

# Assignment 10

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# Problem Statement

## Papoulis 8.10

Among 4000 newborns, 2080 are male. Find the 0.99 confidence interval of the probability  $p = P(\text{male})$

# General

Let,

$$X = \begin{cases} 1 & \text{Baby Boy} \\ 0, & \text{Baby Girl} \end{cases} \quad (1)$$

Let the average value of  $X_i$  for the given sample space be  $\hat{p}$ . We can use  $\hat{p} = 1 - \hat{q}$  to estimate an interval that  $p$  is likely to lie in.

# Solution

Since  $\hat{p} = \frac{\sum X_i}{n}$ , and since  $n$  is large, the sampling distribution of sample proportion can be approximated to a normal distribution, by the Central Limit Theorem.

$$\hat{p} = \text{Mean of the Distribution} \quad (2)$$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \text{Standard Deviation} \quad (3)$$

# Solution

We have

$$\hat{p} = 2080/4000 \quad (4)$$

$$= 0.52 \quad (5)$$

Also,

$$\sigma_{\hat{p}} = \sqrt{\frac{0.52(1 - 0.52)}{4000}} \quad (6)$$

$$= 0.0079 \quad (7)$$

# Solution

To find the interval, we use the  $z$ -score, which tells us the number of standard deviations between the end-points of the confidence interval and the mean. Since we are interested in the 0.99 confidence interval, we have

$$\gamma = 0.99 \quad (8)$$

$$\implies \delta = 1 - \gamma \quad (9)$$

$$= 0.01 \quad (10)$$

Therefore, we have to find  $z$  corresponding to  $\delta = 0.01$ , which from the  $z$ -score table equals 2.58

# Solution

Therefore, it follows that

$$p_u = \mu + z\sigma \quad (11)$$

$$= 0.52 + 2.58 \times 0.0079 \quad (12)$$

$$= 0.54 \quad (13)$$

where  $p_u$  is the upper limit of the interval. Similarly,

$$p_l = \mu - z\sigma \quad (14)$$

$$= 0.52 - 2.58 \times 0.0079 \quad (15)$$

$$= 0.49 \quad (16)$$

Therefore, the 0.99 confidence interval for  $p$  is  $[0.49, 0.54]$