12th Class NCERT Chapter 13 Exercise Question 6

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Outline

Question

There are 3 coins, one is a two headed coin (has both sides head), one is a biased coin that shows head with 75% probability and the last coin is unbiased. A coin is chosen at random and tossed. It shows head, what is the probability that it is a two headed coin?

Solution

Let us call the two headed coin $\mathcal{C}_1$, the biased coin $\mathcal{C}_2$ and the unbiased coin $\mathcal{C}_3$

For C_1

Let the random variable X_{C_1} denote what the coin shows. Then, we see that the sample space is $S = 0, 1$ where 1 is head and 0 is tail. The PMF is given by

$$\Pr\left(X_{C_1} = k\right) = \begin{cases} 1, & k = 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

For C_2

Let the random variable X_{C_2} denote what the coin shows. Then, we see that the sample space is $S = 0, 1$ where 1 is head and 0 is tail. The PMF is given by

$$\Pr\left(X_{C_2} = k\right) = \begin{cases} \frac{3}{4}, & k = 1 \\ \frac{1}{4}, & k = 0 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

For $\mathcal{C}_3$

Let the random variable $X_{\mathcal{C}_3}$ denote what the coin shows. Then, we see that the sample space is $S = 0, 1$ where 1 is head and 0 is tail. The PMF is given by

$$\Pr\left(X_{\mathcal{C}_3} = k\right) = \begin{cases} \frac{1}{2}, & k = 1 \\ \frac{1}{2}, & k = 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Tossing the coin

Let the random variable X denote the coin we picked. Then we see that the same space is $S = 1, 2, 3$ where 1 is $\mathcal{C}_1$, 2 is $\mathcal{C}_2$ and 3 is $\mathcal{C}_3$. The PMF is given by

$$\Pr(X = k) = \begin{cases} \frac{1}{3}, & 1 \leq k \leq 3 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Solving

Given that the coin shows head we have to find the conditional probability that the coin is $\mathcal{C}_1$. This is given by

$$\Pr(X = 1|K) \tag{5}$$

Where K is the condition that the coin shows a head.
Let E be the event : A coin is chosen at random and is tossed, the outcome of this toss is a head and the coin is a two headed coin.

Solving (Contd.)

Now,

$$\Pr(E) = \frac{\Pr(X = 1 \cap C_1)}{\Pr(K)} \tag{6}$$

$$\Pr(X = 1 \cap C_1) = \frac{1}{3} \tag{7}$$

$$\Pr(K) = \sum_{i=1}^{3} \Pr(X = i \cap C_i) \tag{8}$$

$$\implies \Pr(K) = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4} \tag{9}$$

$$\implies \Pr(E) = \frac{4}{9} \tag{10}$$