LiqBoost: Enhancing Liquidity Provision for Blockchain-based Decentralized Exchanges

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Abstract

Liquidity management in decentralized exchanges (DEXs) is a crucial research area in the blockchain industry due to its fundamental role in facilitating efficient trading of cryptocurrencies and fostering market stability. However, existing DEXs often face challenges related to liquidity management, which hinders their widespread adoption and effectiveness. This paper introduces LiqBoost, a novel liquidity provision scheme specifically tailored for Uniswap V3, one of the leading DEX platforms. Our approach focuses on dynamically reallocating the positions of Liquidity Providers (LPs). Simulations using historical transaction data from Uniswap V3 demonstrate that LiqBoost significantly reduces trading costs for traders and enhances incentives for LPs compared to the status quo of Uniswap V3. We detail the system design for on-chain implementation of LiqBoost, leveraging the existing Uniswap V3 framework. This underscores the practicality and effectiveness of LiqBoost within the decentralized finance (DeFi) landscape.

Keywords: Blockchain, Decentralized exchange, System modeling, Incentive analysis, Liquidity management

1. Introduction

Since its surge in popularity in 2022, Decentralized Finance (DeFi) has revolutionized the financial domain (Schueffel, 2021). Built on blockchain technology, DeFi platforms operate as transparent, peer-to-peer networks, providing users with autonomy over their assets (Qin et al.,

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2021; Werner et al., 2022). The total value locked in DeFi platforms currently exceeds $50 billion (DeFi Llama, 2024b), underscoring the ongoing demand for services such as decentralized lending and yield farming. Decentralized exchange (DEX) is one of the pivotal DeFi protocols, facilitating direct cryptocurrency trades via smart contracts implemented on a blockchain network. In contrast to the order book mechanism in traditional financial systems, DEX operates trading without the need for centralized intermediaries, adopting the *Automated Market Maker* (AMM) model. AMM, initially devised to address thin market issues (Hanson, 2003), has gained widespread adoption across various DEX platforms, due to its provision of a cost-effective trading mechanism on the blockchain.

Uniswap, a leading DEX platform (DeFi Llama, 2024a), commenced its evolution in 2018 with the introduction of the Constant Product Market Maker (CPMM) (Uniswap, 2020), as one type of AMM. CPMM enables users to trade assets automatically through the predetermined curve which maintains the product of the amounts of two assets constant. This resolved the primary drawback of traditional limit order books by ensuring the availability of liquidity at any price level. Subsequently, Uniswap V3 introduced the innovative concept of concentrated liquidity Adams et al. (2021), enabling each Liquidity Provider (LP) to set the specific price range for the liquidity provision, to achieve enhanced capital efficiency compared to Uniswap V2.

Throughout the evolution of Uniswap, one key factor remains crucial: the sustainability of DEXs depends on effectively managing liquidity to satisfy both LPs and traders, as they are closely interconnected. When trading volumes decrease, LPs experience a decrease in income, which in turn diminishes their motivation to supply liquidity. This reduction in LP participation subsequently leads to lower liquidity, increasing trading costs. Such a cyclical pattern poses a significant challenge to the long-term sustainability of DEX operations.

To address this issue, prior research focused on enhancing liquidity in DEXs has primarily aimed at boosting capital efficiency, a known factor in reducing trading costs (Canidio and Fritsch, 2023; Jeong et al., 2023; Ramseyer et al., 2021; Singh et al., 2023). However, these studies often overlook the impacts of their proposals on LPs. While the benefits of increased liquidity for traders are evident, the degree to which LPs find new DEX schemes beneficial remains an open question. This gap highlights the necessity for a detailed analysis of LP incentives in emerging DEX models, rejecting the naive assumption that 'What benefits traders may also benefit LPs'.

Therefore, this paper proposes LiqBoost, a novel liquidity allocation scheme designed to reduce trading costs and enhance incentives for LPs than those in Uniswap V3. LiqBoost achieves increased available liquidity utilization by dynamically reallocating inactive positions of LPs. For the quantification of the incentive for LPs, we utilize the Loss-versus-Rebalancing (LVR), recently introduced in the study by Milionis et al. (2022). This metric assesses LPs Profit and Loss by comparing it to their income, derived from swap fees paid by traders. We also introduce implementation details including additional methods for tracking the collected fees among LPs in LiqBoost. This paper is expected to provide a more nuanced understanding of LP incentives in the evolving DEX landscape, thereby informing a more effective and balanced DEX design proposal. The main contributions of this paper are as follows:

(1) Address the limitations of previous studies on DEX design proposals, emphasizing the importance of analyzing both LP incentives and trader costs.
(2) Propose LiqBoost, a DEX liquidity allocation scheme designed to enhance liquidity by dynamically reallocating inactive positions, including a system design for on-chain implementation.
(3) Empirically analyze the outcomes of the proposed scheme by conducting backtests using historical Uniswap V3 transaction data.

The remainder of the paper is organized as follows. Section 2 covers background topics. Section 3 presents relevant literature reviews aligned with our research goals. Section 4 introduces a new liquidity allocation scheme for DEXs. The economic incentives for LPs in this scheme are analyzed in Section 5. Section 6 details the experimental setup and results. Section 7 introduces the implementation details of the proposed scheme. Lastly, Section 8 offers concluding remarks and outlines potential future research directions.

2. Background

2.1. Automated Market Maker (AMM)

The principle underlying DEX involves users depositing their assets into the corresponding liquidity pool, thereby allowing other participants to conduct trades. Those contributing to these deposits are referred to as *liquidity providers* (LPs) who receive compensation in the form of a proportionate fraction of the traded token amount as a fee. Traders executing swap operations via DEX smart contracts receive token amounts determined by the AMM. AMMs are commonly termed Constant Function Market Makers (CFMMs) due to their reliance on predefined constant functions to determine the swapped amount. Figure 1 details the mechanism of liquidity pools in DEX. Uniswap V2 employs the constant product market maker (CPMM), which maintains the constant product of the amounts of two tokens.

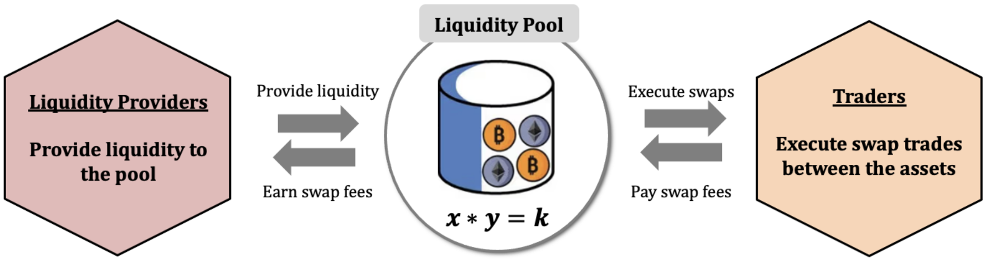

Figure 1: Basic mechanism of liquidity pools in DEXs.

2.2. *Concentrated liquidity AMM*

Uniswap V3, as introduced in the whitepaper (Adams et al., 2021) in 2021, brought the concept of concentrated liquidity, allowing LPs to allocate tokens within specific price ranges. In Uniswap V3, the exchange ratio of two tokens, represented as p, corresponds to a *tick* (t), with $p_t = 1.0001^t$. For simplicity, this paper will use the term **price** interchangeably with **tick**. *Bucket* indicates the minimal unit of liquidity provision, which is denoted as $I_k = [p_k, p_{k+\Delta}]$, where k is a multiple of Δ. A bucket I_k is formed between each p_k and $p_{k+\Delta}$. If a price p falls within I_k, it lies between p_k and $p_{k+\Delta}$, and the liquidity within this bucket remains constant but changes across different buckets.

In Uniswap V3, each LP allocates its assets by creating a *position*. A position is specified by three parameters: the amount of liquidity L, the lower price boundary p_l, and the upper price boundary p_u. This enables LPs to concentrate their liquidity provision within the price range $[p_l, p_u]$, allowing them to earn swap fees solely from trades occurring within $[p_l, p_u]$. We denote the position as $\kappa(l, p_l, p_u)$ in this paper. Expanding on this framework, consider a liquidity pool composed of a pair of tokens, X and Y. The relationship between the quantities of token X and

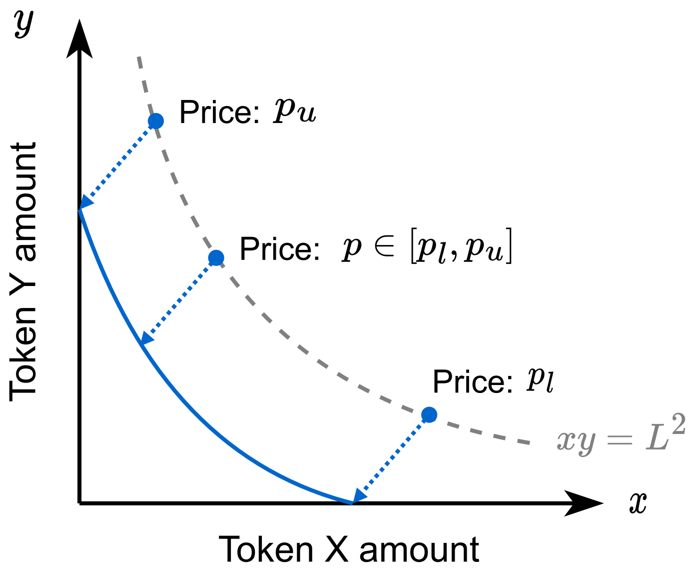

Figure 2: Relationship between token amounts in Uniswap V3.

Y given a price is formulated by Equation 1 and depicted as the blue line in Figure 2. This blue line is a shifted version of the grey line $xy = L^2$.

$$(x + \frac{L}{\sqrt{p_u}})(y + L\sqrt{p_l}) = L^2 \tag{1}$$

Same logic applies to a bucket $I_k = [p_k, p_{k+1}]$. In this context, p_l and p_u are replaced by p_k and p_{k+1}, respectively, in Equation 1. The liquidity within this bucket is denoted as L_k. Consequently, for a given price p within the bucket I_k, the quantities of tokens X and Y are defined by Equation 2 as follows:

$$x(I_k; p) = L_k\left(\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p_{k+1}}}\right), \quad y(I_k; p) = L_k\left(\sqrt{p} - \sqrt{p_k}\right). \tag{2}$$

The trading mechanism in Uniswap V3 follows the CPMM, the same as Uniswap V2. When a trader requests the swap transaction with Δx amount of token X, the resulting amount of token Y (denoted as Δy) is determined by Equation (3). Here, γ represents the fee rate set by the protocol. If the requested swap amount exceeds the available liquidity in the current bucket, a tick cross event occurs, leading to changes in the corresponding bucket. The remainder of the swap transaction is then executed using the parameters in the adjacent bucket, either I_{k+1} or I_{k-1}.

$$(x(I_k; p) + (1 - \gamma)\Delta x) \cdot (y(I_k; p) + \Delta y) = L_k^2 \tag{3}$$

2.3. Loss-versus-Rebalancing (LVR)

Although LPs earn income from swap fees paid by traders, they also face risks associated with liquidity provision. Impermanent Loss (IL), as discussed in (Loesch et al., 2021), is a common metric used to evaluate these risks. IL is the loss incurred when asset prices change in a pool between deposit and withdrawal, compared to just holding the assets. While both IL and LVR analyze the incentives for LPs by comparing them with the collected fees, an important distinction lies in the fact that LVR can have a positive value even when IL equals zero. LVR calculates cumulative value changes resulting from swaps across the period, whereas IL considers only two-time points: the start and end of the period. LVR indicates the value gap between a rebalancing strategy and an LP position. The rebalancing strategy replicates swaps that occurred in the DEX onto the market. Suppose there is a liquidity pool with token pairs X and Y in the DEX. If the price in the DEX is lower than the market, the rebalancing strategy involves selling token X to get more token Y than an LP would. Conversely, if the DEX price is higher, fewer Y tokens are needed to buy the same X amount.

In the theoretical framework proposed by Milionis et al. (2022), the calculation of instantaneous LVR assumes price movements align with Equation (4), displaying proportionality to price volatility and the pool's price. Here, $B_t^{\mathbb{Q}}$ represents a $\mathbb{Q}$-Brownian motion, where $\mathbb{Q}$ denotes a risk-neutral martingale measure. P_t and σ_t denote the price and volatility, respectively.

$$\frac{dP_t}{P_t} = \sigma_t dB_t^{\mathbb{Q}} \tag{4}$$

In our simulation analysis, we utilize the conceptual definition of LVR, focusing on the collective impact on all LPs within the liquidity pool rather than analyzing individual impacts. We assume that LPs employ a weekly rebalancing strategy, wherein the net value of the swapped amount within each week was utilized to calculate the value of the rebalancing strategy. Specifically, LVR in our empirical analysis denotes the absolute difference between $P\Delta X$ and ΔY, where ΔX and ΔY represent the net amount of swapped tokens within a week, and P signifies the price of the liquidity pool at the end of each weekly time unit. The conceptual LVR is mathematically formulated as Equation (5), given N rebalancing events throughout the entire period.

$$\text{Empirical LVR} = \sum_{i=1}^{N} |P_i \Delta X_i - \Delta Y_i| \tag{5}$$

2.4. Slippage

Slippage, as described in Equation 6, occurs when there is a difference between the expected price of a trade (p_{market}) and the actual execution price (p_{executed}) (Investopedia, 2023). Traders in DEXs focus primarily on reducing slippage to minimize the input token amount required for acquiring a specific amount of another token. Ensuring higher liquidity in the DEX is an important aspect of reducing slippage. Additionally, traders should carefully set the slippage level for each swap execution; setting it too low increases the likelihood of transaction failures.

$$\text{Slippage} = \left| \frac{p_{\text{executed}}}{p_{\text{market}}} - 1 \right| \propto \text{Liquidity}^{-1} \tag{6}$$

3. Related work

In terms of related work for LiqBoost, there were many studies on the incentives of being an LP in DEXs. To enhance the predictability of the cryptocurrency markets, many studies focused on forecasting the cryptocurrency prices by using the related blockchain information (Jang and Lee, 2017; Zoumpekas et al., 2020; Kim et al., 2021; Koo and Kim, 2024). Aigner and Dhaliwal (2021) and Heimbach et al. (2022) delved into LP risks, with a particular emphasis on impermanent loss in Uniswap V2 and V3. The FLAIR metric, devised by Milionis et al. (2023b), was utilized to assess LP competitiveness. Furthermore, Kuan (2022) and Chen et al. (2023a) developed payoff functions for LPs and traders, shedding light on their mutual influences.

Aligned with this trend, recent research delved into LP strategies aimed at enhancing their incentives (Bar-On and Mansour, 2023; Fan et al., 2022; Khakhar and Chen, 2022; Milionis et al., 2023a; Yin and Ren, 2021). Among these strategies, the Just-in-Time (JIT) liquidity provision strategy, detailed in Wan and Adams (2022), involved providing liquidity within a tightly concentrated price range for a very short period to yield substantial profits. It can be inferred that LiqBoost mitigates the profit difference between sophisticated LPs and passive LPs who may lack extensive knowledge of price movements. Traders utilizing LiqBoost experience reduced swap slippage, benefiting from the automated strategic allocation of liquidity by LiqBoost, akin to Just-in-Time (JIT) LP strategies.

Several researchers introduced innovative designs for DEXs, aiming to better the experiences of LPs and traders. Jeong et al. (2023) developed a new margin liquidity-providing position, enabling lenders to supply assets to LPs, thus enhancing liquidity and returns. Singh et al. (2023) proposed a unique DEX design with reserve sharing across pools, intended to increase liquidity for less traded pairs. Canidio and Fritsch (2023) put forth a novel AMM model that processed trades in batches at equitable, equilibrium prices, aiming to eradicate LVR and sandwich attacks while improving returns relative to Uniswap V3. Chen et al. (2023b) enhanced DODO Exchange's PMM with the multi-token proactive market maker (MPMM), achieving greater capital efficiency and minimizing impermanent loss.

In our investigation of this field, we discovered that no studies thoroughly examined LP incentives and trader costs in the context of new DEX designs. To address this gap, we propose LiqBoost, a novel liquidity allocation method built on the concentrated liquidity AMM framework, offering a practical solution for the dynamic DEX design landscape.

4. Proposed Method

In an ideal DEX liquidity allocation scheme, positions would be continually adjusted to fit within current buckets, thus ensuring optimized liquidity for traders and increased fee income for LPs. However, in reality, especially during periods of high volatility, DEXs frequently experience consecutive instances of suboptimal liquidity distribution (Uniswap, Accessed: 2024). This situation highlights the challenges in promptly adjusting LP positions in response to rapid price changes. In response to this challenge, we conceived LiqBoost, a novel liquidity allocation scheme designed to enhance liquidity utilization within DEXs.

LiqBoost dynamically draws liquidity from inactive positions and leverages it to the current bucket. This mechanism is designed to improve the efficiency of liquidity utilization within the concentrated liquidity AMM-based DEX, such as Uniswap V3. In Uniswap V3, the quantity of tokens is determined by the formula stated in Equation (2). This implies that when positions are out of the bucket to the left, only Y tokens provide liquidity; to the right, only X tokens are used. Therefore, extra X tokens should be obtained from the inactive positions to the right of I_k, while Y tokens should be drawn from the positions to the left of I_k. A graphical depiction of LiqBoost

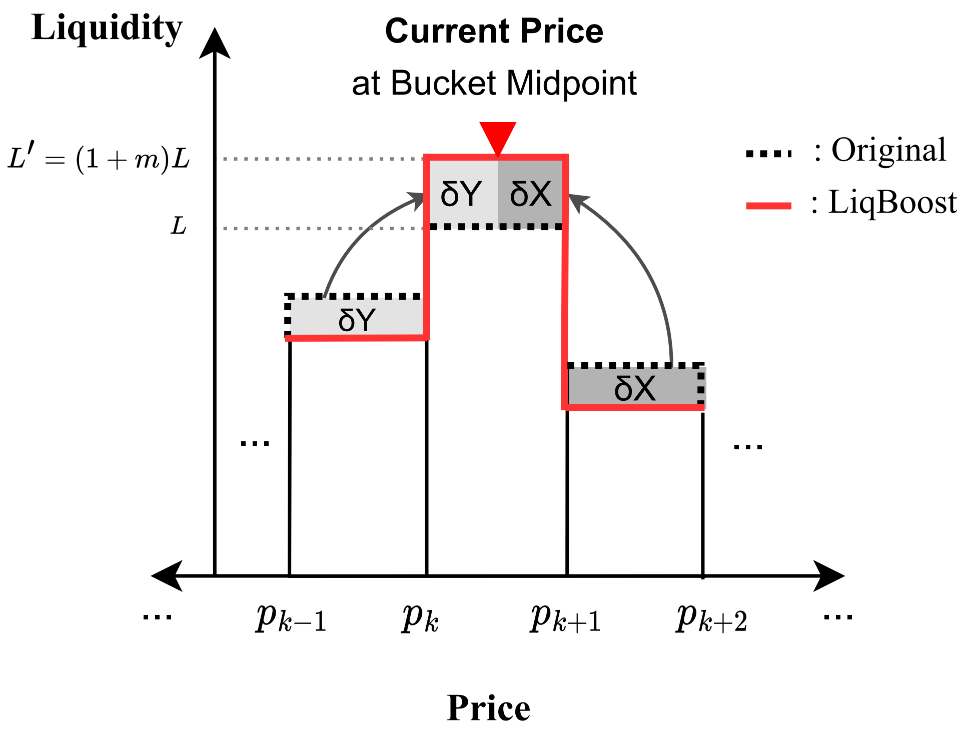

Figure 3: Overview of LiqBoost.

is shown in Figure 3.

LiqBoost defines specific conditions for integrating the extra liquidity provided by the inactive positions with the currently available liquidity. The boosted liquidity is only activated when the current tick precisely aligns with the midpoint of bucket I_k. The reason behind this is that at the midpoint of the bucket, which represents the geometric mean of p_k and p_{k+1}, the values of token X and token Y at I_k are equal, as inferred from $pX = Y$ in Equation (2). This condition ensures an equal contribution from both the left and right sides of the boosted liquidity at I_k.

When designing a new DEX scheme, it is also crucial to ensure the simplicity of implementation. In Uniswap V3, fee calculation for each LP was simple, as the protocol tracked the collected fee per unit liquidity. In LiqBoost, LPs contribute liquidity not only within their initial price range but also across other buckets where their provided liquidity is reallocated. To streamline fee calculations, we have established rules to determine which positions are eligible to participate in the liquidity boost mechanism. Specifically, only the positions with an upper tick at p_k or a lower tick at p_{k+1} can offer additional liquidity at I_k. We provide further details for the fee calculation in Section 7.3.

5. Theoretical analysis

This section aims to analyze the extent of the total LVR reduction in LiqBoost compared to Uniswap V3. The primary focus of this section is Theorem 5.6 while preceding lemmas and propositions serve as supplementary explanations supporting the theorem. To facilitate the analysis, we introduce essential notations. Each liquidity pool operates uniquely with a designated tick space Δ. By defining $c = 1.0001^{-\Delta/2}$ as a constant smaller than 1, we establish a representation of the liquidity pool's exchange rate, p, in terms of its boundary ticks, p_l and p_u, as outlined in Equation (7). This representation allows us to express the time-varying price of the liquidity pool as a function of a single parameter, β, within a given bucket. Thus, a larger β value within the same bucket signifies a higher value for p.

$$\sqrt{p} = \frac{\sqrt{p_l}}{\alpha} = \beta\sqrt{p_u}, \quad \forall\alpha, \beta \in [c, 1] \tag{7}$$

Lemma 5.1 (**Token** X **Amount as a Function of** p **and** V). *In the liquidity pool, where V signifies the aggregated asset value denominated in token Y, p_i denotes the exchange ratio, L represents available liquidity, and X and Y denote the quantities of tokens X and Y, respectively, V and X can be formulated by Equation* (8)*, where* $\varepsilon(t) = \frac{1-t}{2-t-\frac{c}{t}}$.

$$X = \frac{V}{p}\varepsilon(\beta) \tag{8}$$

Proof of Lemma 5.1. The expressions for X_i and Y_i derived from Equation (2) are as follows:

$$X = L\left(\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p_u}}\right) = \frac{L}{\sqrt{p}}(1-\beta)$$

$$Y = L(\sqrt{p} - \sqrt{p_l}) = L\sqrt{p}\left(1 - \frac{c}{\beta}\right)$$

Thus, $V = pX + Y = L\sqrt{p}\left(2 - \beta - \frac{c}{\beta}\right)$, and we can derive:

$$X = \frac{L}{\sqrt{p}}(1-\beta) = \frac{L\sqrt{p}}{p}(1-\beta) = \frac{V}{p}\frac{1-\beta}{2-\beta-\frac{c}{\beta}} = \frac{V}{p}\varepsilon(\beta)$$

□

Note that in Equation (8), the function $\varepsilon(\beta)$ is strictly decreasing and consistently positive for $t \in [c, 1]$. The following Lemma 5.2 represents the formulation of the total LVR utilized in our empirical analysis.

Lemma 5.2 (Total LVR). *Let p_i represent the price of the pool, X_i and Y_i represent the amount of X, and Y tokens respectively at time i. If we assume that there are N number of rebalancing events, then by using two distinct components: $\sigma_{v_i} = \frac{\Delta V_i}{V_{i-1}}$ and $\sigma_{p_i} = \frac{\Delta p_i}{p_{i-1}}$, the total LVR can be formulated as follows:*

$$\textit{Total LVR} = \sum_{i=1}^{N} V_{i-1} |\sigma_{v_i} - \sigma_{p_i} \epsilon(\beta_{i-1})| \tag{9}$$

Proof of Lemma 5.2. By Equation (5) and Equation (8),

$$\begin{aligned}
\text{Total LVR} &= \sum_{i=1}^{N} |p_i \Delta X_i - \Delta Y_i| \\
&= \sum_{i=1}^{N} |p_i X_i + Y_i - p_i X_{i-1} - Y_{i-1}| \\
&= \sum_{i=1}^{N} |\Delta V_i - \Delta p_i X_{i-1}| \\
&= \sum_{i=1}^{N} V_{i-1} \left| \frac{\Delta V_i}{V_{i-1}} - \frac{\Delta p_i}{p_{i-1}} \frac{p_{i-1} X_{i-1}}{V_{i-1}} \right| \\
&= \sum_{i=1}^{N} V_{i-1} |\sigma_{v_i} - \sigma_{p_i} \varepsilon(\beta_{i-1})|
\end{aligned}$$

□

Lemma 5.3 (Unit LVR). *If we define the term within the summation in Equation* (9) *as* ***Unit LVR****, it is formulated by Equation* (10)*. In this lemma, we assume that the price of the liquidity pool remains within the same bucket $[p_{l_i}, p_{u_i}] = [p_l^*, p_u^*]$, so $L_i = L^*$ for $i = 1, 2, \ldots, N$.*

$$\textit{unit LVR} \approx V_{i-1} \varepsilon(\beta_{i-1}) \sigma_{\beta_i}^2 = L^* \sqrt{p_u^*} \left(\frac{1}{\beta_{i-1}} - 1 \right) \Delta \beta_i^{\,2} \tag{10}$$

Proof of Lemma 5.3. By assumption, $p_i = \beta_i^2 p_u^*$. If we define σ_{β_i} as $\frac{\Delta \beta_i}{\beta_{i-1}}$, σ_{v_i} and σ_{p_i} can be expressed as follows:

$$\sigma_{v_i} = \frac{\Delta V_i}{V_{i-1}} \approx 2\sigma_{\beta_i}\varepsilon(\beta_{i-1})$$
$$\sigma_{p_i} = \frac{\Delta p_i}{p_{i-1}} = \sigma_{\beta_i}(2 + \sigma_{\beta_i})$$

By inserting the above terms, we get

$$\text{unit LVR} = V_{i-1}|\sigma_{v_i} - \sigma_{p_i}\varepsilon(\beta_{i-1})| \approx V_{i-1}\varepsilon(\beta_{i-1})\sigma_{\beta_i}^2$$

Using the definition of $\varepsilon(\beta_i)$ and the equations in Lemma 5.1, the unit LVR is formalized solely in terms of β_i, as presented in Equation (10). □

Lemma 5.3 indicates that the unit LVR within the same bucket depends on the initial price of the unit time interval ($\beta_{i-1} \propto \sqrt{p_{i-1}}$) and the price volatility (σ_{β_i}) because both L^* and p_u^* are constant. Thus, in the presence of an equivalent level of price change denoted by $\Delta\beta_i$, it becomes apparent that a lower initial price—signified by a smaller β_{i-1}—results in a higher unit LVR. Utilizing the above lemmas, we aim to compare the total LVR between the original scheme and LiqBoost. First, we consider how the unit LVR differs in two scenarios: Downward price movement and Upward price movement.

Proposition 5.4 (LVR for Downward price movement). *Let p_{i-1} represent the initial price of a given pool and let L^* and $(1 + m)L^*$ denote the liquidity of the original pool and the boosted pool, respectively. Consider a new swap wherein traders sell X tokens and buy Y tokens, leading to price changes from p_{i-1} to p_i and p'_i for each pool ($p_i, p'_i < p_{i-1}$ and $p_{i-1}, p_i, p'_i \in [p_l^*, p_u^*]$). Under this scenario, the LVR for the swap in the original pool is consistently larger than the LVR observed in the boosted liquidity pool.*

Proof of Proposition 5.4. Consider the ratio of the LVR between two pools. The input for the swap should be the same in both pools, allowing it to be formulated as:

$$(1 + m)L^*(\frac{1}{\sqrt{p'_i}} - \frac{1}{\sqrt{p_{i-1}}}) = L^*(\frac{1}{\sqrt{p_i}} - \frac{1}{\sqrt{p_{i-1}}})$$
$$\iff \frac{\Delta\beta_i}{\Delta\beta'_i} = (1 + m)\frac{\beta_i}{\beta'_i}$$

Then, we can derive the LVR ratio using (11).

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = \frac{L^* \sqrt{p_u^*} \frac{(1-\beta_{i-1})}{\beta_{i-1}} \Delta\beta_i{}^2}{(1+m)L^* \sqrt{p_u^*} \frac{(1-\beta_{i-1})}{\beta_{i-1}} \Delta\beta_i'^2}$$
$$= \frac{1}{1+m} \frac{\Delta\beta_i^2}{\Delta\beta_i'^2} = (1+m)\frac{\beta_i^2}{\beta_i'^2}$$

So the LVR Ratio is greater than 1 if and only if $\sqrt{1+m}\beta_i > \beta_i'$.

$$\frac{\beta_i - \beta_{i-1}}{\beta_i' - \beta_{i-1}} = (1+m)\frac{\beta_i}{\beta_i'} \iff \beta_i' = \frac{(1+m)\beta_i\beta_{i-1}}{\beta_{i-1} + m\beta_i}$$

$$\frac{(1+m)\beta_i\beta_{i-1}}{\beta_{i-1} + m\beta_i} < \sqrt{1+m}\beta_i \iff \frac{\sqrt{1+m}-1}{m}\beta_{i-1} < \beta_i$$

Since $\frac{\sqrt{1+m}-1}{m}$ is a decreasing function for $m \in [0,1]$, and $\lim_{x\to 0} \frac{\sqrt{1+m}-1}{m} = \frac{1}{2}$, it suffices to demonstrate that $\frac{1}{2}\beta_{i-1} < \beta_i$. We have assumed that all prices remain within the same bucket, ensuring $\frac{1}{2}\beta_{i-1} < \beta_i$ holds true. □

Proposition 5.5 (LVR for Upward price movement). *Let p_{i-1} represent the initial price of a given pool and let L^* and $(1+m)L^*$ denote the liquidity of the original pool and the boosted pool, respectively. Consider a new swap wherein traders buy X tokens and sell Y tokens, leading to price changes from p_{i-1} to p_i and p_i' for each pool ($p_i, p_i' > p_{i-1}$ and $p_{i-1}, p_i, p_i' \in [p_l^*, p_u^*]$). Under this scenario, the LVR for the swap in the original pool is consistently larger than the LVR observed in the boosted liquidity pool.*

Proof of Proposition 5.5. Consider the ratio of the LVR between two pools. The input for the swap should be the same in both pools, allowing it to be formulated as:

$$(1+m)L^*(\sqrt{p_i'} - \sqrt{p_{i-1}}) = L^*(\sqrt{p_i} - \sqrt{p_{i-1}}) \tag{11}$$
$$\iff \frac{\Delta\beta_i}{\Delta\beta_i'} = 1+m \tag{12}$$

Then, we can derive the LVR ratio using (12).

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = \frac{1}{1+m}\frac{\Delta\beta_i^2}{\Delta\beta_i'^2} = 1+m > 1$$

□

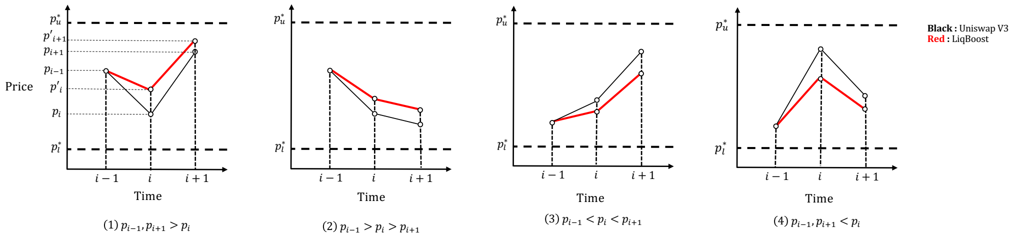

Figure 4: Four trading scenarios with two individual swaps.

To expand our analysis for $N = 2$, we examine four distinct scenarios involving price movements – either increasing or decreasing. Figure 4 illustrates these scenarios. Black lines depict Uniswap V3's price movement, while red lines represent LiqBoost. The comparison of resulting LVR values across these scenarios leads to Theorem 5.6.

Theorem 5.6. *Let p_{i-1}, p_i, p_{i+1} denote the prices along two swaps in the original liquidity pool, and let p_{i-1}, p'_i, p'_{i+1} denote the prices along the boosted liquidity pool. Then for the scenarios depicted in Figure 4, LVR_{boost} is smaller than $LVR_{original}$ when the initial swap induces a downward price movement.*

Proof of Theorem 5.6. For each case, **Down** and **Up** denote the direction of price movement during a single rebalancing event.

Case 1 (**Down** $\Rightarrow$ **Up**): $p_{i-1}, p_{i+1} > p_i$

From Proposition 5.4, we know that the first swap has incurred a larger LVR for the original pool. So if we only compare the LVR of the second swap using Equation (12):

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = (1+m)\frac{\beta'_i(1-\beta_i)}{\beta_i(1-\beta'_i)} > 1+m$$

We can check that both swaps incur larger LVR for the original pool in **Case 1**.

Case 2 (**Down** $\Rightarrow$ **Down**): $p_{i-1} > p_i > p_{i+1}$

From Proposition 5.4, we know that the first swap has incurred a larger LVR for the original pool. So if we only compare the LVR of the second swap using Equation (11) and employing analogous techniques as demonstrated in the proof of Proposition 5.4:

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = \frac{1}{1+m}\frac{\Delta\beta_{i+1}^2}{\Delta\beta'^2_{i+1}}\frac{\beta'_i(1-\beta_i)}{\beta_i(1-\beta'_i)} > 1$$

We can check that both swaps incur larger LVR for the original pool in **Case 2**.

Case 3 (Up ⇒ Up): $p_{i-1} < p_i < p_{i+1}$

From Proposition 5.5, we know that the first swap has incurred a larger LVR for the original pool. So if we only compare the LVR of the second swap using Equation (12):

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = (1+m)\frac{\beta'_i(1-\beta_i)}{\beta_i(1-\beta'_i)}$$

Case 4 (Up ⇒ Down): $p_{i-1}, p_{i+1} < p_i$

From Proposition 5.5, we know that the first swap has incurred a larger LVR for the original pool. So if we only compare the LVR of the second swap using Equation (11):

$$\text{LVR Ratio} = \frac{\text{LVR}_{original}}{\text{LVR}_{boost}} = (1+m)\frac{\beta_{i+1}^2}{\beta'^2_{i+1}}\frac{\beta'_i}{\beta_i}\frac{(1-\beta_i)}{(1-\beta'_i)}$$

For **Case 3** and **Case 4**, whether the unit LVR is larger in LiqBoost depends on m and β_i, with the condition for a strict increase represented by $(1+m)\frac{\beta'_i(1-\beta_i)}{\beta_i(1-\beta'_i)} > 1$. □

Theorem 5.6 highlights that the effectiveness of LiqBoost amplifies notably with an increased occurrence of {Sell X, Buy Y} swaps, which are related to downward price movement. If there are multiple intervals with downward price movement, it is deterministic that the LVR of consecutive intervals will also be consistently smaller. The total LVR, computed through the summation of each interval's unit LVR, will be decreased when multiple intervals record lower unit LVR.

Now we consider the price movements crossing the bucket boundary. Specifically, considering the liquidity of consecutive buckets B_1 and B_2 as L_1 and L_2 respectively, with $L_2 = \gamma L_1$, when the price moves from B_1 to B_2. Following the same logic as above, unit LVR for the tick-cross swap is derived as in Equation (13), using the percent change of the pool value as

$\sigma^c_{v_i} = \left|\gamma(\sigma_{v_i} + 1) - 1\right|$. We can observe the same result as in the non-tick-cross swap by setting $\gamma = 1$.

$$\begin{aligned} \text{unit LVR} &= V_{i-1}\left|\sigma^c_{v_i} - \sigma_{p_i}\epsilon(\beta_{i-1})\right| \\ &= V_{i-1}\epsilon(\beta_{i-1})\sigma_{\beta_i}\left|2(\gamma-1) - \sigma_{\beta_i}\right| \end{aligned} \tag{13}$$

Equation (13) suggests that under the same price volatility σ_{β_i}, having γ closer to $\kappa = 1 + \frac{\sigma_{\beta_i}}{2}$ results in a smaller unit LVR. LiqBoost makes smaller γ, the ratio of liquidity between two buckets. When the bucket of liquidity L_1 is boosted by m, the liquidity of the adjacent bucket becomes $L_2 - \frac{mL_1}{2}$, which makes the new ratio γ' according to Equation (14). If the reduced γ' approaches the value of κ by the proposed boost mechanism, this decrease leads to a smaller absolute value in Equation (13), resulting in an overall decrease in the total LVR. However, if γ was initially small, the liquidity boost leads to a situation where the gap between γ' and κ widens. As a result, the unit LVR increases, potentially causing an unfavorable outcome for the tick-cross swaps.

$$\gamma' = \frac{L_2 - \frac{m}{2}L_1}{(1+m)L_1} = \frac{2\gamma - m}{2(1+m)} < \gamma \tag{14}$$

6. Empirical Analysis

6.1. Experimental Setup

6.1.1. Data

We analyzed historical Uniswap V3 swap transactions and LP positions to identify the effect of LiqBoost. Spanning approximately 10 months, from June 21, 2022, to April 7, 2023, the data were collected from Ethereum blocks within the range of 15,000,000 to 17,000,000. Transactions associated with Uniswap V3 were extracted from the Ethereum blockchain using an Erigon client (Ledgerwatch, 2023). On-chain transactions have log data, and we filtered out those without swap, mint (add liquidity), and burn (remove liquidity) events. Mint and burn data include lower tick, upper tick, and liquidity amounts of positions. The swap data encompasses the liquidity pool's liquidity and price immediately after the swap, along with the exchanged amounts of each token.

Two liquidity pools were used for simulation: WBTC/ETH (0.3% fee)[1] and SHIB/ETH (1% fee)[2]. WBTC represents wrapped Bitcoin and has the same value as Bitcoin. SHIB represents the Shiba Inu token. The former is the largest in Total Value Locked (TVL) on Uniswap V3, while the latter is one of the smallest, having lower liquidity and fewer swap occurrences compared to the former. The numbers of collected data from each liquidity pool are described in Table 1.

Table 1: Summary of Collected Data from Liquidity Pools

Liquidity Pool	**Mint**	**Burn**	**Swap**	**TVL**
WBTC/ETH (0.3%)	8,649	10,308	83,190	$186.69M
SHIB/ETH (0.1%)	861	959	8,560	$698.41K

Table 2: Summary statistics for liquidity positions in each liquidity pool.

	WBTC/ETH (0.3%)		**SHIB/ETH (1%)**	
	Liquidity	Width	Liquidity	Width
Mean	1.79×10^{17}	9.32×10^{3}	6.52×10^{25}	4.92×10^{4}
Std	1.39×10^{18}	7.63×10^{4}	9.24×10^{26}	1.89×10^{5}
Min	1.00	6.00×10^{1}	1.77×10^{16}	2.00×10^{2}
Median	1.00×10^{14}	2.88×10^{3}	1.41×10^{22}	1.02×10^{4}
Max	3.09×10^{19}	1.77×10^{6}	1.83×10^{28}	1.77×10^{6}

Notes. 'Width' represents the difference between the upper and lower ticks of the position.

Table 2 provides details regarding the liquidity amounts and the length of price intervals for the minted positions in each pool. Note that a direct comparison of liquidity amounts between two pools lacks significance due to their difference in token decimals. When observing the position lengths of liquidity positions, LPs in the WBTC/ETH pool tend to mint shorter positions compared to the LPs in the SHIB/ETH pool. Additionally, the strategies of LPs in the SHIB/ETH pool appear to be more diverse, resulting in a higher standard deviation.

Table 3 displays the volume of swap transactions in each liquidity pool. Swaps occur more frequently in the WBTC/ETH pool, indicating higher trading activity. The WBTC/ETH pool also shows a higher mean swap volume (57.9 ETH) compared to the SHIB/ETH pool (3.2 ETH). Ap-

[1] Pool contract address: 0xCBCdF9626bC03E24f779434178A73a0B4bad62eD

[2] Pool contract address: 0x5764a6F2212D502bC5970f9f129fFcd61e5D7563

Table 3: Summary statistics for swap volume.

	WBTC/ETH (0.3%)	SHIB/ETH (1%)
Buy ETH Ratio	0.48	0.52
Mean	5.785×10^{19}	3.207×10^{18}
Std	1.489×10^{20}	6.262×10^{18}
Min	1.516×10^{7}	9.00
Median	1.527×10^{19}	2.220×10^{18}
Max	1.156×10^{22}	2.482×10^{20}

Notes. 'Buy ETH Ratio' represents the proportion of swaps involving the purchase of ETH relative to the total number of swaps. Descriptive statistics (Mean, Std, Min, Median, Max) are represented in Wei, where 1 ETH equals to 10^{18} Wei.

proximately 48% of swaps in the WBTC/ETH pool involve buying ETH, which leads to downward price movement in the liquidity pool. In the SHIB/ETH pool, swaps that involved buying ETH accounted for 51.7%. The ratio above 50% indicates that rebalancing events associated with downward price movements outnumber those with upward movement. This finding aligns with our theoretical analysis in Section 5, which demonstrates that more downward price movements lead to more significant benefits from LiqBoost, *i.e.,* a larger reduction in the total LVR compared to Uniswap V3.

6.1.2. Simulation

For simulation of LiqBoost, we initially implemented the concentrated liquidity AMM by constructing functions to calculate the swapped token amounts and predict the subsequent price movements based on the liquidity distribution in each tick. Then, we incorporated the boost function, utilizing boosting ratios selected from the set $\{0.03, 0.07, 0.1, 0.15, 0.2\}$.

In actual trading scenarios, disparities between prices on Centralized Exchanges (CEX) and DEX trigger arbitrages. As available liquidity for each swap in LiqBoost simulation deviates from real-world price movements, we added extra arbitrage actions when the simulated price diverged by more than 10 ticks from the price observed on DEX. As described in Section 4, our liquidity boost protocol triggers only when prices precisely align with the midpoint of the current bucket. However, in our simulation, we allowed a 2-tick margin to mitigate the impact of random occurrences on the activation of the boost event. Before activating the boost, the following four criteria must be met:

1. **Price Stability Requirement:** The price should remain within the same bucket for more than 5 swaps to trigger the boost.
2. **Maintaining Minimum Liquidity:** There should not be any bucket where the liquidity drops below the minimum value after the boost.
3. **Controlled Liquidity Difference:** The liquidity difference between consecutive buckets should be less than x times the ratio of the current bucket's liquidity.
4. **Restriction on Liquidity Movements:** The liquidity of adjacent ticks must have been moved less than y times before.

Table 4: Simulation settings.

	Criteria 3 (x)	**Criteria 4** (y)
Simulation 1	0.5	10
Simulation 2	0.7	3

We conducted two simulations by adjusting the parameters x and y in criteria (3) and (4). The simulation settings for the experiment are described in Table 4. Simulation 2 restricts liquidity boost compared to Simulation 1, aiming to achieve a more consistent distribution of liquidity and avoid extreme fluctuations. Our liquidity boost scheme is outlined in the following pseudo-code in Algorithm 1.

6.2. Empirical Results and Discussion

6.2.1. Incentive Analysis of LiqBoost

Table 5: Empirical results (Simulation 1): WBTC/ETH (0.3%)

Boost ratio (m)		0	0.03	0.07	0.10	0.15	0.20
Boost counts		-	345	342	349	352	341
Realized Volatility (1e-5)		2.82	**2.81**	2.85	2.94	3.12	3.32
Slippage (bp)		15.83	**15.46**	**15.46**	**15.46**	15.47	15.48
LP's profit (Fee - LVR)	Weekly	-227.64	-233.34	-227.16	-204.20	-240.72	**-176.39**
	Monthly	-934.86	-933.36	-908.63	-816.81	-962.88	**-705.58**
	Long-Term	-4674.28	-4666.82	-4543.17	-4084.06	-4814.40	**-3527.89**

The empirical results from the experiments conducted on the WBTC/ETH (0.3%) and SHIB/ETH (1%) liquidity pools are outlined in Table 5 through Table 8. The tables display the total num-

Algorithm 1 Pseudo-code for Liqboost

```
function SIMUL_LIQBOOST(m, data)
    Initialize liquidity distribution and block number
    Store swap_info
    for swap in data do
        liq, ct ← Get liquidity distribution and current tick
        if conditions for arbitrage are met then
            Make arbitrage
        end if
        if previous boost == True and all boost criteria met then
            Update liq by boost mechanism
        end if
        Liquidity history ← Liquidity history ∪ {liq}
        new_tick ← SWAP_EXECUTION(swap_info, liq)
        Tick history ← Tick history ∪ {new_tick}
        if new_tick is in bucket's middle point then
            boost ← True
        end if
    end for
    return results
end function
```

Table 6: Empirical results (Simulation 2): WBTC/ETH (0.3%)

Boost ratio (m)		0	0.03	0.07	0.10	0.15	0.20
Boost counts		-	107	106	108	108	106
Realized Volatility (1e-5)		**2.82**	**2.82**	2.83	2.85	2.89	2.86
Slippage (bp)		15.83	**15.46**	**15.46**	**15.46**	**15.46**	**15.46**
LP's profit (Fee - LVR)	Weekly	-227.64	-233.96	-233.45	-230.57	-228.03	**-220.93**
	Monthly	-934.86	-935.85	-933.81	-922.30	-912.11	**-883.74**
	Long-Term	-4674.28	-4679.25	-4669.07	-4611.49	-4560.57	**-4418.69**

Table 7: Empirical results (Simulation 1): SHIB/ETH (1%)

Boost ratio (m)		0	0.03	0.07	0.10	0.15	0.20
Boost counts		-	98	95	105	95	86
Realized Volatility (1e-3)		2.85	2.85	2.74	2.68	2.73	**2.46**
Slippage (bp)		59.81	**59.69**	**59.69**	**59.69**	59.76	59.99
LP's profit (Fee - LVR)	Weekly	1.83	1.84	1.88	1.89	1.93	**2.04**
	Monthly	7.72	7.77	7.96	8.01	8.16	**8.61**
	Long-Term	50.27	50.50	**50.68**	50.66	50.32	48.69

Table 8: Empirical results (Simulation 2): SHIB/ETH (1%)

Boost ratio (m)		0	0.03	0.07	0.10	0.15	0.20
Boost counts		-	32	31	33	33	30
Realized Volatility (1e-3)		2.85	2.85	2.70	2.63	2.49	**2.40**
Slippage (bp)		59.81	59.78	59.50	59.36	59.11	**58.99**
LP's profit (Fee - LVR)	Weekly	1.83	1.59	**2.94**	1.60	2.17	2.20
	Monthly	7.72	6.71	**12.80**	6.74	9.60	9.36
	Long-Term	50.27	42.60	**80.90**	50.69	61.28	53.26

ber of boost executions, realized volatility of each of the underlying assets, average slippage for traders, and the LP incentive calculated by subtracting LVR from the collected fees weekly. LP incentive in the tables represents the averages derived from the cumulative sums across different time intervals (weekly, monthly, long-term) spanning the entire period. Here, long-term denotes five months. For both pools, Simulation 1 consistently showed approximately three times larger numbers of boost executions across all ratios compared to Simulation 2, attributed to the stronger boost constraint of Simulation 2. Overall, the introduction of LiqBoost resulted in an increase in economic incentives for traders and LPs, particularly observed in reduced slippage and increased profits of LPs.

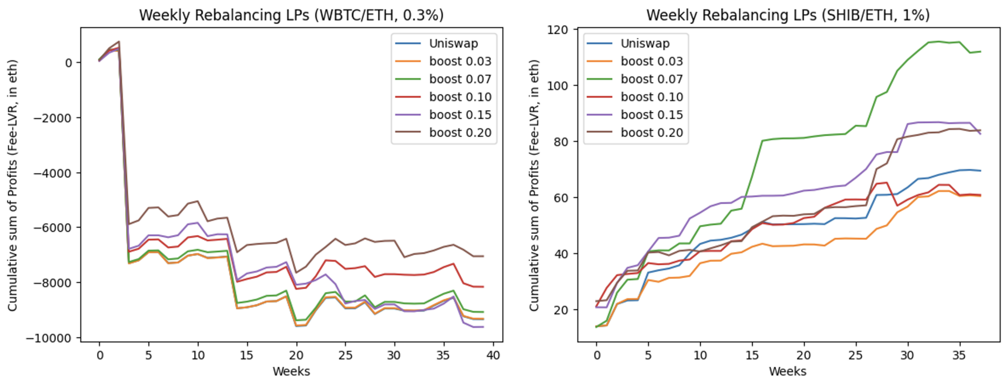

Figure 5: Cumulative profit of LPs for 40 weeks.

In terms of realized volatility, the SHIB/ETH pool experienced a more pronounced boost effect compared to the WBTC/ETH pool. When comparing the results of the same pool with different simulation settings, we observed smaller realized volatility with fewer boost executions in both pools. However, a significant difference emerged between the two distinct pools. As the boost ratio increases, the WBTC/ETH pool shows an increase in realized volatility, whereas the SHIB/ETH pool demonstrates a decrease in realized volatility. We interpret this as empirical evidence of our findings on tick-cross swaps in Section 5, showing that depending on the characteristics of the liquidity pool, excessive use of the boosted liquidity might result in unfavorable outcomes as it generates a larger liquidity gap between the buckets, leading to higher price volatility.

Moreover, there was not a significant difference in the improvement of slippage among different boost ratios observed in both pools. However, both pools demonstrated overall enhancement in slippage compared to scenarios without boost, with the WBTC/ETH pool exhibiting a more significant improvement. In the SHIB/ETH pool, slippage was improved except for $m = 0.2$ in Simulation 1. It's important to note that in a liquidity pool with low liquidity and highly volatile assets, a consistent boost effect is not guaranteed and may result in a lack of liquidity in certain price intervals, thereby leading to increased slippage for traders.

Table 5 and Table 6 indicate that in the WBTC/ETH pool, $m = 0.2$ demonstrates the most significant cost enhancement for LPs compared to the other boost ratios. Simulation 1 resulted in higher profits than Simulation 2 (stronger boost constraints). Table 7 and Table 8 indicate in the SHIB/ETH pool, the boost ratio $m = 0.07$ appears to yield the highest profits for LPs. We attribute the variation in the best-performing setting for LP profitability between the pools to differences in slippage enhancement levels. A lower slippage indicates that trading has been conducted under more favorable conditions for regular traders; therefore, it also implies that LPs have executed trades under more unfavorable conditions. Consequently, for the settings where LP profits were highest, it can be observed that slippage for traders is generally higher. Meanwhile, the cumulative profits of LPs for the entire 40-week period in both liquidity pools are depicted in Figure 5. While cumulative profits in the WBTC pool are negative, in contrast to the positive profits in the SHIB pool, both pools demonstrate enhancements with LiqBoost compared to the Uniswap V3.

6.2.2. Liquidity Enhancement in LiqBoost

To quantify the degree of liquidity enhancement by LiqBoost, we computed the mean difference in liquidity of Uniswap v3 and LiqBoost for different boost ratios within each liquidity pool. The outcomes of these comparisons are presented in Table 9. Positive values in Table 9 indicate that LiqBoost leads to higher liquidity utilization compared to Uniswap V3. Across all ratios, a consistent trend of a larger difference with higher boost ratios is observed in both pools. The most significant enhancement in liquidity occurred at the boost ratio $m = 0.2$ for both pools.

7. Design concept for an on-chain implementation

To guarantee the practicability of the proposed scheme, we illustrate the design concept by introducing protocol variables and the way LPs collect their fees by

7.1. Additional Variables for LiqBoost

In LiqBoost, we introduce two new position variables, `APL` and `APU`, specifically designed to track additional fee income resulting from boosted liquidity. Additionally, we refine the original `LiquidityGross` variable in Uniswap V3 by splitting it into two distinct variables: `LGL` for the lower tick and `LGR` for the upper tick. When a new position is minted, `LiquidityGross` for both the lower tick and upper tick is initially increased. This adjustment ensures that `LGL` for a specific tick represents the total liquidity provided by positions referencing that tick as their lower tick.

Table 9: Liquidity Enhancement by LiqBoost

	WBTC/ETH(0.3%)	SHIB/ETH(1%)
Boost ratio (m)	$H_0 : \mu_1 = \mu_2$	$H_0 : \mu_1 = \mu_2$
0.03	0.25×10^{16} (0.78)	0.52×10^{23} (0.07)
0.07	0.81×10^{16} (2.50*)	0.90×10^{23} (0.12)
0.1	1.31×10^{16} (4.09**)	1.04×10^{23} (0.14)
0.15	3.13×10^{16} (9.56**)	2.88×10^{23} (0.38)
0.2	3.28×10^{16}(10.19**)	5.29×10^{23} (0.69)

Notes. Each entry represents the mean difference between the liquidity pools (Original vs. LiqBoost). The value in the parentheses indicates the T-statistics. * and ** denotes the statistical significance at the 1% and 0.1% level, respectively.

The tick variables in our scheme are detailed in Table 10. Notably, `LiquidityNet` in LiqBoost remains consistent with Uniswap V3. In Uniswap V3, the tick variable `LiquidityNet` achieves gas efficiency by capturing consecutive tick liquidity changes instead of storing all positions contributing to the current bucket. This approach calculates the available liquidity at a specific price by summing the `LiquidityNet` values for all ticks below that price.

When additional liquidity is provided in proportion to the current liquidity (L increasing by a certain percentage m), the token amount to be added is determined by Equation (15), where L_1 is from the left buckets and L_2 is from the right buckets relative to the current bucket.

$$L_1 = \frac{mL(\sqrt{p} - \sqrt{p_k})}{\sqrt{p_k} - \sqrt{p_{k-1}}}, \quad L_2 = \frac{mL(\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p_{k+1}}})}{\frac{1}{\sqrt{p_{k+1}}} - \frac{1}{\sqrt{p_{k+2}}}} \tag{15}$$

After the liquidity from out-of-range position $\kappa(l, p_l, p_u)$ is utilized to enhance contributions to I_k, the position adjusts to $\kappa(l - \delta l, p_l, p_u)$ and gets involved with either $\kappa(mL, p_k, p_{k+\Delta})$ or $\kappa(mL, p_{k-\Delta}, p_k)$. Here, p_k can denote either p_l or p_u. The decreased amount of liquidity for each position at a given time, $\delta l(\kappa(l, p_l, p_u))$, is described in (16). The newly allocated position, denoted as κ, continues to exist within I_k, and each LP involved in κ can burn a portion of their position based on their liquidity.

Table 10: State Variables in LiqBoost

Type	Name	Description
Tick-Indexed	`LGL (LGU)`**	Liquidity Gross by lower (upper) tick
	ΔL	`LiquidityNet`
	f_o	`feeGrowthOutside` of each token
Position-Indexed	`APL (APU)`**	Additional liquidity provision by lower (upper) tick
	`Z`**	Option for position reallocation (Binary)
	l	Liquidity provision
	$f_r(t_0)$	`feeGrowthInside` of each token

Note. ** denotes the newly added variable for LiqBoost compared to the Uniswap V3.

$$\delta l(\kappa) = \begin{cases} \frac{lL_1}{\mathtt{LGU}(p_k)} & \text{if } p_k = p_l \\ \frac{lL_2}{\mathtt{LGL}(p_k)} & \text{if } p_k = p_u \end{cases} \tag{16}$$

7.2. Mint and Burn

In V3, LPs mint their positions by setting liquidity, upper tick, and lower tick parameters. In LiqBoost, they also have the option to automatically reallocate their positions when specific conditions are met during the holding period. These choices of LPs are tracked by the binary variable Z of the contract. When Z is set to 1, the system updates the position's upper and lower tick's tick variables, `LGL` and `LGU` as well as ΔL. Otherwise, only ΔL of both lower and upper ticks are updated, as in Uniswap V3's original scheme. When burning a position, it's crucial to consider that multiple liquidity providers (LPs) from outside the range have contributed to the liquidity within the current bucket I_k. The first element of position variables `APU` and `APL` tracks the reallocated liquidity amounts for each LP. The burned amount of each token is calculated by multiplying `APU[0]` (or `APL[0]`) by both amounts of token X and Y in I_u (or $I_{l-\Delta}$) and then dividing by 2. This equal division ensures that both LPs on each side contribute equally to enhancing liquidity by combining additional liquidity within the current bucket as the tick crosses its midpoint. It's important to note that the liquidity of the original position within the range $[p_l, p_u]$ has decreased by the reallocated amounts.

7.3. *Collected Fees Calculation*

LPs who set $Z = 1$ can earn extra fees when the liquidity pool price slightly goes outside the current range. For example, in Figure 6, suppose there is a position with the price range $[p_l, p_u]$ minted at t_0 and burned at t_7, with liquidity boosted at t_2 by the upper tick, and at t_5 by the lower tick. The added liquidity, described in Equation (15) for each position, is saved as the first element of the position variable `APU` or `APL`. While positions with the same price range in the current Uniswap V3 scheme are only active during the period depicted by the red line in Figure 6, positions in our scheme also benefit during the period depicted by the blue line. Calculating extra fees during the time within the blue lines follows a similar method as Uniswap V3. In Uniswap V3, collected fees are calculated by the difference between $f_r(t_0)$ and $f_r(t_7)$, where t_0 is the time of minting a position, and t_7 is the time of calculating collected fees (Adams et al., 2021). Similarly, in our scheme, we define $g_r^u(t)$ in Equation (17), representing the difference between the `feeGrowthInside` of the original position and that of the position in the right bucket $[p_u, p_{u+\Delta}]$. The same logic applies to the definition of $g_r^l(t)$ within the bucket $[p_{l-\Delta}, p_l]$.

$$g_r^u(t) = f_r^u(t) - f_r(t) = f_r(p_u, p_{u+\Delta}; t) - f_r(p_l, p_u; t) \tag{17}$$

The values of g_r^u and g_r^l can be calculated directly using the tick variables at the time of position burning. The computed value is then used to update the second element of `APU` (or `APL`), whenever extra liquidity is added to the current bucket.

The computation of additional fees, depicted as the blue box in Figure 6, is based on the position variables `APU` and `APL`, which are updated every time extra liquidity is added to the current time interval. These variables are tuples with the first element (`[0]`) indicating the added liquidity amount, and the second value (`[1]`) representing the newly calculated g_r^u (or g_r^l) at the time of updating. In summary, the extra collected fees for a single position in our scheme can be formulated as $\frac{1}{2}\{\mathtt{APU[0]}(g_r^u(t_7) - \mathtt{APU[1]}) + \mathtt{APL[0]}(g_r^l(t_7) - \mathtt{APL[1]})\}$.

8. Conclusion

In this work, we propose a simple-to-implement DEX liquidity allocation scheme based on concentrated liquidity AMM, aiming to enhance liquidity by automatically reallocating inactive LP positions. This scheme highlights the importance of analyzing the economic incentives of

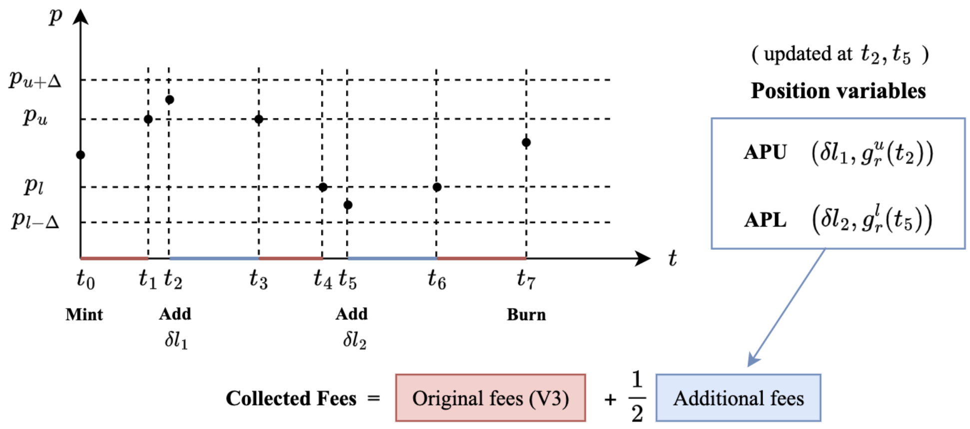

Figure 6: Calculation for the collected fees in LiqBoost

both traders and LPs within DEX ecosystems. Utilizing historical transaction data from Uniswap V3, our empirical analysis validates the practical impact of LiqBoost on both LPs and traders. Overall, LiqBoost enhances economic incentives for both LPs and traders, as evidenced by reduced slippage and increased profits for LPs.

Further research could expand into four critical directions. First, while the paper primarily examines the incentives of the entire LPs within the pool, exploring the individual LP incentive by tracking the specific position within the liquidity pool will provide a comprehensive understanding of the motivations driving their participation as LP. Considering the impact of LiqBoost on individual LPs' liquidity provision strategies, conducting the game-theoretical analysis of the interaction between traders and LPs would also be meaningful. Second, exploring a spectrum of diverse liquidity pools encompassing stable/risky pairs (e.g., USDC/ETH) and major/minor pairs (e.g., ETH/PEPE) might offer valuable insights into the operational dynamics and characteristics of various liquidity pools in DEX. Third, optimizing the boost ratio and identifying suitable boost conditions for LiqBoost aims to refine and maximize the effectiveness of the boost mechanism, ultimately enhancing the liquidity provision. Lastly, integrating the proposed scheme into Uniswap V4 through hook contracts would present a practical avenue for advancing decentralized exchange operations (Adams et al., 2023). This integration provides LPs with the opportunity to customize their liquidity pools within the upcoming Uniswap V4, enabling them to maximize profitability.

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