ICSE 2017 Q8 b

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Question 21(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = 200 - \frac{x}{400}$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

Solution. Let the total price $p(x) = P.x$

$$\implies \frac{p(x)}{x} = 200 - \frac{x}{400} \tag{0.1}$$

$$c(x) = \frac{x^2}{100} + 100x + 40 \tag{0.2}$$

$$Profit = p(x) - c(x) \tag{0.3}$$

Now let $Profit = y = f(x)$
We need to optimize $profit$ given that $x > 0$.
Let us find $\nabla f(x)$

$$\implies \frac{dy}{dx} = 100 - \frac{x}{40} \tag{0.4}$$

$$\implies f'(x) = 100 - \frac{x}{40} \tag{0.5}$$

We will be able to find x for maximum $f(x)$ by iterating the following equation till $f'(x_{k-1})$ approaches 0.

$$x_k = x_{k-1} - (\alpha f'(x_{k-1})) \tag{0.6}$$

When $f'(x_{k-1}) = 0$, we have

$$f'(x_{k-1}) = 100 - \frac{x_{k-1}}{40} = 0 \tag{0.7}$$

$$\implies x_{k-1} = 100.40 = 4000 \tag{0.8}$$

This means that profit is maximum at $x = 4000$ and the corresponding values are in the table

Symbol	**Value**	**Description**
x	4000	Number of Units
$P(x)$	760000	Total Price
P	190	Price per unit
$C(x)$	560040	Total Cost
$Profit$	199960	Total Profit

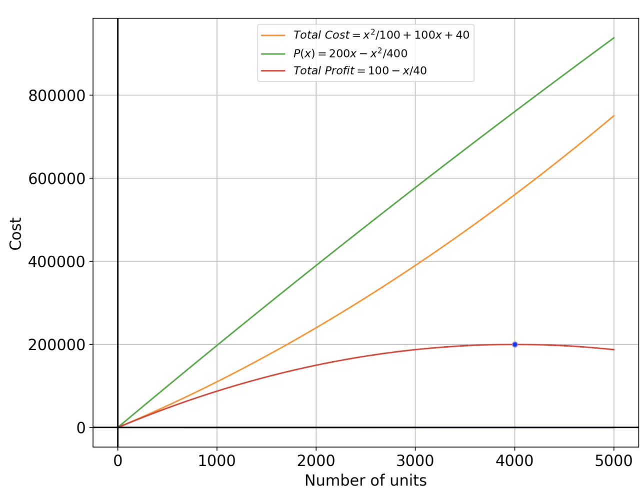

Fig. 0.1. Graph shows Total Profit, Total cost, Total profit with x