# 15MAT213- PROBABILITY & RANDOM PROCESSES

## ANSWER KEY

## FIRST ASSESSMENT-DEC 2019

1) $E_i$ - person passing the $i^{th}$ test

E - event that he is selected.

$P(E) = P(E_1)P(E_2|E_1) +$
$P(E_1).P(\bar{E}_2|E_1)P(E_3|\bar{E}_2) +$
$P(\bar{E}_1)P(E_2|\bar{E}_1)P(E_3|E_2)$ (2)

$= p.p + p(1-p)p/2 + (1-p)p/2\,p$

$= 2p^2 - p^3$ — (3)

2) $\sum p(x=k) = 1$

(i) $\sum_{k=0}^{5} \frac{(k+1)}{2^k} a = 1 \Rightarrow a = \frac{4}{15}$ (2m)

$= 0.267$

(ii) $P(x \le 3) = p(0) + p(1) + p(2) + p(3)$

$= 13/15 = 0.867$ (3m)

3) (i) $P(x \ge 12.6) = \int_{12.6}^{\infty} 20 e^{-20(x-12.5)} dx$

$e^{-2}$

$= 0.135$ (3m)

(ii) $P(x \le 12.6) = 1 - P(x > 12.6)$

$= 0.865$ (2m)

(iii) $F(x) = 1 - e^{-20(x-12.5)}$, $x \ge 12.5$ (2m)

4) M.G.F of Binomial dist'n

$M_x(t) = \sum_{x=0}^{n} e^{tx}\, nC_x\, p^x q^{n-x}$

$= (q+pe^t)^n$ — (3)

Mean $= E(x) = M_x'(0) = np$ — (2m)

$E(x^2) = M_x''(0) = np + n^2p^2 - np^2$

$Var = E(x^2) - [E(x)]^2$

$= npq$. (3m)

5) A – test indicates corrosion

$P(A|E_1) = 0.7$ ; $P(A|E_2) = 0.2$

$P(E_1) = 0.1$ ; $P(E_2) = 0.9$ (2)

(a) $P(A) = 0.25$ — (3m)

(b) $P(E_1|A) = \frac{P(E_1).P(A|E_1)}{\sum P(E_i).P(A|E_i)}$ (3m)

$= \frac{0.07}{0.25} = 0.28$.

$\frac{7}{25}$

6) Let $x$ denote no. of companies to which the engineer is called.

$p = 0.1,\ n = 4,\ q = 0.9$

$p(x) = 4C_x (0.1)^x (0.9)^{4-x},\ x = 0, 1, \ldots 4$ — (1).

(i) $P(X=4) = (0.1)^4 = 0.0001$ — 1

(ii) $P(X \geq 3) = p(3) + p(4)$

.3439 $= 0.0037$ — 2

(iii) $P(X=4 / X \geq 1) = \dfrac{P(X=4)}{P(X \geq 1)}$ — 2

$= 0.0003$

(iv) mean $= np = 0.4$

var $= npq = 0.36$ — 2

7) Let X denote the no. of winning tickets.

$n = 100\ ;\quad p = \dfrac{100}{10{,}000} = 0.01$ — (2)

$\lambda = np = 1$

$p(x) = \dfrac{e^{-\lambda}\lambda^x}{x!} = \dfrac{e^{-1}(1)^x}{x!} = \dfrac{e^{-1}}{x!}$ — (2)

(i) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - p(0) = 1 - e^{-1}$

$= 0.632$ — (2)

(ii) $P(X \geq 1) \geq 0.95$

$1 - e^{-\lambda} \geq 0.95$

$e^{-\lambda} < 0.05$

$\lambda \geq 3$

$np \geq 3$

$n \geq 300.$ — (2)

One should buy atleast 300 tickets to have 95% confident of having a winning ticket.