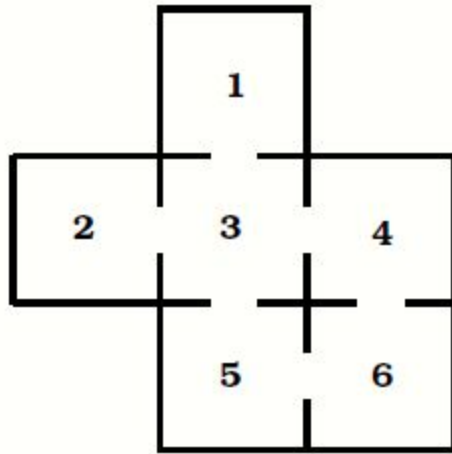# Assignment Problems for ASM

1. A rat runs through the maze shown below. At each step it leaves the room it is in by choosing at random one of the doors out of the room.

   (a) Give the transition matrix P for this Markov chain. (b) Show that it is irreducible but not aperiodic. (c) Find the stationary distribution (d) Now suppose that a piece of mature cheddar is placed on a deadly trap in Room 5. The mouse starts in Room 1. Find the expected number of steps before reaching Room 5 for the first time, starting in Room 1. (e) Find the expected time to return to room 1.

2. Show that a Markov chain with transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$$

   has more than one stationary distributions. Find the matrix that P^n converges to, as $n \to \infty$, and verify that it is not a matrix all of whose rows are the same.

3. Build a Markov chain as follows: When in state k (k = 1, 2, 3, 4, 5, 6), roll a die k times, take the largest value and move to that state. (i) Compute the transition probabilities and write down the transition probability matrix. (ii) Is the chain aperiodic? (iii) Does it have a unique stationary distribution? (iv) Can you find which state will be visited more frequently on the average?

4. Simulate the famous Buffon's needle problem using monte carlo method. Buffon's needle problem asks to find the probability that a needle of length 'l' will land on

a line, given a floor with equally spaced parallel lines a distance 'd' apart. The problem was first posed by the French naturalist Buffon in 1733 (Buffon 1733, pp. 43-45), and reproduced with solution by Buffon in 1777 (Buffon 1777, pp. 100-104).

5.   Determine the value of 'e' using Monte Carlo method.