1.1)20. determine whether each of these conditional statements is true or false.

a) if 1+1=3, then unicorn exist. (false)

b) if 1+1=3, then dogs can fly. (false)

c) if 1+1=2, then dogs can fly. (false)

d) if 2+2=4, then 1+2=3. (true)

1.2)10. Are these system specifications consistent? “Whenever the system software is being upgraded, users cannot access the file system. If user can access the file system, then they can save the new files. If users cannot save new files, then the system software is not being upgraded.” (No)

1.3)31. Show that p ⬄ q and (p->q) (q->p) are logically equivalent.

|  |  |  |
| --- | --- | --- |
| p | q | p⬄q |
| T | T | T |
| F | T | F |
| T | F | F |
| F | F | T |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | p->q | q->p | (p->q)(q->p) |
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

1.4)24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Everyone in your class has a cellular phone

P (x, y): x has y

T (y): y is cellular phone

b) Somebody in your class has seen a foreign movie.

P (x, y): x watches y

T (y): y is foreign movie

c) There is a person in your class who cannot swim.

P(x): x in your class

T (x): x cannot swim

d) All students in your class can solve quadratic equations.

P(x): Student in your class.

T(x): Students can solve quadratic equations.

e) Some student in your class does not want to be rich.

P(x): Student in your class

T(X): Student want to be rich

1.5/1. Translate these statements into English, where the domain for each variable consists of all real numbers.

a) ∀x∃y(x < y)

All real numbers have a greater number

b) ∀x∀y(((x ≥ 0) ∧ (y ≥ 0)) → (xy ≥ 0)).

All 2 real numbers if greater than 0 then their product will greater than 0.

c) ∀x∀y∃z(xy = z).

There a real number will equal the product of 2 real number

1.6/4. What rule of inference is used in each of these arguments?  
a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials. (Modus ponens)

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous. (Disjunctive syllogism)

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard. (Modus ponens)

d) Steve will work at a computer company this summer.  
Therefore, this summer Steve will work at a computer  
company or he will be a beach bum. (Addition)

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material. (Hypothetical syllogism)

2.1/31. What is the Cartesian product *A* × *B*, where *A* is the set of courses offered by the mathematics department at a  
university and *B* is the set of mathematics professors at  
this university? Give an example of how this Cartesian  
product can be used.

A={a,b,c}

B={1,2,3}

* A x B = {{a,1},{a,2},{a,3},{b,1},{b,2},{b,3},{c,1},{c,2},{c,3}}

2.2/16. Let *A* and *B* be sets. Show that

A= {1,2,5,6,8,12,13,15,16,19}

B= {2,3,7,8,9,12,13,14,16,19}  
**a)** (*A* ∩ *B*) *⊆ A*.

(*A* ∩ *B*) = {2,8,12,13,16,19}

A= {1,2,5,6,8,12,13,15,16,19}

**b)** *A ⊆* (*A* ∪ *B*).

(*A* ∪ *B*) = {1,2,3,5,6,7,8,9,12,13,14,15,16,19}

A= {1,2,5,6,8,12,13,15,16,19}   
**c)** *A* - *B ⊆ A*.

*A* – *B= {1,5,6,15}*

A= {1,2,5,6,8,12,13,15,16,19}

**d)** *A* ∩ (*B* - *A*) = ∅.

(*B* - *A*) = {3,7,9,14}

A= {1,2,5,6,8,12,13,15,16,19}  
**e)** *A* ∪ (*B* - *A*) = *A* ∪ *B*.

(*B* - *A*) = {3,7,9,14}

A = {1,2,5,6,8,12,13,15,16,19}  
*A* ∪ (*B* - *A*) = {1,2,3,5,6,7,8,9,12,13,14,15,16,19}

(*A* ∪ *B*) = {1,2,3,5,6,7,8,9,12,13,14,15,16,19}

2.3/2. Determine whether *f* is a function from **Z** to **R** if  
**a)** *f* (*n*) = ±*n*.  
**b)** *f* (*n*) = √*n*2 + 1.  
**c)** *f* (*n*) = 1∕(*n*2 - 4).

1. Given: f(x)=±n  
   f is not a function, because every element x∈Z (except for x=0) has been assigned to more than one value.  
   For example: if x=2, then f(x)=2 and f(x)=−2. A function should have only one value for f(x) when x=0.  
   b) Given f(n)=n2+1  
   f is defined for all n ∈ Z and the function f maps every element of Z to exactly one element in R, thus f is a function.  
   c) Given: f(n)=1n2−4  
   We note that at n=2 or n=-2, the function is not defined, because we cannot divide by 0. This then means that f cannot divide by 0. This then means that f cannot be a function from Z to R, since 2∈Z and −2∈Z

A picture containing text, watch

Description automatically generated

2.4/30. What are the values of these sums, where *S* =  
{1*,* 3*,* 5*,* 7}?  
**a/**

**b/**

**c/**

**d/**

2.5/1. Determine whether each of these sets is finite, countably  
infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.  
**a)** the negative integers (countably infinite)

**b)** the even integers (countably infinite)

**c)** the integers less than 100 (countably infinite)

**d)** the real numbers between 0 and 1/2 (uncountably infinite)  
**e)** the positive integers less than 1,000,000,000. (countable)  
**f )** the integers that are multiples of 7 (countably infinite)

3.2/10. Show that *x*3 is *O*(*x*4) but that *x*4 is not *O*(*x*3)

F(x)=x3

G(x)=x4

Suppose k=1, thus x>1.

|f(x)| < |g(x)| always true. Therefore, *x*3 is *O*(*x*4) but that *x*4 is not *O*(*x*3)

3.3/16. What is the largest n for which one can solve within a day using an algorithm that requires f(n) bit operations, where each bit operation is carried out in 10−11 seconds, with these functions f(n)?

t=24\*60\*60=86400

T=10-11

t/T=8.64x1015

a) log n:

log n =8.64x1015

n=108.64x10^15

b) 1000n

1000n=8.64x1015

n= =8.64x1012

c) n2

n2=8.64x1015

n=

d) 1000n2

1000n2=8.64x1015

n=

e) n3

n3=8.64x1015

n=

f ) 2n

2n=8.64x1015

n=

g) 22n

h) 22^n

4.1/30. Find the integer a such that

a) a ≡ 43 (mod 23) and −22 ≤ a ≤ 0.

a=23n+20

a=23\*(-1) + 20=-3

a=23\*(-2) + 20=-26

Because −22 ≤ a ≤ 0

=> a=-3

b) a ≡ 17 (mod 29) and −14 ≤ a ≤ 14.

a=29n+17

a=29\*(-1) + 17=-12

a=29\*(-2) + 17 = -41

a=29\*0 + 17 = 17

Because −14 ≤ a ≤ 14

=> a=-12

c) a ≡ −11 (mod 21) and 90 ≤ a ≤ 110.

a=21n + 10

a=21\*1 + 10=31

a=21\*2 + 10=52

a=21\*3 + 10=73

a=21\*4 + 10=94

a=21\*5 + 10=115

Because 90 ≤ a ≤ 110

=>a=94

4.2/20. Explain how to convert from binary to base 64 expansions and from base 64 expansions to binary expansions and from octal to base 64 expansions and from base 64 expansions to octal expansions

**Convert from binary to base 64 expansions**

First: convert from binary to decimal by 2n

Second: convert from decimal to base 64 dividing 64

**Convert from base 64 expansions to binary expansions**

First: convert from base 64 to decimal by 64n

Second: convert from decimal to binary dividing 2

**Convert from octal to base 64 expansions**

First: convert from octal to decimal by 8n

Second: convert from decimal to base 64 dividing 64

**Convert from base 64 expansions to octal expansions**

First: convert from base 64 to decimal by 64n

Second: convert from decimal to octal dividing 8

4.3/17.Determine whether the integers in each of these sets are pairwise relatively prime.

a) 11, 15, 19:

11 = {1;11}

15 = {1;3;5;15}

19 = {1;19}

This set is pairwise relatively prime

b) 14, 15, 21:

14 = {1;2;7;14}

15 = {1;3;5;15}

21 = {1;3;7;21}

This set is not pairwise relatively prime

c) 12, 17, 31, 37:

12 = {1;2;3;6;12}

17 = {1;17}

31 = {1;31}

37 = {1;37}

This set is pairwise relatively prime

d) 7, 8, 9, 11

7 = {1;7}

8 = {1;2;4;8}

9 = {1;3;9}

11 = {1;11}

This set is pairwise relatively prime

4.5/15. The first nine digits of the ISBN-10 of the European version of the fifth edition of this book are 0-07-119881. What is the check digit for that book?

0-07-119881: 1\*0+2\*0+3\*7+4\*1+5\*1+6\*9+7\*8+8\*8+9\*1 mod 11

= 213 mod 11

= 4

So the check digit for the book is 4

4.6/3. Encrypt the message WATCH YOUR STEP by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.

W:23,A:1,T:20,C:3,H:8,Y:25,O:15,U:21,R:18,S:19,E:5,P:16

a) f(p) = (p + 14) mod 26

f(23) = (23+14) mod 26 = 37 mod 26 = 11

f(1) = (1+14) mod 26 = 15 mod 26 = 15

f(20) = (20+14) mod 26 = 34 mod 26 = 8

f(3) = (3+14) mod 26 = 17 mod 26 = 17

f(8) = (8+14) mod 26 = 22 mod 26 = 22

f(25) = (25+14) mod 26 = 39 mod 26 =13

f(15) = (15+14) mod 26 = 29 mod 26 = 3

f(21) = (21+14) mod 26 = 35 mod 26 =9

f(18) = (18+14) mod 26 = 32 mod 26 = 6

f(19) = (19+14) mod 26 = 33 mod 26 = 7

f(5) = (5+14) mod 26 = 19 mod 26 = 19

f(16) = (16+14) mod 26 = 30 mod 26 = 4

=>KOHQV MCIF GHSD

b) f(p) = (14p + 21) mod 26

f(23) = (14\*23+21) mod 26 = 343 mod 26 = 5

f(1) = (14\*1+21) mod 26 = 35 mod 26 = 9

f(20) = (14\*20+21) mod 26 = 301 mod 26 = 15

f(3) = (14\*3+21) mod 26 = 63 mod 26 = 11

f(8) = (14\*8+21) mod 26 = 133 mod 26 = 3

f(25) = (14\*25+21) mod 26 = 371 mod 26 = 7

f(15) = (14\*15+21) mod 26 = 231 mod 26 = 23

f(21) = (14\*21+21) mod 26 = 315 mod 26 = 3

f(18) = (14\*18+21) mod 26 = 273 mod 26 = 13

f(19) = (14\*19+21) mod 26 = 287 mod 26 = 1

f(5) = (14\*5+21) mod 26 = 91 mod 26 = 13

f(16) = (14\*16+21) mod 26 = 245 mod 26 = 11

=> EIOKC GWCM AOMK

c) f(p) = (−7p + 1) mod 26

f(23) = (−7\*23 + 1) mod 26 = -160 mod 26 = 22

f(1) = (−7\*1 + 1) mod 26 = -6 mod 26 = 20

f(20) = (−7\*20 + 1) mod 26 = -139 mod 26 = 17

f(3) = (−7\*3 + 1) mod 26 = -20 mod 26 = 6

f(8) = (−7\*8 + 1) mod 26 = -55 mod 26 = 23

f(25) = (−7\*25 + 1) mod 26 = -174 mod 26 = 8

f(15) = (−7\*15 + 1) mod 26 = -104 mod 26 = 26

f(21) = (−7\*21 + 1) mod 26 = -146 mod 26 = 10

f(18) = (−7\*18 + 1) mod 26 = -125 mod 26 = 5

f(19) = (−7\*19 + 1) mod 26 = -132 mod 26 = 24

f(5) = (−7\*5 + 1) mod 26 = -34 mod 26 = 18

f(16) = (−7\*16 + 1) mod 26 = -111 mod 26 = 19

=> VTQFW HZJE XQRS

5.1/16. Prove that for every positive integer n,

1 ⋅ 2 ⋅ 3 + 2 ⋅ 3 ⋅ 4 + ⋯ + n(n + 1)(n + 2) = n(n + 1)(n + 2)(n + 3)∕4.

n=1 => 1\*2\*3 = (1\*2\*3\*4)/4 (True)

Suppose:

1 ⋅ 2 ⋅ 3 + 2 ⋅ 3 ⋅ 4 + ⋯ + k(k + 1)(k + 2) = k(k + 1)(k + 2)(k + 3)∕4

For :

1 ⋅ 2 ⋅ 3 + 2 ⋅ 3 ⋅ 4 + ⋯ + k(k + 1)(k + 2) + (k+1)(k+2)(k+3)

= + (k+1)(k+2)(k+3)

=

=

5.3/41. When does a string belong to the set A of bit strings defined recursively by

𝜆 ∈ A

0x1 ∈ A if x ∈ A,

where 𝜆 is the empty string?

A bit string length n belong to the set A of bit strings defined recursively when the first half is (n/2) number 0 second half is (n/2) number 1.

5.3/3. Find f(2), f(3), f(4), and f(5) if f is defined recursively by f(0) = −1, f(1) = 2, and for n = 1, 2, …

a) f(n + 1) = f(n) + 3f(n − 1).

f(2)=f(1)+3f(0) = 2+3\*-1=-1

f(3)=f(2)+3f(1)=-1+3\*2=5

f(4)=f(3)+3f(2)=5+3\*-1=2

b) f(n + 1) = f(n) 2 f(n − 1).

f(2)=f(1)2f(0)=22\*-1=-4

f(3)=f(2)2f(1)=-42\*2=32

f(4)=f(3)2f(2)=322\*-4=-1024

c) f(n + 1) = 3f(n) 2 − 4f(n − 1)2.

f(2)=3f(1)2-4f(0)2=3\*22-4\*(-1)2=8

f(3)=3f(2)2-4f(1)2=3\*64-4\*4=176

f(4)=3f(3)2-4f(2)2=3\*1762-4\*64=92672

d) f(n + 1) = f(n − 1)∕f(n).

f(2)=f(1)/f(0)=2/-1=-2

f(3)=f(2)/f(1)=-2/2=-1

f(4)=f(3)/f(2)=-1/-2=1/2

5.4/21. Prove that the recursive algorithm that you found in Exercise 7 is correct

a1=x

an=nx

an-1=(n-1)x

an-an-1=nx-(n-1)x=x

6.1/43. How many 4-element RNA sequences

a) contain the base U?

44-34=175

b) do not contain the sequence CUG?

44-4-4=248

c) do not contain all four bases A, U, C, and G?

44-4! = 232

d) contain exactly two of the four bases A, U, C, and G?

44-4!-34-4=147

6.1/12. How many bit strings are there of length six or less, not counting the empty string?

27-6=126

6.1/48.In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

6\*9\*8\*7\*6\*5=90720

b) both the bride and groom must be in the picture?

15\*2\*8\*7\*6\*5=50400

c) exactly one of the brides and the groom is in the picture?

10P6-8\*7\*6\*5\*4\*3-50400=80640

8.1/7. a) Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive 0s.

an=an-1+an-2+2n-2

b) What are the initial conditions?

a0=a1=0

c) How many bit strings of length seven contain two consecutive 0s?

a3=a2+a1+21 =1+0+2=3

a4=a3+a2+22=3+1+4=8

a5=a4+a3+23=8+3+8=19

a6=a5+a4+24=19+8+16=43

a7=a6+a5+25=43+19+32=94

8.3/1. How many comparisons are needed for a binary search in a set of 64 elements?

From 1 to 6 comparisons.

10.1/26. a) Explain how graphs can be used to model electronic mail messages in a network. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

b) Describe a graph that models the electronic mail sent in a network in a particular week.

10.2/8. Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph

A close-up of a stethoscope

Description automatically generated with medium confidence

The number of vertices: 4

The number of edges: 8

|  |  |  |
| --- | --- | --- |
| Vertices | In-degree | Out-degree |
| a | 2 | 2 |
| b | 3 | 4 |
| c | 2 | 1 |
| d | 1 | 1 |

10.3/28.Determine whether an undirected graph is sparse, dense, or neither, and explain your answer, if it is used to mode

a) The street network in a city (where the vertices are street intersections).

Dense,

b) Whether buildings in a city are within two miles of one another.

Dense

c) Whether two people in the world are siblings.

Sparse, because two people are siblings account for 0.025% of about 7,753 milliards citizens in the world

d) Whether two people work for the same company

Sparse, because two people work for the same company account for a small percentage of citizen in a city.

10.4/4. whether the given graph is connected

Chart

Description automatically generated

The given graph is connected

10.5/49.

a) Removing an edge from a cycle leaves a path, which is still connected.

b) Removing an edge from the cycle portion of the wheel leaves that portion still connected as in part (a), and the central vertex is clearly still connected to it as well. Removing a spoke leaves the cycle intact and the central vertex still connected to it as well.

c) Let u, v, a, b be any four vertices of Km,n with u and v in one part and a and bin the other. They are connected by the 4-cycle uavb. Removing one edge will not disconnect this 4-cycle, so these vertices are still connected, and the entire graph is therefore still connected. Note that we needed m, n ?: 2 for this to work (and for the statement to be true).

d) Think of Qn as two copies of Qn-l with corresponding vertices joined by an edge. Without loss of generality we can assume that the removed edge is one of the edges joining corresponding vertices. Since each Qn-l is connected and at least one edge remains joining the two copies, the resulting graph is connected.

10.6/1. For each of these problems about a subway system, describe a weighted graph model that can be used to solve the problem.

a) What is the least amount of time required to travel between two stops?

We will put an edge from A to B whenever there is a train that travels from A to B without intermediate stops. The weight of that edge will be the time required for the trip, including half the stopping time at each end station. This model is not perfect. For example, the time may depend on the time of day. Also, it is not clear that allocating the waiting time at each station in this way is the best way to model the system.

b) What is the minimum distance that can be traveled to reach a stop from another stop?

We assume that distance refers to the distance along the subway tracks. If so, this model is straightforward and similar to part(a). We put an edge from A to B whenever there is a train that travels from A to B without intermediate stops. The weight of that edge will be the distance the train travels on that trip.

c) What is the least fare required to travel between two stops if fares between stops are added to give the total fare?

Under the assumption stated, we can model this problem in a manner like the previous parts. We put an edge from A to B whenever there is a train that travels from A to B without intermediate stops. The weight of that edge will be the fare required for that trip. Very few subway systems operate under this assumption.

11.1/11. a) How many nonisomorphic unrooted trees are there with three vertices?

- 0

b) How many nonisomorphic rooted trees are there with three vertices (using isomorphism for directed graphs)?

11.2/6. How many weighings of a balance scale are needed to find a lighter counterfeit coin among four coins? Describe an algorithm to find the lighter coin using this number of weighings

2 times:

First you weight 2 coins with 2 coins which one is lighter separate 2 coins and weight them.

11.3/10. In which order are the vertices of the ordered rooted tree in Exercise 7 visited using an inorder traversal?

The left subtree of the root comes first, namely the tree rooted at b. There again the left subtree comes first, so the list begins with d. After that comes b, the root of this subtree, and then the right subtree of b, namely (in order) f , e, and g . Then comes the root of the entire tree and finally its right child. Thus the answer is d, b, f, e, g, a, c.

11.4/50.

Note that a “lower” level is further down the tree, i.e., further from the root and therefore having a larger value. (So “lower” really means “greater than”!) This is similar to Exercise 34. Again notice that the order in which vertices are put into (and therefore taken out of) the list L is level-order. In other words, the root of the resulting tree comes first, then the vertices at level 1 (put into the list while processing the root), then the vertices at level 2 (put into the list while processing vertices at level 1), and so on. Now suppose that uv is a directed edge not in the tree. First assume that the algorithm processed u before it processed v . Other words, u entered the list L before v did.) Since the edge uv is not in the tree, it must be the case that v was already in the list L when u was being processed. In order for this to happen, the parent p of v must have already been processed before u. Note that p’s level in the tree is one less than v’s level. Therefore u’s level is greater than or equal to p’s level but less than or equal to v’s level, so this directed edge goes from a vertex at one level to a vertex either at the same level or one level below. Next suppose that the algorithm processed v before it processed u. Then v’s level is at or above u’s level, and there is nothing else to prove.

11.5/25.

a) First we need to find the least expensive edges incident to each vertex. These are the links from New York to Atlanta, Atlanta to Chicago, and Denver to San Francisco. The algorithm tells us to choose all of these edges. At the end of this first pass, then, we have a forest of two trees, one containing the three eastern cities, the other containing the two western cities. Next we find the least expensive edge joining these two trees, namely the link from Chicago to San Francisco, and add it to our growing forest. We now have a spanning tree, and the algorithm has finished. Note, incidentally, that this is the same spanning tree that we obtained in Example 1; by the result of Exercise 19, since the weights in this graph are all different, there was only one minimum spanning tree. b) On the first pass, we choose all the edges that are the minimum weight edges at each vertex. This set consists of {a,b}, {b,f}, {c,d}, {a,e}, {c,g}, {g,h}, {i,j}, {f,j}, and {k,l}. At this point the forest has three components. Next we add the lowest weight edges connecting these three components, namely { h, l} and {b, c}, to complete our tree.