Summary / Cheat Sheet

- Use Bayesian Methods to combine prior info w/ data. Lose long run performance gntees
- Bayesian Method:
  1. choose prior $f(\theta)$
  2. chose stat model: $f(x|\theta)$
  3. update belief w/ posterior: $f(\theta|x^n) \propto f(x^n|\theta) \cdot f(\theta)$ } "posterior is proportional to likelihood times prior"

- With posterior we can form
  - posterior mean $\triangleq \int \theta f(\theta|x^n) d\theta \approx \frac{1}{B} \sum_i^B \theta_i$; $\theta_i \sim f(\theta|x^n)$
  - $1-\alpha$ interval estimate $\triangleq (a, b)$ $\approx (\theta_{\frac{\alpha}{2}}, \theta_{1-\frac{\alpha}{2}})$

    s.t. $\int_{-\infty}^{a} f(\theta|x^n) d\theta =$
    $= \int_{b}^{+\infty} f(\theta|x^n) d\theta = \frac{\alpha}{2}$

    - percentile of sampled data from posterior

- Functions of parameters: $\tau = g(\theta)$

  analytically: $A_\tau(z) \triangleq \{\theta : g(\theta) \leq z\}$

  $F_\tau(z) \triangleq \int_{A_\tau(z)} f(\theta|x^n) d\theta$

  $f_\tau(z) \triangleq \frac{dF_\tau(z)}{dz}$ ← pdf of posterior of $\tau = g(\theta)$ $f_\tau(z|x^n)$

  simulation: let $\theta_1, \dots, \theta_B \sim f(\theta|x^n)$
  $\tau_i \triangleq g(\theta_i)$ → then $\tau_1, \dots, \tau_B \sim f(\tau|x^n) - f(g(\theta)|x^n)$

  post. mean $= \bar{\tau}_n \triangleq E[\tau|x^n] \approx \frac{1}{B} \sum_i^B \tau_i$

  post $1-\alpha$ interval $= (\tau_{\frac{\alpha}{2}}, \tau_{1-\frac{\alpha}{2}}) \approx$ (percentile$(\tau, \frac{\alpha}{2})$, " $_{1-\frac{\alpha}{2}}$)

- For large samples: MLE $\hat{\theta}$ $\approx$ Bayes Estimate $\bar{\theta} \approx N(\hat{\theta}, \hat{se}^2)$