ICSE 2017 Q8 b

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Question 21(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = 200 - \frac{x}{400}$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

Solution. Let the total price $p(x) = P.x$

$$\implies \frac{p(x)}{x} = 200 - \frac{x}{400} \tag{0.1}$$

$$c(x) = \frac{x^2}{100} + 100x + 40 \tag{0.2}$$

$$Profit = p(x) - c(x) \tag{0.3}$$

Now let $Profit = y = f(x)$
We need to optimize $profit$ given that $x > 0$.
because $x > 0$ we set $x = t^2$
for this lets use the Lagrange multipliers,

$$\mathcal{L}(x, \lambda) = f(x) - \lambda g(x). \tag{0.4}$$

Where $g(x)$ is the constraint function, here we do not have any constraints as $x = t^2$ is always going to be positive.
Let,

$$k(t) = f(t^2) \tag{0.5}$$

$$\mathcal{L}(t, \lambda) = k(t) - \lambda g(t). \tag{0.6}$$

Now,

$$\nabla(\mathcal{L}) = 0 \tag{0.7}$$

$$\implies dx(f'(x) - \lambda g'(x))\hat{i} + d\lambda(-g(x))\hat{j} = 0 \tag{0.8}$$

$$\implies f'(x) - \lambda g'(x) = 0 \tag{0.9}$$

$$\implies -g(x) = 0; \tag{0.10}$$

Now, from the above equations

$$k'(t) = 0 \tag{0.11}$$

$$\implies 2tf'(t^2) = 0 \tag{0.12}$$

$$\implies f'(t^2) = 0 \tag{0.13}$$

$$\implies 100 - \frac{t^2}{100} = 0 \tag{0.14}$$

$$\implies t = \sqrt{4000} \tag{0.15}$$

This means that profit is maximum at $x = 4000$ and the corresponding values are in the table

Symbol	Value	Description
x	4000	Number of Units
$P(x)$	760000	Total Price
P	190	Price per unit
$C(x)$	560040	Total Cost
$Profit$	199960	Total Profit

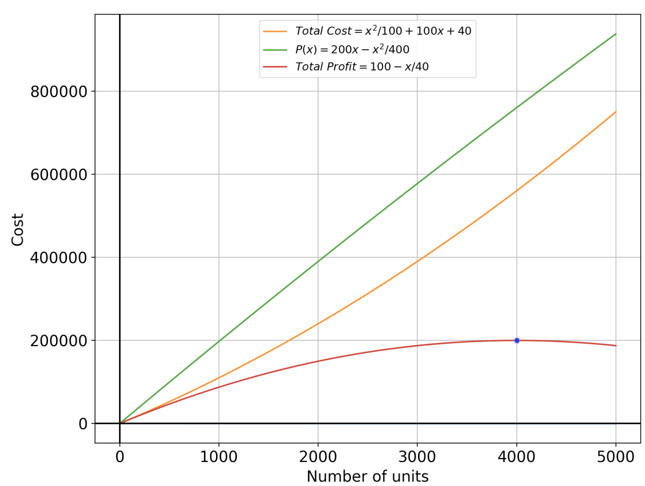

Fig. 0.1. Graph shows Total Profit, Total cost, Total profit with x