Assignment Set for Laboratory 3

- ATSC 409: Hand-in an answers to Question 1 from the Lab.
- EOSC 511/ATSC 506: Hand-in an answer to the supplementary Question, question #5 below

5. Consider a long hallway in an office building. If we assume that any cigarrette smoke, mixes across the width of the hallway and vertically through the depth of the hallway much faster than it mixes along the hallway, we can write the diffusion of cigaratte smoke as an equation

$$\frac{\partial S}{\partial t} = \kappa \frac{\partial^2 S}{\partial x^2} - \gamma S + \alpha(x)$$

where S is the concentration of smoke, κ is the rate of diffusion of smoke, γ is the rate at which the smoke sticks to the walls or otherwise leaves the system, $\alpha(x)$ is the sources of smoke, t is the time and x is distance along the hallway.

   (a) Write the appropriate equation for the steady state.

   (b) Discretize the hall into N segments and write the equation for the steady state as a matrix equation.

   (c) Taking $\alpha(x) = 0.005\delta(x_*)$ kg m^{-1}s^{-1} where you can choose the point x_*, $\kappa = 0.05m^2s^{-1}$, $\gamma = (3600 \text{ s})^{-1}$, find the solution for your choice of N between 5 and 15. Take the length of the hall as 20 m.

   (d) What is the condition number of the matrix?

   (e) If γ is 0 what is the condition number of the matrix? Physically why is there no single solution?

   (f) If γ is 0 and α is 0, why physically is there no single solution?

$\delta(x) = 1$ at x and 0 everywhere else