MATH1005/MATH6005 Discrete Mathematical Models

Adam Piggott

Preface to the course

An acknowledgment of country

We acknowledge and celebrate the First Australians on whose traditional lands we live, work and study. We pay our respects to the elders past and present. In particular, we acknowledge the Ngunnawal and Ngambri people, the Traditional Owners of the land upon which the University's Acton campus is located.

Credits

This course has been developed by a number of mathematicians over more than a decade. Contributing authors include:

- Judy-anne Osborne
- Pierre Portal
- Malcolm Brooks
- Adam Piggott

The mathematics in this course has been developed over millennia. Credit for discovery will be given only occasionally.

Learning Outcomes

Upon successful completion of MATH1005, students will have the knowledge and skills to:

1. Recall, invent, interpret examples of motivation for mathematical constructs used in discrete mathematics as models of processes in the world.
2. Recognise, define, explain and use terminology and notation from discrete mathematics.
3. Identify the logical structure of a statement, and then identify the logical structure of an argument that may be used to prove or disprove the statement.
4. Perform mathematical calculations in discrete mathematics using methods presented in the course.
5. Write simple proofs/construct explicit counterexamples for statements relating to discrete mathematics topics covered in the course.

Learning Outcomes

Upon successful completion of MATH6005, in addition to the knowledge and skills attained by students completing MATH1005, students will have the knowledge and skills to:

6. Use their deep knowledge and understanding of the material presented in the course to formulate responses to complex concrete and abstract problems.
7. Communicate their understanding and skills in discrete mathematics with colleagues and non-experts and apply their knowledge in an occupational situation.

Reference materials

The lecture notes for the course are intended to be self-contained. Notes will be available through the course Wattle page immediately after the lecture, if not before. For most students, these notes, the workshop materials and the web will be sufficient.

If you would like an alternative presentation of the material, our optional text is Susanna S. Epp. *Discrete Mathematics with Applications: Metric Version*. Cengage Learning, 5th edition, 2019. ISBN: 9780357114087.

- An ebook copy of the text is available through the ANU library.

Other notes

- You must check the Wattle page, particularly the announcements, often.
- MATH1005 Assessment: workshop quizzes, workshop participation, three assignments and a final exam.
- MATH6005 Assessment: workshop quizzes, three assignments, and a final exam.
- All students should sign up for a workshop through the course Wattle page.

Learning design

Lectures (3 hours per week): The discussion progresses and there is something new to wrestle with, but you probably won't understand it just from listening.

Processing the lectures ($\sim$3 hours per week) You spend time with the new material, unpack definitions, work examples from class yourself, try other examples.

Workshops (2 hours per week) A short quiz, then new examples to try with your classmates and demonstrators to assist. This is the most useful time of the week provided you have prepared for it.

Assignments ($\sim$2 hours per week) Give you a chance to measure your skills and understanding. They are NOT designed to communicate everything you need to take out of the lectures.

Final Exam Summative assessment items in which you demonstrate the extent to which you have obtained the learning outcomes.

Why study discrete mathematical models?

What are discrete mathematical models?

Discrete mathematical models are abstract representations of processes and objects, the steps or units of which can be indexed by the positive integers. In particular, we avoid continua (like the open interval $(0, 1)$).

Discrete mathematics is the study of discrete mathematical models.

Sudoku?

Q: Is completing a Sudoku puzzle an exercise in discrete mathematics?

Example: Which webpages are the most interesting?

Problem: A search on the internet can produce an overwhelming number of "hits." We need an effective and efficient method to order the hits from a web search so that the most interesting pages appear first.

Google's PageRank algorithm is an excellent solution to this problem that has made its inventors very wealthy. The algorithm combines a simple discrete mathematical model of the internet with some linear algebra discovered circa 1910. We will learn this algorithm in our course.

Computing and discrete mathematical models

Discrete mathematics and computer science go hand-in-hand because ...

Q: Describe an abstract model of the memory of a computer. That is, how do you think about memory?

Q: Do you think about computers working in continuous time or discrete time ("clock cycles")?

Q: Can the interval $(0, 1)$ be modeled inside a computer?

Discrete mathematics and computer science

In 1974, Knuth wrote:

> "Discrete mathematics, especially combinatorial theory, has been given an added boost by the rise of computer science, in addition to all the other fields in which discrete mathematics is currently being extensively applied."

Donald E. Knuth. Computer science and its relation to mathematics. *Amer. Math. Monthly*, 81:323-343, 1974.

Discrete mathematics and computer scientists

Let's consider the educational background of some A. M. Turing Award laureates.

- Donald Knuth (1974)
- Andrew Chi-Chih Yao (2000)
- Barbara Liskov (2008)
- Shafi Goldwasser(2012)
- David Patterson (2017)

Some optional activities

- (Optional) Read the first 4 sections (pp. 323-329) of Knuth's paper referenced on slide #-2 (the paper is available in Wattle), and take notes.
- (Optional) Play Sudoku at Sudoku.com. Ask yourself the following questions: Can you describe your strategy? Can you guarantee that your strategy will work every time? Is your strategy "the best"? Are these questions examples of discrete mathematics?