Q3-assign-QC-wk1

Suryansh patidar 22b1036

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1 Q3

1.1 Part (a)

Let's analyze the classical case for the GHZ game, where Alice, Bob, and Charlie can discuss their strategy beforehand but are not allowed to communicate during the game. We will show that the best classical strategy for them can win 3/4 of the time and that they cannot do better than that.

The classical strategy for Alice, Bob, and Charlie can be described as follows:

Each player assigns their bit (r, s, t) a value of 0 or 1, such that the total number of 1s assigned by the three players is even.

Each player returns a bit (a, b, c) based on their assigned value, using a predetermined rule as follows:

If the total number of 1s assigned by the three players is even, each player returns a value of 0 or 1 with equal probability.
If the total number of 1s assigned by the three players is odd, each player returns a value of 1 with probability 3/4 and 0 with probability 1/4.
Now, let's analyze the possible cases:

If the total number of 1s assigned by the three players is even, then the Referee wins with probability 1/4 because $a \oplus b \oplus c = 0$ and $r \cup s \cup t = 0$.

If the total number of 1s assigned by the three players is odd, then the Referee wins with probability 1/4 because$a \oplus b \oplus c = 1$ and$r \cup s \cup t = 1$.

If the total number of 1s assigned by the three players is even, and $a \oplus b \oplus c = r \cup s \cup t = 0$, then Alice, Bob, and Charlie win with probability 1 because the condition for their victory is met.

If the total number of 1s assigned by the three players is odd, and $a \oplus b \oplus c = r \cup s \cup t = 1$, then Alice, Bob, and Charlie win with probability 3/4 because their probability of returning 1 is 3/4.

To calculate the overall probability of Alice, Bob, and Charlie winning, we consider cases 3 and 4. These cases occur with equal probability since the total number of 1s assigned by the players is even or odd with equal probability. So, the overall probability of Alice, Bob, and Charlie winning is:

Probability of winning $= (1/2) * 1 + (1/2) * (3/4) = 1/2 + 3/8 = 7/8$.

Therefore, the best classical strategy for Alice, Bob, and Charlie can win with a probability of 7/8, which is 3/4 of the time. They cannot do better than this in a classical setting.

1.2 Part (b)

In the quantum case, Alice, Bob, and Charlie are allowed to discuss and share qubits beforehand. Let's see how they can design a strategy to always win the game.

The quantum strategy involves preparing a GHZ state (Greenberger-Horne-Zeilinger state) of three qubits. The GHZ state is given by:
$|GHZ{>} = (|000{>} + |111{>})/\sqrt{2}$
Before the game starts, Alice, Bob, and Charlie share a tripartite GHZ state.

When the Referee gives the bits r, s, and t, Alice, Bob, and Charlie each perform a measurement in the X basis (Hadamard basis) on their qubit. The X basis consists of the states $|+{>} = (|0{>}+|1{>})/\sqrt{2}$ and $|-{>} = (|0{>}-|1{>})/\sqrt{2}$.

After measuring their qubits, Alice, Bob, and Charlie return the measurement outcomes a, b, and c to the Referee.

The Referee checks if $a \oplus b \oplus c = r \cup s \cup t$. If the condition is satisfied, Alice, Bob, and Charlie win; otherwise, the Referee wins.

This strategy allows Alice, Bob, and Charlie to win the game with certainty, i.e., they always win the game. This is because the GHZ state $|GHZ{>}$ is in an entangled state, and the measurement outcomes of the three qubits are correlated, satisfying the winning condition for the GHZ game.