Homework 5
(due Tuesday, March 12th, 2019, 02:00)

1. Two-dimensional patterns with $x = (x^{(1)}, x^{(2)})$ are placed at $x_1 = (1, 0)$, $x_2 = (-1, 0)$, $x_3 = (0, 1)$ and $x_4 = (0, -1)$. You are given the classification $x_1, x_2 \in C_1$ and $x_3, x_4 \in C_2$. In addition, we also add $100 - 4$ random 2D patterns (giving us a total of 100 patterns with 50 patterns in each class) obeying the criteria

$$\begin{aligned} &\text{If } |x^{(1)}| > 1 \text{ and } |x^{(2)}| < 1, \text{ then } x \in C_1, \\ &\text{If } |x^{(1)}| < 1 \text{ and } |x^{(2)}| > 1, \text{ then } x \in C_2, \\ &\qquad \text{Other locations are not permitted.} \end{aligned}$$

Implement a fully connected multi-layer backpropagation network from scratch using sigmoidal units (including in the output layer and using cross-entropy as the output measure). (You may follow the excellent tutorial using notation very similar to the class in `https://www.bogotobogo.com/python/python_Neural_Networks_Backpropagation_for_XOR_using_one_hidden_layer.php`.) Report on the minimum number of layers required for obtaining 100% classification while using three hidden units per layer. Also report all weights and the corresponding hidden unit rules (for arbitrary inputs). Other parameters (sigmoidal slope parameter, learning rate, number of epochs for convergence etc.) should also be recorded and reported. Compare your results with TensorFlow/Keras using a close to identical network.