

# Assignment 3 (NCERT Class 12 Probability)

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**Abstract**—This document contains the solution to Question 6 of Exercise 3 of Chapter 13 (Probability) in the NCERT Class 12 Textbook.

**Example 3.** There are 3 coins, one is a two headed coin (has both sides head), one is a biased coin that shows head with 75% probability and the last coin is unbiased. A coin is chosen at random and tossed. It shows head, what is the probability that it is a two headed coin?

**Solution:** Let us call the two headed coin  $C_1$ , the biased coin  $C_2$  and the unbiased coin  $C_3$ .

For  $C_1$  :

Let the random variable  $X_{C_1}$  denote what the coin shows. Then, we see that the sample space is  $S = \{0, 1\}$  where 1 is head and 0 is tail. The PMF is given by

$$\Pr(X_{C_1} = k) = \begin{cases} 1, & k = 1 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

For  $C_2$  :

Let the random variable  $X_{C_2}$  denote what the coin shows. Then, we see that the sample space is  $S = \{0, 1\}$  where 1 is head and 0 is tail. The PMF is given by

$$\Pr(X_{C_2} = k) = \begin{cases} \frac{3}{4}, & k = 1 \\ \frac{1}{4}, & k = 0 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

For  $C_3$  :

Let the random variable  $X_{C_3}$  denote what the coin shows. Then, we see that the sample space is  $S = \{0, 1\}$  where 1 is head and 0 is tail. The PMF is given by

$$\Pr(X_{C_3} = k) = \begin{cases} \frac{1}{2}, & k = 1 \\ \frac{1}{2}, & k = 0 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Let the random variable  $X$  denote the coin we

picked. Then we see that the same space is  $S = \{1, 2, 3\}$  where 1 is  $C_1$ , 2 is  $C_2$  and 3 is  $C_3$ . The PMF is given by

$$\Pr(X = k) = \begin{cases} \frac{1}{3}, & 1 \leq k \leq 3 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Given that the coin shows head we have to find the conditional probability that the coin is  $C_1$ . This is given by

$$\Pr(X = 1|K) \quad (5)$$

Where  $K$  is the condition that the coin shows a head.

Let  $E$  be the event : A coin is chosen at random and is tossed, the outcome of this toss is a head. The coin is a two headed coin.

Now,

$$\Pr(E) = \frac{\Pr(X = 1 \cap C_1)}{\Pr(K)} \quad (6)$$

$$\Pr(X = 1 \cap C_1) = \frac{1}{3} \quad (7)$$

$$\Pr(K) = \sum_{i=1}^3 \Pr(X = i \cap C_i) \quad (8)$$

$$K = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4} \quad (9)$$

$$\Rightarrow \Pr(E) = \frac{4}{9} \quad (10)$$