

```
In [3]: #Author: Suryoday Basak  
#suryodaybasak.info  
import pandas as pd  
import numpy as np  
import matplotlib.pyplot as plt  
import matplotlib as mpl  
plt.style.use('ggplot')  
mpl.rcParams['figure.figsize'] = (10,8)
```

```
In [4]: #Reading the data  
df = pd.read_csv('../datasets/cars/mtcars.csv')  
print(df)
```

	model	mpg	cyl	disp	hp	drat	wt	qsec	vs
am \									
0	Mazda RX4	21.0	6	160.0	110	3.90	2.620	16.46	0
1									
1	Mazda RX4 Wag	21.0	6	160.0	110	3.90	2.875	17.02	0
1									
2	Datsun 710	22.8	4	108.0	93	3.85	2.320	18.61	1
1									
3	Hornet 4 Drive	21.4	6	258.0	110	3.08	3.215	19.44	1
0									
4	Hornet Sportabout	18.7	8	360.0	175	3.15	3.440	17.02	0
0									
5	Valiant	18.1	6	225.0	105	2.76	3.460	20.22	1
0									
6	Duster 360	14.3	8	360.0	245	3.21	3.570	15.84	0
0									
7	Merc 240D	24.4	4	146.7	62	3.69	3.190	20.00	1
0									
8	Merc 230	22.8	4	140.8	95	3.92	3.150	22.90	1
0									
9	Merc 280	19.2	6	167.6	123	3.92	3.440	18.30	1
0									
10	Merc 280C	17.8	6	167.6	123	3.92	3.440	18.90	1
0									
11	Merc 450SE	16.4	8	275.8	180	3.07	4.070	17.40	0
0									
12	Merc 450SL	17.3	8	275.8	180	3.07	3.730	17.60	0
0									
13	Merc 450SLC	15.2	8	275.8	180	3.07	3.780	18.00	0
0									
14	Cadillac Fleetwood	10.4	8	472.0	205	2.93	5.250	17.98	0
0									
15	Lincoln Continental	10.4	8	460.0	215	3.00	5.424	17.82	0
0									
16	Chrysler Imperial	14.7	8	440.0	230	3.23	5.345	17.42	0
0									
17	Fiat 128	32.4	4	78.7	66	4.08	2.200	19.47	1
1									
18	Honda Civic	30.4	4	75.7	52	4.93	1.615	18.52	1
1									
19	Toyota Corolla	33.9	4	71.1	65	4.22	1.835	19.90	1
1									
20	Toyota Corona	21.5	4	120.1	97	3.70	2.465	20.01	1
0									
21	Dodge Challenger	15.5	8	318.0	150	2.76	3.520	16.87	0
0									
22	AMC Javelin	15.2	8	304.0	150	3.15	3.435	17.30	0
0									
23	Camaro Z28	13.3	8	350.0	245	3.73	3.840	15.41	0
0									
24	Pontiac Firebird	19.2	8	400.0	175	3.08	3.845	17.05	0
0									
25	Fiat X1-9	27.3	4	79.0	66	4.08	1.935	18.90	1
1									
26	Porsche 914-2	26.0	4	120.3	91	4.43	2.140	16.70	0
1									
27	Lotus Europa	30.4	4	95.1	113	3.77	1.513	16.90	1

1										
28	1	Ford Pantera L	15.8	8	351.0	264	4.22	3.170	14.50	0
29	1	Ferrari Dino	19.7	6	145.0	175	3.62	2.770	15.50	0
30	1	Maserati Bora	15.0	8	301.0	335	3.54	3.570	14.60	0
31	1	Volvo 142E	21.4	4	121.0	109	4.11	2.780	18.60	1
	1									

	gear	carb
0	4	4
1	4	4
2	4	1
3	3	1
4	3	2
5	3	1
6	3	4
7	4	2
8	4	2
9	4	4
10	4	4
11	3	3
12	3	3
13	3	3
14	3	4
15	3	4
16	3	4
17	4	1
18	4	2
19	4	1
20	3	1
21	3	2
22	3	2
23	3	4
24	3	2
25	4	1
26	5	2
27	5	2
28	5	4
29	5	6
30	5	8
31	4	2

```

In [5]: data_mat = df.values
        r,c = np.shape(data_mat)
        print(np.shape(data_mat))
        print(r)
        ones_stub = np.ones((r,))
        data_mat = np.c_[data_mat, ones_stub]
        print(data_mat)

(32, 12)
32
[['Mazda RX4' 21.0 6 160.0 110 3.9 2.62 16.46 0 1 4 4 1.0]
 ['Mazda RX4 Wag' 21.0 6 160.0 110 3.9 2.875 17.02 0 1 4 4 1.0]
 ['Datsun 710' 22.8 4 108.0 93 3.85 2.32 18.61 1 1 4 1 1.0]
 ['Hornet 4 Drive' 21.4 6 258.0 110 3.08 3.215 19.44 1 0 3 1 1.0]
 ['Hornet Sportabout' 18.7 8 360.0 175 3.15 3.44 17.02 0 0 3 2 1.0]
 ['Valiant' 18.1 6 225.0 105 2.76 3.46 20.22 1 0 3 1 1.0]
 ['Duster 360' 14.3 8 360.0 245 3.21 3.57 15.84 0 0 3 4 1.0]
 ['Merc 240D' 24.4 4 146.7 62 3.69 3.19 20.0 1 0 4 2 1.0]
 ['Merc 230' 22.8 4 140.8 95 3.92 3.15 22.9 1 0 4 2 1.0]
 ['Merc 280' 19.2 6 167.6 123 3.92 3.44 18.3 1 0 4 4 1.0]
 ['Merc 280C' 17.8 6 167.6 123 3.92 3.44 18.9 1 0 4 4 1.0]
 ['Merc 450SE' 16.4 8 275.8 180 3.07 4.07 17.4 0 0 3 3 1.0]
 ['Merc 450SL' 17.3 8 275.8 180 3.07 3.73 17.6 0 0 3 3 1.0]
 ['Merc 450SLC' 15.2 8 275.8 180 3.07 3.78 18.0 0 0 3 3 1.0]
 ['Cadillac Fleetwood' 10.4 8 472.0 205 2.93 5.25 17.98 0 0 3 4 1.0]
 ['Lincoln Continental' 10.4 8 460.0 215 3.0 5.4239999999999995 17.82 0
0
 3 4 1.0]
 ['Chrysler Imperial' 14.7 8 440.0 230 3.23 5.345 17.42 0 0 3 4 1.0]
 ['Fiat 128' 32.4 4 78.7 66 4.08 2.2 19.47 1 1 4 1 1.0]
 ['Honda Civic' 30.4 4 75.7 52 4.93 1.615 18.52 1 1 4 2 1.0]
 ['Toyota Corolla' 33.9 4 71.1 65 4.22 1.835 19.9 1 1 4 1 1.0]
 ['Toyota Corona' 21.5 4 120.1 97 3.7 2.465 20.01 1 0 3 1 1.0]
 ['Dodge Challenger' 15.5 8 318.0 150 2.76 3.52 16.87 0 0 3 2 1.0]
 ['AMC Javelin' 15.2 8 304.0 150 3.15 3.435 17.3 0 0 3 2 1.0]
 ['Camaro Z28' 13.3 8 350.0 245 3.73 3.84 15.41 0 0 3 4 1.0]
 ['Pontiac Firebird' 19.2 8 400.0 175 3.08 3.845 17.05 0 0 3 2 1.0]
 ['Fiat X1-9' 27.3 4 79.0 66 4.08 1.935 18.9 1 1 4 1 1.0]
 ['Porsche 914-2' 26.0 4 120.3 91 4.43 2.14 16.7 0 1 5 2 1.0]
 ['Lotus Europa' 30.4 4 95.1 113 3.77 1.5130000000000001 16.9 1 1 5 2
1.0]
 ['Ford Pantera L' 15.8 8 351.0 264 4.22 3.17 14.5 0 1 5 4 1.0]
 ['Ferrari Dino' 19.7 6 145.0 175 3.62 2.77 15.5 0 1 5 6 1.0]
 ['Maserati Bora' 15.0 8 301.0 335 3.54 3.57 14.6 0 1 5 8 1.0]
 ['Volvo 142E' 21.4 4 121.0 109 4.11 2.78 18.6 1 1 4 2 1.0]]

```

```
In [6]: #Define x and y
x = data_mat[:,2:]
y = data_mat[:,1]

print('The x is:')
print(x)
print('')
print('The y is:')
print(y)
```

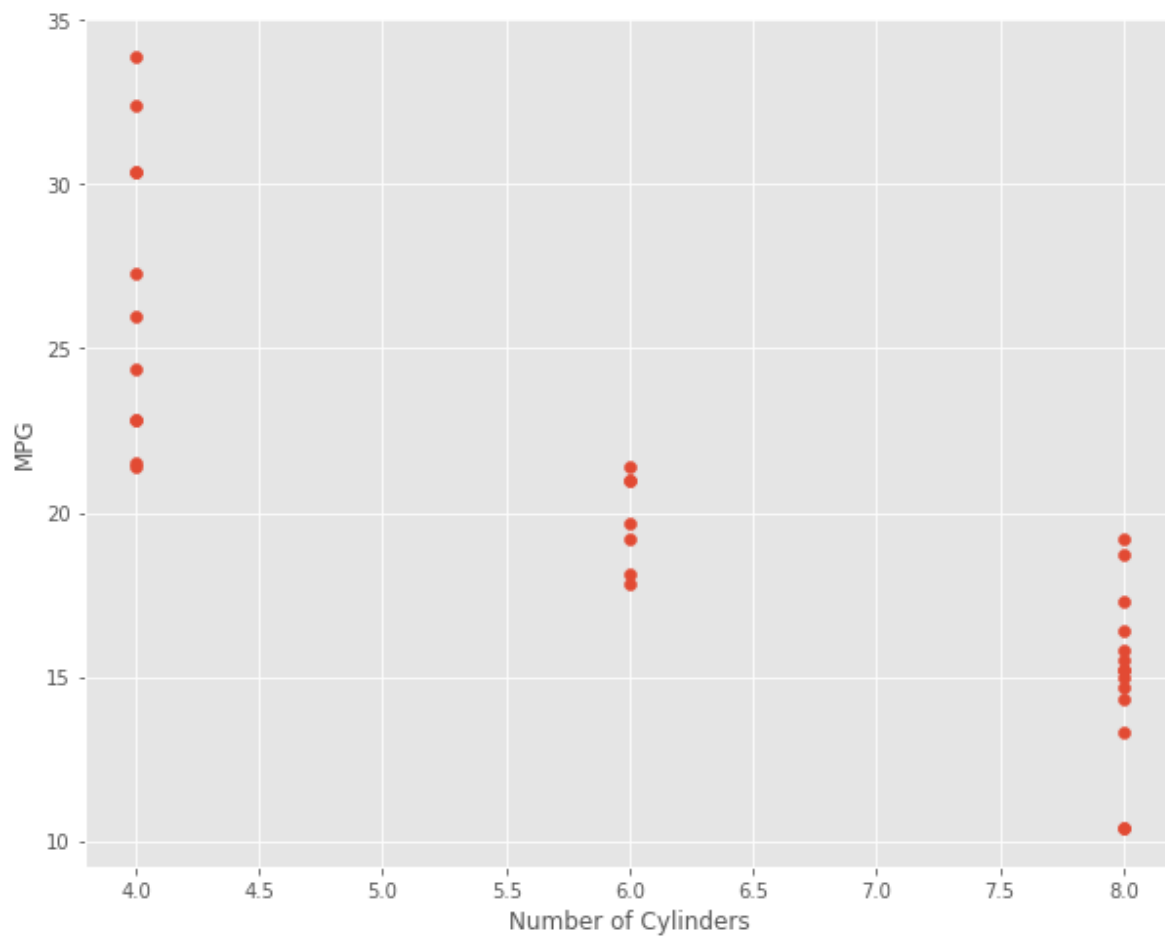
The x is:

```
[[6 160.0 110 3.9 2.62 16.46 0 1 4 4 1.0]
 [6 160.0 110 3.9 2.875 17.02 0 1 4 4 1.0]
 [4 108.0 93 3.85 2.32 18.61 1 1 4 1 1.0]
 [6 258.0 110 3.08 3.215 19.44 1 0 3 1 1.0]
 [8 360.0 175 3.15 3.44 17.02 0 0 3 2 1.0]
 [6 225.0 105 2.76 3.46 20.22 1 0 3 1 1.0]
 [8 360.0 245 3.21 3.57 15.84 0 0 3 4 1.0]
 [4 146.7 62 3.69 3.19 20.0 1 0 4 2 1.0]
 [4 140.8 95 3.92 3.15 22.9 1 0 4 2 1.0]
 [6 167.6 123 3.92 3.44 18.3 1 0 4 4 1.0]
 [6 167.6 123 3.92 3.44 18.9 1 0 4 4 1.0]
 [8 275.8 180 3.07 4.07 17.4 0 0 3 3 1.0]
 [8 275.8 180 3.07 3.73 17.6 0 0 3 3 1.0]
 [8 275.8 180 3.07 3.78 18.0 0 0 3 3 1.0]
 [8 472.0 205 2.93 5.25 17.98 0 0 3 4 1.0]
 [8 460.0 215 3.0 5.4239999999999995 17.82 0 0 3 4 1.0]
 [8 440.0 230 3.23 5.345 17.42 0 0 3 4 1.0]
 [4 78.7 66 4.08 2.2 19.47 1 1 4 1 1.0]
 [4 75.7 52 4.93 1.615 18.52 1 1 4 2 1.0]
 [4 71.1 65 4.22 1.835 19.9 1 1 4 1 1.0]
 [4 120.1 97 3.7 2.465 20.01 1 0 3 1 1.0]
 [8 318.0 150 2.76 3.52 16.87 0 0 3 2 1.0]
 [8 304.0 150 3.15 3.435 17.3 0 0 3 2 1.0]
 [8 350.0 245 3.73 3.84 15.41 0 0 3 4 1.0]
 [8 400.0 175 3.08 3.845 17.05 0 0 3 2 1.0]
 [4 79.0 66 4.08 1.935 18.9 1 1 4 1 1.0]
 [4 120.3 91 4.43 2.14 16.7 0 1 5 2 1.0]
 [4 95.1 113 3.77 1.5130000000000001 16.9 1 1 5 2 1.0]
 [8 351.0 264 4.22 3.17 14.5 0 1 5 4 1.0]
 [6 145.0 175 3.62 2.77 15.5 0 1 5 6 1.0]
 [8 301.0 335 3.54 3.57 14.6 0 1 5 8 1.0]
 [4 121.0 109 4.11 2.78 18.6 1 1 4 2 1.0]]
```

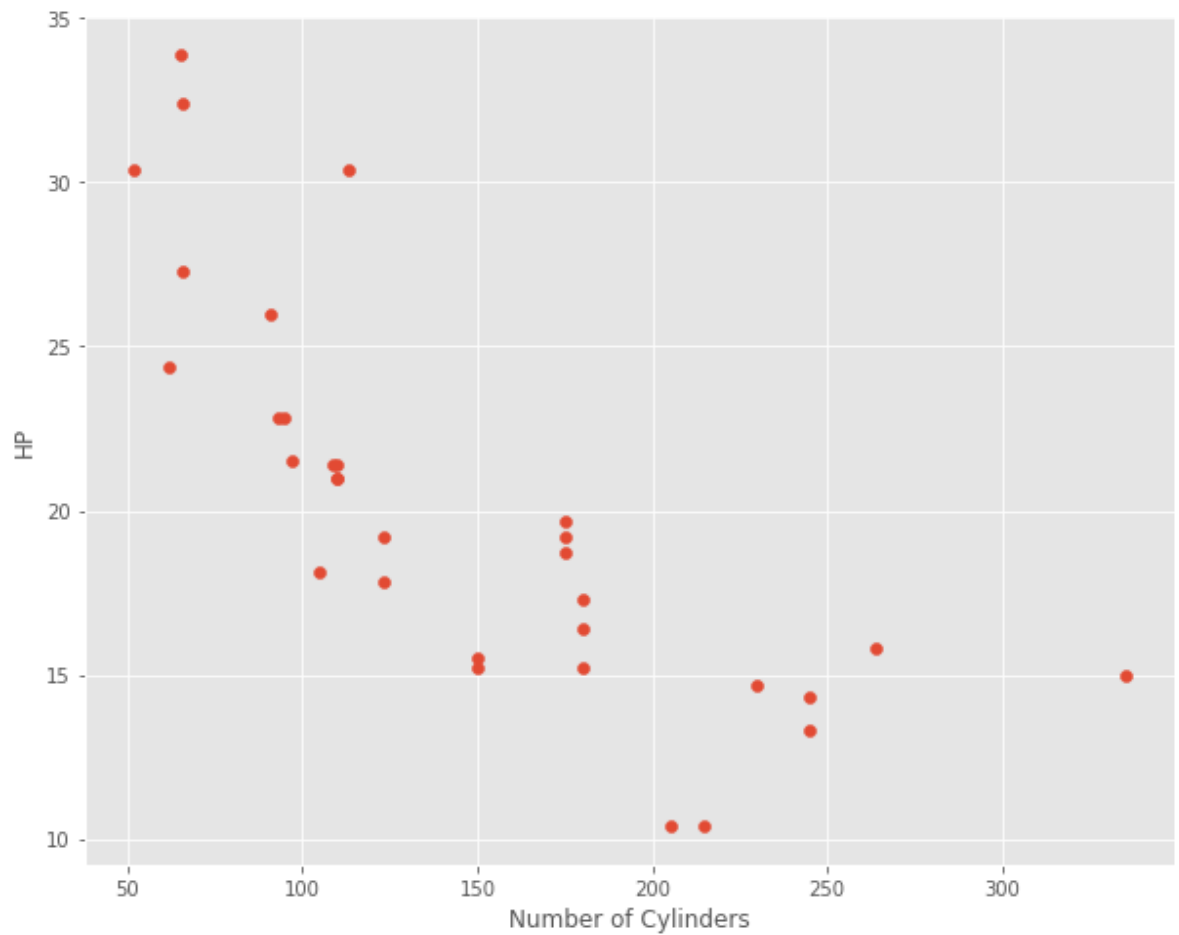
The y is:

```
[21.0 21.0 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 17.8 16.4 17.3 15.2
 10.4 10.4 14.7 32.4 30.4 33.9 21.5 15.5 15.2 13.3 19.2 27.3 26.0 30.4
 15.8 19.7 15.0 21.4]
```

```
In [7]: #Visualize cyl vs mpg  
plt.scatter(x[:,0], y)  
plt.xlabel('Number of Cylinders')  
plt.ylabel('MPG')  
plt.show()
```



```
In [8]: #Visualize hp vs mpg
plt.scatter(x[:,2], y)
plt.xlabel('Number of Cylinders')
plt.ylabel('HP')
plt.show()
```



```
In [12]: #Find the number of features
_, n = np.shape(x)
print(n)
```

11



```
In [13]: #Our matrix system will have n+1 entries, with the 1 addition correspo  
nding to the intercept  
A = np.zeros((n,n))  
print(A)
```

```
[[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
```

```
In [17]: for i in range(0,n):
          for j in range(0,n):
              A[i,j] = np.sum(np.multiply(x[:,i],x[:,j]))

#Populating the last row of the matrix
          for j in range(0,n):
              A[n-1,j] = np.sum(sum(x[:,j]))

print(A)
```

```
[ [1.32400000e+03 5.18724000e+04 3.22040000e+04 6.91400000e+02
   6.79404000e+02 3.47556000e+03 6.40000000e+01 6.60000000e+01
   7.10000000e+02 6.04000000e+02 1.98000000e+02]
  [5.18724000e+04 2.17962747e+06 1.29136440e+06 2.50947960e+04
   2.70914888e+04 1.28801504e+05 1.85440000e+03 1.86590000e+03
   2.56503000e+04 2.32161000e+04 7.38310000e+03]
  [3.22040000e+04 1.29136440e+06 8.34278000e+05 1.63722800e+04
   1.64717440e+04 8.10921600e+04 1.27900000e+03 1.64900000e+03
   1.71120000e+04 1.57760000e+04 4.69400000e+03]
  [6.91400000e+02 2.50947960e+04 1.63722800e+04 4.22790700e+02
   3.58718960e+02 2.05691400e+03 5.40300000e+01 5.26500000e+01
   4.32950000e+02 3.21260000e+02 1.15090000e+02]
  [6.79404000e+02 2.70914888e+04 1.64717440e+04 3.58718960e+02
   3.60901070e+02 1.82809458e+03 3.65580000e+01 3.13430000e+01
   3.66582000e+02 3.10502000e+02 1.02952000e+02]
  [3.47556000e+03 1.28801504e+05 8.10921600e+04 2.05691400e+03
   1.82809458e+03 1.02934802e+04 2.70670000e+02 2.25680000e+02
   2.09746000e+03 1.54767000e+03 5.71160000e+02]
  [6.40000000e+01 1.85440000e+03 1.27900000e+03 5.40300000e+01
   3.65580000e+01 2.70670000e+02 1.40000000e+01 7.00000000e+00
   5.40000000e+01 2.50000000e+01 1.40000000e+01]
  [6.60000000e+01 1.86590000e+03 1.64900000e+03 5.26500000e+01
   3.13430000e+01 2.25680000e+02 7.00000000e+00 1.30000000e+01
   5.70000000e+01 3.80000000e+01 1.30000000e+01]
  [7.10000000e+02 2.56503000e+04 1.71120000e+04 4.32950000e+02
   3.66582000e+02 2.09746000e+03 5.40000000e+01 5.70000000e+01
   4.52000000e+02 3.42000000e+02 1.18000000e+02]
  [6.04000000e+02 2.32161000e+04 1.57760000e+04 3.21260000e+02
   3.10502000e+02 1.54767000e+03 2.50000000e+01 3.80000000e+01
   3.42000000e+02 3.34000000e+02 9.00000000e+01]
  [1.98000000e+02 7.38310000e+03 4.69400000e+03 1.15090000e+02
   1.02952000e+02 5.71160000e+02 1.40000000e+01 1.30000000e+01
   1.18000000e+02 9.00000000e+01 3.20000000e+01]]
```

```
In [19]: #This vector will store the RHS of the matrix system
          b = np.zeros((n,))

          for i in range(0,n):
              b[i] = np.sum(x[:,i]*y)
          #b[n] = np.sum(y) #This term will hold sum(yi)

          print(b)
```

```
[ 3693.6    128705.08    84362.7    2380.277    1909.7528    11614.745
   343.8         317.1         2436.9         1641.9         642.9    ]
```

```
In [20]: #Finding the inverse
A_inv = np.linalg.inv(A)
print(A_inv)
```

```
[[ 1.55487569e-01 -7.07183997e-04 -5.95395085e-04  6.70914657e-02
   3.18058726e-02  2.98592741e-02  1.00440083e-01  7.98098197e-02
   7.83313749e-02 -2.81337389e-02 -1.87424239e+00]
 [-7.07183997e-04  4.54030479e-05 -2.90503410e-05 -4.81865226e-04
  -3.70558863e-03  5.30481853e-04  5.35944692e-04  1.40983761e-04
  -2.98197687e-04  1.42126468e-03 -8.41481423e-04]
 [-5.95395085e-04 -2.90503410e-05  6.74689335e-05  4.40699441e-04
  1.41161399e-03  2.45554276e-04 -1.77604959e-03 -3.02647283e-04
  -4.14002931e-04 -1.35265847e-03 -3.78868814e-03]
 [ 6.70914657e-02 -4.81865226e-04  4.40699441e-04  3.80782830e-01
  7.42871900e-02  6.18475667e-03 -1.45767070e-02 -7.48973106e-02
  -2.60486727e-02 -3.98232345e-02 -1.84263491e+00]
 [ 3.18058726e-02 -3.70558863e-03  1.41161399e-03  7.42871900e-02
  5.10967881e-01 -9.99519610e-02  4.81275585e-02  5.23321257e-02
  7.30403152e-02 -1.55368805e-01  4.49345889e-01]
 [ 2.98592741e-02  5.30481853e-04  2.45554276e-04  6.18475667e-03
 -9.99519610e-02  7.60490849e-02 -7.99725516e-02  5.86241152e-02
  1.25790869e-02  2.31673742e-02 -1.50159397e+00]
 [ 1.00440083e-01  5.35944692e-04 -1.77604959e-03 -1.45767070e-02
  4.81275585e-02 -7.99725516e-02  6.30587107e-01  1.29246808e-01
 -1.94917069e-02  2.30188288e-02  5.19141666e-01]
 [ 7.98098197e-02  1.40983761e-04 -3.02647283e-04 -7.48973106e-02
  5.23321257e-02  5.86241152e-02  1.29246808e-01  6.02233193e-01
 -1.37190334e-01  1.50552284e-02 -1.26497276e+00]
 [ 7.83313749e-02 -2.98197687e-04 -4.14002931e-04 -2.60486727e-02
  7.30403152e-02  1.25790869e-02 -1.94917069e-02 -1.37190334e-01
  3.17478643e-01 -7.47704807e-02 -1.61711918e+00]
 [-2.81337389e-02  1.42126468e-03 -1.35265847e-03 -3.98232345e-02
 -1.55368805e-01  2.31673742e-02  2.30188288e-02  1.50552284e-02
 -7.47704807e-02  9.77897589e-02  2.58652641e-01]
 [-1.87424239e+00 -8.41481423e-04 -3.78868814e-03 -1.84263491e+00
  4.49345889e-01 -1.50159397e+00  5.19141666e-01 -1.26497276e+00
 -1.61711918e+00  2.58652641e-01  4.98835322e+01]]
```

```
In [21]: #Computing the slopes corresponding to each variable
theta = np.matmul(A_inv,b)
print(theta)
```

```
[-0.11144048  0.01333524 -0.02148212  0.78711097 -3.71530393  0.8210407
 5
 0.31776281  2.52022689  0.65541302 -0.19941925 12.30337416]
```

```

In [26]: #Finding the error and displaying the difference between original and
          estimated
          error = 0.0
          r, c = np.shape(x)
          for i in range(0,r):
              i_est = np.sum(x[i,:]*theta)
              error += (i_est - y[i])**2 #(Estimated - original)^2
              print('Original:\t', y[i], '\tEstimated:\t',i_est)

          error/=n
          print("The mean squared error is:\t\t", error)
          print("The root means squared error is:\t", error**(0.5))

```

Original:	21.0	Estimated:	22.599505761262435
Original:	21.0	Estimated:	22.111886079356715
Original:	22.8	Estimated:	26.250644084799433
Original:	21.4	Estimated:	21.23740454667545
Original:	18.7	Estimated:	17.693434028696203
Original:	18.1	Estimated:	20.383039035671622
Original:	14.3	Estimated:	14.38625625277921
Original:	24.4	Estimated:	22.496011884889107
Original:	22.8	Estimated:	24.419089897563087
Original:	19.2	Estimated:	18.699029942339234
Original:	17.8	Estimated:	19.191654392144116
Original:	16.4	Estimated:	14.172162110442295
Original:	17.3	Estimated:	15.599573596008746
Original:	15.2	Estimated:	15.742224699462273
Original:	10.4	Estimated:	12.034013415125738
Original:	10.4	Estimated:	10.936437710853777
Original:	14.7	Estimated:	10.493629361832134
Original:	32.4	Estimated:	27.77290580777931
Original:	30.4	Estimated:	29.89673891131207
Original:	33.9	Estimated:	29.51236909573993
Original:	21.5	Estimated:	23.643103441605476
Original:	15.5	Estimated:	16.943053221175177
Original:	15.2	Estimated:	17.732181497828407
Original:	13.3	Estimated:	13.30602197619969
Original:	19.2	Estimated:	16.691678988690626
Original:	27.3	Estimated:	28.29346869344556
Original:	26.0	Estimated:	26.15295396086912
Original:	30.4	Estimated:	27.63627258279828
Original:	15.8	Estimated:	18.870040802863176
Original:	19.7	Estimated:	19.693828154474943
Original:	15.0	Estimated:	13.941118382058537
Original:	21.4	Estimated:	24.368267683244035
The mean squared error is:			13.40858454696824
The root means squared error is:			3.661773415569052