Assignment Set for Laboratory 3

Fall 2007
ATSC 409: Hand-in answers from the Questions in the Lab pdf numbered: 1 and 2a,c
EOSC 511/ATSC 506: Hand-in answers from the Questions in the Lab pdf numbered: 2 and 3 and the supplementary question 5 below.

5. Consider a long hallway in an office building. If we assume that any cigarette smoke, mixes across the width of the hallway and vertically through the depth of the hallway much faster than it mixes along the hallway, we can write the diffusion of cigarette smoke as an equation

$$\frac{\partial S}{\partial t} = \kappa \frac{\partial S^2}{\partial x^2} - \gamma S + \alpha(x)$$

$$\frac{\partial S}{\partial x} = 0 \text{ at } x = 0, L$$

where S is the concentration of smoke, κ is the rate of diffusion of smoke, γ is the rate at which the smoke sticks to the walls or otherwise leaves the system, $\alpha(x)$ is the sources of smoke, t is the time, x is distance along the hallway and L is total length of the hallway.

   (a) Write the appropriate ordinary differential equation and boundary conditions for the steady state.

   (b) Discretize the hall into N segments and write the equation for the steady state as a matrix equation.

   (c) Taking $\alpha(x) = 0.005\delta(x_*)$ kg m^{-1} s^{-1} where you can choose the point x_*, $\kappa = 0.05$ m^2 s^{-1}, $\gamma = 0.003$ s^{-1}, find the solution for your choice of N between 5 and 15. Take the length of the hall as $L = 20$ m.

   (d) What is the condition number of the matrix?

   (e) If γ is 0 what is the condition number of the matrix? Physically why is there no single solution?

   (f) If γ is 0 and α is 0, why physically is there no single solution? What would you need to specify to get a solution?