Efficient Search with 2 Balls

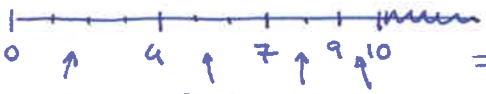
Is 21 drops the best we can do? We did make the assumption that the floors we would drop from were evenly distributed, k, 2k, 3k, etc. It turns out that distributing the floors carefully and unevenly can shrink the number of drops required below 21.

Worst case is when the first ball breaks at the last interval. A better strategy is to equalize the number of ~~drop~~ total drops in each interval.

$$\text{drops}_1 + \text{drops}_2 = \text{cte}$$

each interval should be smaller than the previous one

Example for $N = 10$ floors



drops $1+3$ $2+2$ $1+3$ $4+0$

$\Longrightarrow$ size of first interval (k) must be equal to number of intervals (n) *

Recurrence formulas for intervals

$$f(0) = 0$$

$$f(1) = k$$

$$f(2) = k + k - 1 = 2k - 1$$

$$f(3) = k + k - 1 + k - 2 = 3k - 3$$

...

$$\boxed{f(n) = f(n-1) + k - (n-1)} = f(n-1) + k + 1 - n$$

Unfolding

$$= f(n-2) + 2(k+1) - (n-1) - n$$

$$= f(n-3) + 3(k+1) - (n-2) - (n-1) - n$$

...

$$= f(0) + n(k+1) - [\underbrace{1 + 2 + \dots + (n-2) + (n-1) + n}]$$

S_n : Sum of first n positive integers

$$S_n = 1 + 2 + \dots + (n-1) + n$$

$$+ \; S_n = n + (n-1) + \dots + 2 + 1$$

$$2S_n = (n+1) + (n+1) + \dots + (n+1) + (n+1)$$

$$= n(n+1)$$

$$\boxed{S_n = \frac{n(n+1)}{2}}$$

$$= n(k+1) - \frac{n(n+1)}{2} \overset{* \; k=n}{=} n(n+1) - \frac{n(n+1)}{2} = \boxed{\frac{n(n+1)}{2}}$$

$$f(n) = \frac{n(n+1)}{2} > N$$

for $N = 128$ $\Rightarrow$ $\boxed{n = \begin{cases} 15'508 \\ -16'508 \end{cases}}$

$$\boxed{n = \left\lceil \frac{\sqrt{8N+1} - 1}{2} \right\rceil = k}$$