

ICSE 2017 Q8 b

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Question 21(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = 200 - \frac{x}{400}$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

Solution. Let the total price $p(x) = P \cdot x$

$$\Rightarrow \frac{p(x)}{x} = 200 - \frac{x}{400} \quad (0.1)$$

$$c(x) = \frac{x^2}{100} + 100x + 40 \quad (0.2)$$

$$Profit = p(x) - c(x) \quad (0.3)$$

Now let $Profit = y = f(x)$

We need to optimize *profit* given that $x > 0$.

For this lets use the Lagrange multipliers,

$$\mathcal{L}(x, \lambda) = f(x) - \lambda g(x). \quad (0.4)$$

Now,

$$\nabla(\mathcal{L}) = 0 \quad (0.5)$$

$$\Rightarrow dx(f'(x) - \lambda g'(x))\hat{i} + d\lambda(-g(x))\hat{j} = 0 \quad (0.6)$$

$$\Rightarrow f'(x) - \lambda g'(x) = 0 \quad (0.7)$$

$$\Rightarrow -g(x) = 0; \quad (0.8)$$

For our case we have $g(x) = x - |x| = 0$

$$g'(x) = 0 \quad \forall x > 0 \quad (0.9)$$

$$g'(x) = -2 \quad \forall x < 0 \quad (0.10)$$

Now plugging the values,

$$\Rightarrow g(x) = 0 \Rightarrow x > 0 \quad (0.11)$$

$$\Rightarrow g'(x) = 0 \quad (0.12)$$

$$\Rightarrow f'(x) = 0 \quad (0.13)$$

$$\Rightarrow \frac{x}{40} - 100 = 0 \quad (0.14)$$

$$\Rightarrow x = 4000 \quad (0.15)$$

This means that profit is maximum at $x = 4000$ and the corresponding values are in the table

Symbol	Value	Description
x	4000	Number of Units
$P(x)$	760000	Total Price
P	190	Price per unit
$C(x)$	560040	Total Cost
$Profit$	199960	Total Profit

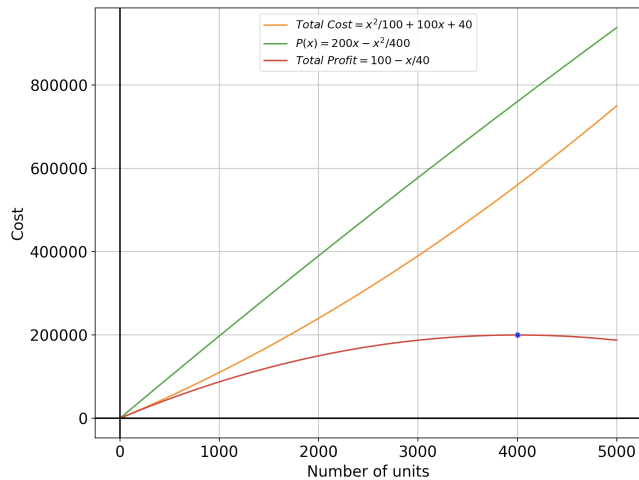


Fig. 0.1. Graph shows Total Profit, Total cost, Total profit with x