Module-3

3.1

Pigeon Hole Principle

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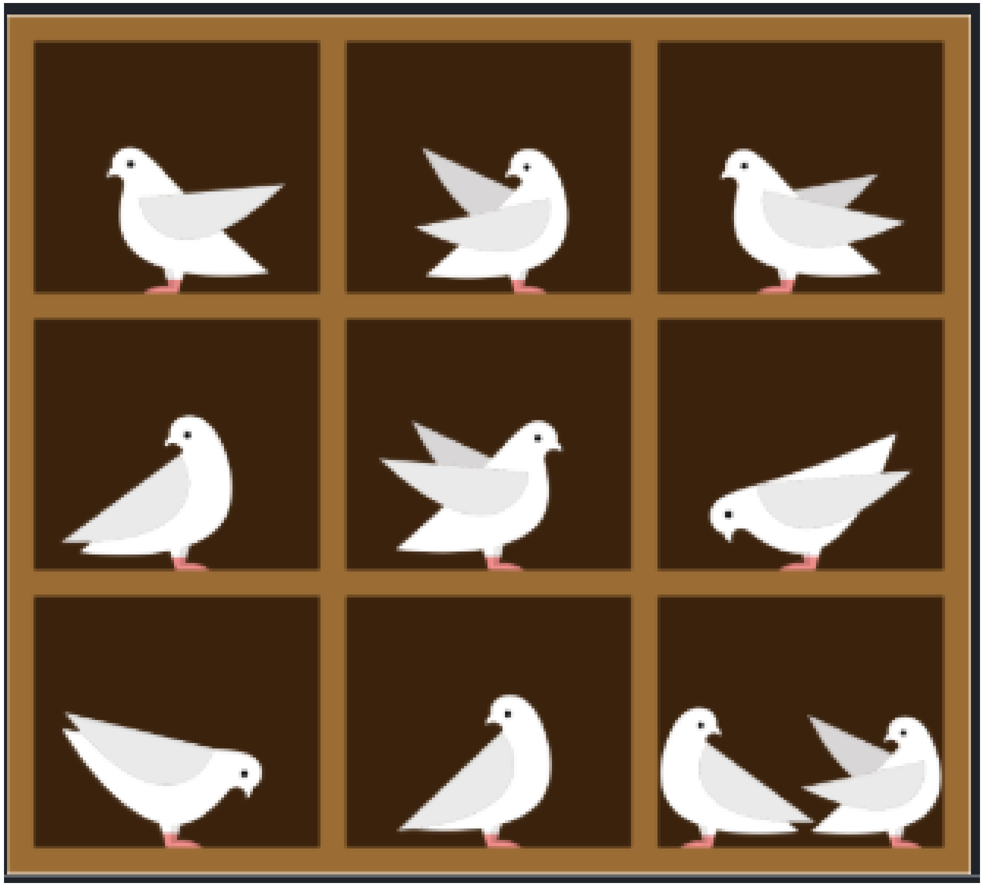
Pigeon hole principle

when there are k pigeonholes and k+1 pigeons, then there will be 1 pigeonhole with at least 2 pigeons.

In General, If n pigeons are assigned to m Pigeonholes and $m < n$ then at least one pigeonhole contains more than one pigeons.



Example 1

If 13 people are involved in a survey to determine the month of their birthday, then there are at least 2 people who will have their birthday falling in the same month.

Ans. As we all know, there are 12 months in a year, thus, even if the first 12 people have their birthday from the month of January to the month of December, the 13th person has to have his birthday in any of the month of January to December.

13 pigeons and 12 pigeonholes, 13 > 12 Thus, by PHP we are right to say that there are at least 2 people who have their birthday falling in the same month.

Example:2

Given any distinct 6 integers from 1 to 10, show that there is at least one pair such that sum of the numbers is 11.

Solution: We construct 5 sets of two elements each from 1 to 10 such that their sum is 11,

these are Possible pigeonholes: {1,10},{2,9},{3,8},{4,7},{5,6}

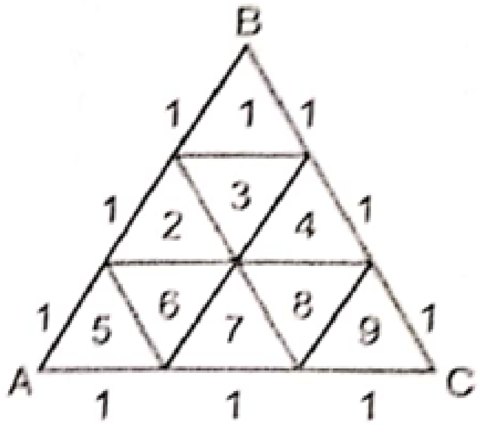
There are 5 pigeonholes and 6 pigeons,

hence By PHP, there exists a pigeonhole with 2 pigeons. i.e. 2 numbers from same set and their sum will be 11

EXAMPLE:3

If 10 points are to be chosen in an equilateral triangle of side three units then show that there are at least 2 points at a distance less than one unit .

Let ΔABC be an equilateral triangle of side 3 units.
Divide each side into 3 equal parts and mark the points.
Join these points as shown in the figure.



So there are 9 equilateral triangles of side 1 unit in the ΔABC.
And we have to choose 10 points in the ΔABC.
So using pigeonhole principle ,to put 10(points) pigeons into 9(equilateral triangles of side 1 unit)pigeonholes there will be 1 pigeonhole with at least 2 pigeons i.e. there will be 1 equilateral triangle of side 1 unit with at least 2 points .So distance between these 2 points will be less than or equal to 1 unit

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Exercise

- If 7 points are to be chosen in a regular hexagon of side one unit then show that there are atleast 2 points at a distant less than one unit .
- If 5 points are to be chosen in a square of side 2 units then show that there are atleast 2 points at a distant less than $\sqrt{2}$ units .
- If 7 positive integers with distinct unit places are chosen then show that there are 2 positive integers whose sum is divisible by 10.

Extended Pigeon hole principle

$\left[\frac{n}{m}\right]$ denotes quotient obtained by dividing n by m. e.g. $\left[\frac{63}{8}\right] = 7$

EPHP - If there are m pigeonholes and n pigeons with m < n then one of the pigeonhole contains at least $\left[\frac{n-1}{m}\right] + 1$ pigeons

Example: Show that in a group of 50 students at least 5 are born in same month.

Solution: Consider 12 months as pigeonholes (m = 12) and 50 students as pigeons (n = 50), 12 < 50

Then By EPHP,

There one of the pigeonhole (month) contains birthdate of $\left[\frac{50-1}{12}\right] + 1 = 5$ pigeons (students).

Therefore at least 5 are born in same month.

Example: Find the least number of students in a group such that at least 5 are born in same month.

Solution: Consider 12 months as pigeonholes (m = 12) and we want at least 5 (k) are born in same month. n is unknown

Then By EPHP,

Least number is given by $(k-1)m+1$

= (5-1)(12) + 1 = 49

Therefore at least 49 students should be in a group so that at least 5 are born in same month.