

ICSE 2017 Q8 b

Suryaansh Jain*

Question 21(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = 200 - \frac{x}{400}$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

Solution. Let the total price $p(x) = P \cdot x$

$$\Rightarrow \frac{p(x)}{x} = 200 - \frac{x}{400} \quad (0.1)$$

$$c(x) = \frac{x^2}{100} + 100x + 40 \quad (0.2)$$

$$Profit = p(x) - c(x) \quad (0.3)$$

For maximum profit $\frac{dProfit}{dx} = 0$

$$\Rightarrow 100 - \frac{x}{40} = 0 \quad (0.4)$$

$$\Rightarrow x = 4000 \quad (0.5)$$

The total production level $x = 4000$

Price per unit $= P(x) = 190$

$$Profit = 199960 \quad (0.6)$$

Total profit = 199960

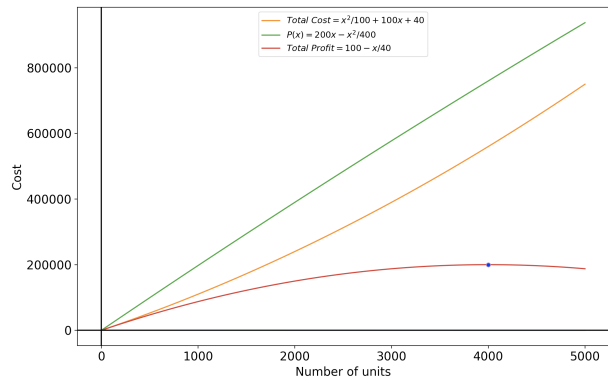


Fig. 0.1. Graph shows Total Profit, Total cost, Total profit with x

| Symbol | Description | Value |
|----------|---------------|--------|
| x | NumberofUnits | 4000 |
| $P(x)$ | TotalPrice | 760000 |
| P | Priceperunit | 190 |
| $C(x)$ | TotalCost | 560040 |
| $Profit$ | TotalProfit | 199960 |