AI1110: Probability and Random Variables

Assignment 12: Papoulis-Pillai Example 15.10

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Outline

Problem

Let N represent the combined population of two cities A and B. Suppose migration occurs between the cities one at a time with probability proportional to the population of the city. and let the population of A determine the state of the system. Find the equilibrium state of the system.

Solution

A can move into either $e_k - 1$ or $e_k + 1$ with probabilities k/N or $1 - k/N$ respectively. Thus the transition matrix looks like

$$
\mathbf{P} = \begin{pmatrix}
0 & 1 & 0 & . & . & . & . & 0 \\
p & 0 & 1-p & 0 & . & . & . & 0 \\
0 & 2p & 0 & 1-2p & . & . & . & 0 \\
0 & 0 & 3p & 0 & 1-3p & 0 & . & 0 \\
. & . & . & . & . & . & . & . \\
0 & 0 & 0 & . & . & 1-p & 0 & p \\
0 & 0 & 0 & . & . & . & 1 & 0
\end{pmatrix} \tag{1}
$$

where $p = 1/N$. We can also think of this model as a random walk with totally reflective barriers where the probability of a step varies with the position or state. If $k < N/2$ the particle is more likely to move to the right, and if $k > N/2$ the particle will move to the left and therefore any particle will move to the center which is the Equilibrium state.