Models for Aircraft Landing Optimization

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***Abstract*—Due to an anticipated increase in air traffic during the next decade, air traffic control in busy airports is one of the main challenges confronting controllers in the near future. Since the runway is often a bottleneck in an airport system, there is great interest in optimizing usage of the runway. Our study first presents a brief review of the aircraft landing problem. A model for the problem is then introduced, and possible solution approaches are discussed.**

I. Introduction

Airport capacity (and hence runways and gates) is increasingly becoming a limiting factor in meeting the rising demand for more flights. One of the main factors that determines the runway throughput at airports is the required separation between aircraft during take-off and landing. Dependency of separation on the leading and trailing aircraft and the type of aircraft add to the challenge of solving the sequencing and scheduling problem. Due to its complexity, it is hard to find the optimal solution to the problem in most cases. Thus, it draws significant attention from different scientific communities with numerous research studies carried out on modelling and developing algorithms to increase capacity at an airport.

Since the appearance of a paper of Blumstein [1] on estimating the capacity of an arrivals runway, there have been a variety of studies on airport runway optimization. Although the literature during the last three decades contains more than sixty publications on aircraft landing optimization, the majority of the proposed methods have never been implemented.

The Aircraft Landing Problem (ALP), which is the focus of our work, is to sequence landing aircraft onto the available runways at an airport and to assign each aircraft a landing time, subject to variety of operational constraints. The simple way of sequencing and scheduling of landing aircraft on a single runway is using the First-Come-First-Served (FCFS) approach which assigns scheduled landing time to each aircraft based upon the sequence implied by the earliest time that the aircraft can land. It has been found that FCFS is rarely the best sequencing order in terms of capacity, average delay or even average passenger delay [2].

The prime responsibility of the air traffic controllers is safety of the flights. Standard vertical and horizontal separations which keep flights apart provide one of the main Air Traffic Control (ATC) safety devices. Vortices generated by the aircraft as a consequence of their lift are one of the reasons for imposing separations. The minimum required separation governs the minimum permissible distance interval between aircraft lined up in sequence on the approach to landing on the runway. Generally, the Wake Vortex (WV) separation required between consecutive aircraft depends on the type of the aircraft, and therefore it is sequence dependent. During peak capacity operations, the WV is often a major concern. It effectively determines runway throughput and thus limits airport capacity in the terminal airspace. Significant asymmetries in minimum required separations can offer an opportunity to reduce airborne delays by shifting aircraft positions in the landing sequence, although finding the best way to achieve this presents a challenge.

There are well-known procedures for making an aircraft wait to land, such as using Vector-for-Space (VFS) or Holding Pattern (HP) [3]. Nevertheless, reassigning an aircraft to landing time far from its initial place in FCFS sequence is not always feasible because of the operational constraints in practice. The *Constrained Position Shifting* (CPS) concept is introduced by Dear [4] for the ALP to prevent the final positions of aircraft in landing sequence from differing from the FCFS order by more than a pre-specified number, called the *Maximum Position Shift* (MPS). Furthermore, when the MPS is small, it maintains fairness among the aircraft by not deviating too far from the FCFS sequence. *Relative Position Shifting* (RPS) is a variant of MPS that takes into account the closeness of the aircraft to the runway when specifying the MPS.

This paper focuses on the techniques and tools of operational research and management science (OR/MS) for solving the ALP. Section II is a brief review of the literature on the use of OR/MS techniques for tackling the ALP. A mixed integer programming (MIP) model is proposed in Section III. Finally, future research directions are discussed in Section IV.

II. Literature Review

Most previous research on the ALP considers a static (off-line) environment based on a given set of aircraft operating over a predefined time horizon. However, a more realistic model considers a dynamic (on-line or real-time) environment, where solutions are revised as aircraft arrive over time and new information becomes available.

A variety of OR/MS techniques have been used to model the problem, such as mixed integer programming (MIP) and queueing theory, while commonly used approaches for machine scheduling and travelling salesman problems (TSP) are also useful in the development of solution algorithms. Various search techniques have been also applied to solve the problem such as Dynamic Programming (DP), Branch-and-Bound, Branch-and-Price, Genetic Algorithms/Programming, Ant Colony Optimization, etc.

Based on a DP approach for solving the TSP, Psaraftis [5] develops three algorithm for the static case of the ALP to examine two alternative objectives, the landing time of the last aircraft, and the total passenger delay with respect to FCFS discipline. The CPS concept is also incorporated within the algorithm. Bianco et al. [6] point out the relationship between the ALP, a machine scheduling problem (denoted by $1|r_j, \text{seq-dep}\,|\sum C_j)$ with n jobs and the cumulative TSP with ready times. A dynamic programming formulation and three lower bounds are proposed for the scheduling problem.

The scheduling of aircraft landing is formulated as a single machine scheduling problem in Bayen et al. [7]. A DP approach and a linear programming relaxation and rounding are used in the main algorithm. Two approximation algorithms for the sum of arrival times of all aircraft and the arrival time of the last aircraft have performance bounds of 5 and 3, respectively. Because different classes of aircraft are not considered, the required separation between landings is independent of the aircraft type.

Recently, Balakrishnan and Chandran [8] present a DP-based approach to maximize the runway throughput (equivalently, minimizing the landing time of the last aircraft). The problem of scheduling landing aircraft in a CPS environment is considered subject to various operational constraints imposed by arrival time windows, minimum aircraft separation requirements, and precedence relations. The problem is formulated as a modified shortest path problem on a network with $O(n(2k+1)^{2k+2})$ arcs, where n is the number of aircraft and k is the maximum position shift.

Chandran and Balakrishnan [9] also propose a DP algorithm to compute the tradeoff curve between the robustness (reliability of a schedule) and throughput based on their earlier work [8]. More recently, Lee and Balakrishnan [10] extend the previous framework [8], [9] and present a DP algorithm for minimizing the total delay costs of an arrival schedule. Also, the problem of minimizing the fuel costs of the arrival schedule as the main operating cost for most airline, has been studied using the proposed algorithm by allowing the earliest landing time to be less than the estimated landing times which is known as Time Advance (TA). Several polynomial-time DP algorithms for the ALP based on machine scheduling concepts are presented in [11], [12]. Six sequencing algorithms which include three DPs, two FCFS rules, and a heuristic that represents a potential algorithm for an operational AMAN (Arrival Manager) system are implemented. Moreover, four delay sharing strategies include all delay in hold, delay as late as possible, delay as early as possible, and delay evenly throughout the route strategies are implemented.

The literature also contains several branch-and-bound algorithms for the ALP. For example, in addition to an extensive literature overview on the ALP, Beasley et al. [13] develop algorithms for both single and multiple runways. The model is based on an earlier MIP formulation presented in [14].

Different metaheuristic approaches have been examined for scheduling landing aircraft. One of the first and the simplest application of a GA in minimizing the earliness/lateness for the ALP is investigated by Stevens [15]. Based on his work on the permutation-based approach, Ciesielski and Scerri [16], [17] compare two GAs for a real-time dynamic ALP in terms of the percentage of valid solutions and best fitness by specifying a 30-second time slot and variable times between landings. Cheng et al. [18] design four different GAs for solving the multiple runway ALP.

A Population Heuristic (PH) is developed by Beasley et al. [19] to improve the utilization of a single runway. The algorithm aims to minimize the squared deviations from estimated landing time in the presence of five separation criteria. Later, Hansen [20] examines the efficiency and effectiveness of various genetic approaches for the ALP. Regarding the objective function, three different formulations are presented by Capri and Ignaccolo [2] with respect to minimizing the delays which depend on the aircraft classes, maximizing the system capacity, and minimizing the sum of landing times.

The Receding Horizon Control (RHC)-based GA that is introduced by Hu and Chen [21] minimizes the airborne delay, which is the deviation of actual landing time from estimated landing time in a dynamic environment. Hu and Paolo [22], [23] experiment with alternative solution representations in a single and multiple runway ALP. Two different population heuristics, Scatter Search and the Bionomic Algorithm, are applied by Pinol and Beasley [24] to the multiple runway ALP. Both linear and non-linear objective functions are considered.

Ant Colony Optimization (ACO), which is a constructive metaheuristic technique with biological foundation, has been applied to the ALP by Randall [25]. His algorithm aims to minimize the difference between an estimated landing time and the actual landing time for each aircraft, subject to a specified time window and separation criteria.

Dynamic programming exhibits the best performance among exact methods because of its enumerative nature. However, the ability to control run time in local search methods such as GAs makes them serious candidates for use in decision support tools for air traffic controllers.

III. Problem Definition

The majority of research on the ALP considers sequencing the aircraft in the Terminal Manoeuvering Area (TMA). However, sequencing aircraft further away from the airport (such as Extended TMA) may produce better results. The problem is divided into three time stages.

- Stage 1 (*Sequencing Stage*): The first stage starts by entering the aircraft into the airport landing planner's radar range about 40 minutes before touchdown.
- Stage 2 (*Modifying Schedule Stage*): The second stage starts 11 minutes before landing and takes 8 minutes and

includes the final approach step.

- Stage 3 (*Freezing Stage*): The last stage consists of the runway controller's range of operation which is 3 minutes long.

Sequencing and scheduling of arrival aircraft are performed in *stage 1*. As time progresses and new aircraft enter the sequencing stage, the sequence and schedule have to be updated, which is done every five minutes. In *stage 2*, the sequence is not usually changed, with only minor adjustments to the schedules being made. As the aircraft is so close to the runway in *stage 3*, neither the sequence nor the schedule can be modified.

A. Notation

Decision Variables

- SLT_i The scheduled landing time of each aircraft i.
- X_{ij} Defined to be 1 if aircraft i lands before (not necessarily immediately) aircraft j, and 0 otherwise.

Parameters

- n The number of aircraft to be scheduled.
- A The set of available aircraft for landing, $A = \{1, \dots, n\}$, which is updated every 5 minutes.

The parameters below are defined for each aircraft i, for $i = 1, \dots, n$.

- TLT_i The target (or expected) landing time of aircraft i based on the assigned time slot which is normally specified in flight plan.
- Est_i The estimated (or predicted) landing time of aircraft i calculated by trajectory synchronizer equipment after entering the aircraft into the radar range, and is normally based on the FCFS sequence.
- Al_i^- The allowed earliness for aircraft i to land.
- Al_i^+ The allowed lateness for aircraft i to land.
- ELT_i The earliest possible landing time for aircraft i, subject to technical and operational restrictions.
- LLT_i The latest possible landing time for aircraft i which is usually determined from fuel limitation, maximum allowed delay, or meeting a connecting flight.
- E_i The earliness penalty cost per unit for aircraft i to be advanced more than Al_i^-.
- L_i The lateness penalty cost per minute for aircraft i to be delayed more than Al_i^+.
- P_{ij} Defined to be 1 if aircraft i must land before (not necessarily immediately) aircraft j, and 0 otherwise.
- S_{ij} The minimum time separation between aircraft i and j, if aircraft i lands before aircraft j.
- TS_i The time shifting of aircraft i, which is the maximum time deviation of this aircraft from/to Est_i in the landing sequence.
- FB_i^+ Average required fuel burn cost per minute for aircraft i to be delayed.
- FB_i^- Average required fuel burn cost per minute for aircraft i to be advanced.

B. Objective Function

Choosing an appropriate objective function for the ALP is controversial and not all stakeholders (ATC, airport, airlines, and government) agree on the selection process. However, the following multi-criteria objective function can potentially satisfy the interests of all the parties.

- Minimizing the *average delay* which includes the lateness and earliness.

$$\sum_{i=1}^{n} (L_i \max\{(\mathrm{SLT}_i - \mathrm{TLT}_i - \mathrm{Al}_i^+), 0\} + E_i \max\{(\mathrm{TLT}_i - \mathrm{SLT}_i - \mathrm{Al}_i^-), 0\}). \quad (1)$$

- Maximizing the *runway throughput* (or runway utilization). Equivalently, the average of the landing times can be minimized rather than maximizing the number of aircraft landing on the runway (throughput or lead time).

$$\frac{1}{n} \sum_{i=1}^{n} \mathrm{SLT}_i. \quad (2)$$

- Minimizing the *fuel burn cost* (and hence minimizing carbon dioxide emissions). Fuel cost is almost 50% of the operating cost. The fuel burn depends on different factors such as pilot flying techniques, altitude, air speed, aircraft model, aircraft weight (including passengers and cargo), and fuel in the tank. Consequently, the extra fuel burn cost caused by lateness and earliness has to be considered.

$$\sum_{i=1}^{n} (\mathrm{FB}_i^+ \max\{(\mathrm{SLT}_i - \mathrm{TLT}_i), 0\} + \mathrm{FB}_i^- \max\{(\mathrm{TLT}_i - \mathrm{SLT}_i), 0\}). \quad (3)$$

Since the ALP may involve the simultaneous optimization of various correlated dependent objectives that are not necessarily aligned, a trade-off among the objectives is required. Therefore, they need to be optimized in the form of a weighted multi-criteria objective function.

C. Constraints

A variety of operational constraints can be imposed for the ALP, the most typical of which are the following.

- *Runway Usage*: Each runway can be used by at most one aircraft at a time so either aircraft i lands before j or vice versa.

$$X_{ij} + X_{ji} = 1 \quad \forall i, j \in A,\ i \neq j. \quad (4)$$

- *Wake Vortex (WV) Separation*: Aircraft have to observe a separation distance to avoid turbulence caused by preceding aircraft.

$$\mathrm{SLT}_i + S_{ij} \leq \mathrm{SLT}_j + M(1 - X_{ij}) \quad \forall i, j \in A,\ i \neq j,\ M \gg 0. \quad (5)$$

- *Time Constraint*: Based on operational and technical considerations such as limited fuel, airspeed, etc., each aircraft has a maximum and a minimum allowable airborne time which have to be treated as hard constraints.

$$\mathrm{ELT}_i \leq \mathrm{SLT}_i \leq \mathrm{LLT}_i \quad \forall i \in A. \tag{6}$$

A time slot (or time window) assigned to each landing aircraft which typically starts 5 minutes before TLT_i and ends 10 minutes after TLT_i does not necessarily coincide with the time constraint.

- *Time Shifting*: There is limited flexibility in moving the aircraft's landing time either forward or backward in time relative to its estimated landing time. Time shifting is considered rather than position shifting in re-sequencing the aircraft since it can be dependent on aircraft type.

$$(\mathrm{Est}_i - \mathrm{TS}_i) \leq \mathrm{SLT}_i \leq (\mathrm{Est}_i + \mathrm{TS}_i) \quad \forall i \in A. \tag{7}$$

- *Precedence Constraint*: Airline preferences may dictate that one aircraft should land before another.

$$\mathrm{SLT}_i P_{ij} < \mathrm{SLT}_j \quad \forall i, j \in A,\ i \neq j. \tag{8}$$

IV. Future Research Directions

Although many research papers on the ALP have been published during the last three decades, the majority have not developed methods that have been implemented. The reason could be because the methods may relax or dismiss hard (critical) operational constraints, have unreasonable algorithm runtime, study a static rather than dynamic environment, ignore the requirements of the various stakeholders, or depend on features of a specific airport. Existing research generally considers some of the common and obvious constraints. This research aims to capture vital operational constraints that have been observed from the daily work of controllers in our model building.

Solution approaches for the model remain to be developed. The solutions must be obtained quickly to be of use to air traffic controllers. Since the problem is complex (it is NP-hard), heuristic methods including local search algorithms may be more appropriate than enumerative methods such as dynamic programming which can be computationally demanding.

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