Papoulis Question 2.23

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Outline

Question

Box 1 contains 1 white and 999 red balls. Box 2 contains 1 red and 999 white balls, A ball is picked from a randomly selected box. If the ball is red what is the probability that it came from box 1

Solution

Let us call the first box $\mathcal{B}_1$ and the second box $\mathcal{B}_2$

For $\mathcal{B}_1$

Let the random variable $X_{\mathcal{B}_1}$ denote the colour of the ball picked. Then, we see that the sample space is $S = 0, 1$ where 1 is red and 0 is white. The PMF is given by

$$\Pr\left(X_{\mathcal{C}_1} = k\right) = \begin{cases} \frac{999}{1000}, & k = 1 \\ \frac{1}{1000}, & k = 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

For $\mathcal{B}_2$

Let the random variable $X_{\mathcal{B}_2}$ denote the colour of the ball picked. Then, we see that the sample space is $S = 0, 1$ where 1 is red and 0 is white. The PMF is given by

$$\Pr\left(X_{\mathcal{C}_2} = k\right) = \begin{cases} \frac{999}{1000}, & k = 0 \\ \frac{1}{1000}, & k = 1 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Picking a box

Let the random variable X denote the box we picked. Then we see that the same space is $S = 1, 2$ where 1 is $\mathcal{B}_1$ and 2 is $\mathcal{B}_2$. The PMF is given by

$$\Pr(X = k) = \begin{cases} \frac{1}{2}, & 1 \leq k \leq 2 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Solving

Given that the ball is red we have to find the conditional probability that the box is $\mathcal{B}_1$. This is given by

$$\Pr(X = 1|K) \tag{4}$$

Where K is the condition that the ball is red.
Let E be the event : A box is chosen at random and a ball is picked, the ball is red and it is from box 1.

Solving (Contd.)

Now,

$$\Pr(E) = \frac{\Pr(X = 1, \mathcal{B}_1)}{\Pr(K)} \tag{5}$$

$$\Pr(X = 1, \mathcal{B}_1) = \frac{999}{2000} \tag{6}$$

$$\Pr(K) = \sum_{i=1}^{2} \Pr(X = i, \mathcal{B}_i) \tag{7}$$

$$\implies \Pr(K) = \frac{999}{2000} + \frac{1}{2000} = \frac{1}{2} \tag{8}$$

$$\implies \Pr(E) = \frac{999}{1000} \tag{9}$$