1. You are going to build a retail bank. Bank's job is to maintain right balance between asset and liability. It takes common people's money and pay them interest. On the other side, it issues loans to business world and others for higher interest rate and thus sustain in the banking system. We are aware that the greatest threat to a bank now is NPA (non-performing asset). There are many components to it but for simplicity let's assume that only the loans which cant be recovered goes to NPA. It is highly important for the banks to keep the right balance between the loans that will be returned in full and NPAs. Let's assume the loans that your bank will issue can be categorized in four classes.

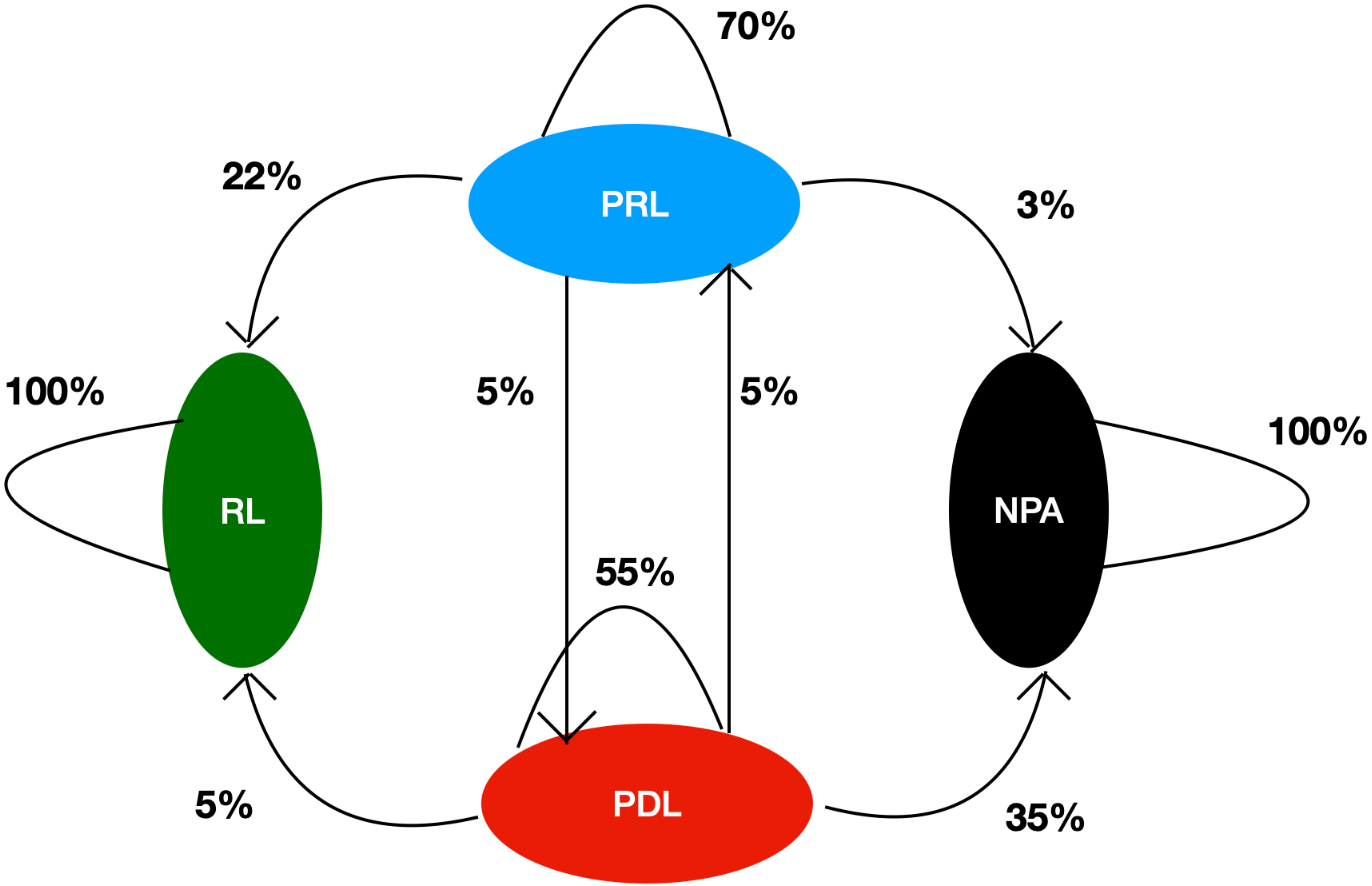
1. Potentially Recoverable Loan (PRL): These loans are given to potentially well earning and profit making individuals. These loans have high probability of getting fully paid in time.

2. Potentially Defaultable Loan (PDL): These loans are provided to individuals who are not so convincing in returning. These loans have a good probability that they might go to NPAs.

3. NPAs: These loans wont ever be paid off.

4. Recovered Loans (RL): These loans have already been fully recovered in time.

You hired a market research consultant to do the financial research for you on historical trends of loan transitions and the Markov Chain they came up with in this scenario is shown below (time period for transition: 1 year):



**Use simulation of Markov Chain to find a sustainable initial probability distribution. Please remember that initially only PRL and PDL loans can be provided. So suggest a sensible initial probability vector [p1,p2,0,0], where Prob(PRL)=p1, Prob(PDL)=p2 by trying different initial probability vector in simulation and seeing how the steady state probability looks like.**

2. Use simulation to support the following interesting phenomenon regarding Prime numbers and Poisson distribution.

THEOREM 2.9. *(Prime Number Theorem): The number of primes $p \leq x$ has size about $x/\log x$, where here the log is to the base-e. That is, if $\pi(x)$ denotes this number of primes less then or equal to $x$, then*

$$\pi(x) \sim x/\log x.$$

*This means that*

$$\lim_{n\to\infty} \frac{\pi(x)}{x/\log x} = 1.$$

In what follows, we will use the notation $\pi(I)$ to denote the number of primes in a given interval $I$.

Now suppose that $x$ is a "really large" number , say $x = 10^{100}$. Then, if you pick a number $n \leq x$ at random, there is about a $1/100\log(10)$ (about 0.43 percent) chance that it will be prime; and we expect that a typical interval $[n, n+\log n] \subset [1, x]$ will contain about one prime.

In fact, much more is true. If we pick $n \leq x$ at random, and choose $\lambda > 0$ and $j$ not too large (say $\lambda, j \leq 20$), then the number of primes in $[n, n+\lambda \log n]$ roughly obeys a Poisson distribution:

$$P\{\pi([n, n+\lambda \log n]\} = j \approx \frac{e^{-\lambda}\lambda^j}{j!}.$$

Notice that we do not have an equality; in order to get an equality we would have to let $x \to \infty$ in some way. Certainly the larger we take $x$ to be, the close the above probability comes to

$$\frac{e^{-\lambda}\lambda^j}{j!}$$

3. Assuming that Corona count in the world for a day follow Poisson Process, try to give a reasonable fit to the available Corona count data into a Poisson Process. Show graphs for different simulation to support your model. Try finding λ by trial and error method.

Hint: Try simulating different Poisson processes and see for which λ the data looks a reasonable fit for a day. Take available data from internet for any day you like or prepare a reasonable real time corona count data in the world for any given day. The problem is very open for imagination, you are free to consider any reasonable assumption. Following documents are provided to help in simulation.

http://www.math.uchicago.edu/~may/VIGRE/VIGRE2010/REUPapers/Mcquighan.pdf

https://transp-or.epfl.ch/courses/OptSim2012/slides/05b-poisson.pdf