G_{\mathrm{c}}(s) = 12\frac{(1+0.339s)}{s(s+1)(1+0.1178s)}$$

11.4.7 Design of Lead Compensator Using Root Locus Technique

The desired time-domain specifications that can be specified for designing the lead compensator are peak overshoot, settling time, rise time, damping ratio and undamped natural frequency of the system. The step-by-step procedure for designing the lead compensator for