10.2_Numpy

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1 Introduction to Python for Open Source Geocomputation

- Instructor: Dr. Wei Kang

Content:

- Numpy
- A new data type: `numpy.array`
  - How to create an array
  - Array operations

2 What is Numpy?

- The fundamental package for scientific computing with Python
- Nearly every scientist working in Python draws on the power of NumPy.
- NumPy brings the **computational power** of languages like C and Fortran to Python, a language much easier to learn and use. With this power comes **simplicity: a solution in NumPy is often clear and elegant**.
- Essential in many different realms:
  - NumPy lies at the core of a rich ecosystem of **data science** libraries
  - NumPy forms the basis of powerful **machine learning** libraries like scikit-learn, SciPy, TensorFlow, and PyTorch
  - NumPy is an essential component in the burgeoning Python **visualization landscape**, which includes Matplotlib, Seaborn, Plotly, Altair, Bokeh, Holoviz, Vispy, Napari, and PyVista, to name a few.

2.1 What makes Numpy so important?

arrays: A very powerful data type essential to numerical computing: * sequences of data all of the *same type* * behave a lot like lists, except for the constraint in the type of their elements. * There is a huge efficiency advantage when you know that **all elements of a sequence are of the same type**—so equivalent methods for arrays execute a lot **faster** than those for lists.

2.2 Numpy `Array` (or `ndarray`)

- homogeneous multidimensional array
  - a table of elements (usually numbers), all of the same type, indexed by a tuple of non-negative integers
    - for the data types accepted in Numpy. Read the docs: Data type objects.
  - dimensions are called *axes*
- An Example: points' coordinates
  - one single point: one-dimensional array: `np.array([1,2])`
  - two or more points: two-dimensional array:
    - two points: `np.array([[1,2], [3,4]])`
    - five points: `np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])`

[1]:
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import numpy as np
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Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2 (Intel(R)
SSE4.2) enabled only processors has been deprecated. Intel oneAPI Math Kernel
Library 2025.0 will require Intel(R) Advanced Vector Extensions (Intel(R) AVX)
instructions.
Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2 (Intel(R)
SSE4.2) enabled only processors has been deprecated. Intel oneAPI Math Kernel
Library 2025.0 will require Intel(R) Advanced Vector Extensions (Intel(R) AVX)
instructions.
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a1 = np.array([1,2])
a1
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[2]:
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array([1, 2])
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[3]:
```
a2 = np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])
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[3]:
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array([[ 1,  2],
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       [ 9, 10]])
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2.2.1 Motivation (1): What can a Numpy array used for?

- An array can contain:
  - values of an experiment/simulation at discrete time steps, e.g., income, air pollution, crime rate, animal/plant occurrence

- pixels of an image, grey-level or colour
- signal recorded by a measurement device, e.g. sound wave
- 3-D data measured at different X-Y-Z positions, e.g. MRI scan, digital elevation model

2.2.2 Motivation (2): Efficiency of Numpy array - an example

- Problem description: Write a python program that calculate the square of each number in a list, such that $x_i = i^2$, for $0 \leq i < n$.

Two data types: * Python built-in type: list * Numpy array

We use `%timeit` to calculate the time execution of a Python statement or expression.

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L = list(range(1000)) #produce a list of integers from 0 to 999
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 998,
 999]
```

[7]:

```
L = list(range(1000))
for i in range(len(L)):
```

```
    L[i] = L[i] **2
```

list comprehension

[8]:
```
%timeit -n 1000 [i**2 for i in L]
```

```
187 µs ± 2.54 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

[9]:
```
import numpy as np
a = np.arange(1000) #produce an array of integers from 0 to 999
# a
```

[10]:
```
%timeit -n 1000 a**2
```

```
1.04 µs ± 248 ns per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

3 Importing Numpy

import numpy as np

[11]:
```
import numpy as np
```

[12]:
```
dir(np) #function dir gives you the package's attributes and functions.
```

[12]:
```
['ALLOW_THREADS',
 'AxisError',
 'BUFSIZE',
 'CLIP',
 'ComplexWarning',
 'DataSource',
 'ERR_CALL',
 'ERR_DEFAULT',
 'ERR_IGNORE',
 'ERR_LOG',
 'ERR_PRINT',
 'ERR_RAISE',
 'ERR_WARN',
 'FLOATING_POINT_SUPPORT',
 'FPE_DIVIDEBYZERO',
 'FPE_INVALID',
 'FPE_OVERFLOW',
 'FPE_UNDERFLOW',
 'False_',
 'Inf',
 'Infinity',
 'MAXDIMS',
 'MAY_SHARE_BOUNDS',
 'MAY_SHARE_EXACT',
```

```
'ModuleDeprecationWarning',
'NAN',
'NINF',
'NZERO',
'NaN',
'PINF',
'PZERO',
'RAISE',
'RankWarning',
'SHIFT_DIVIDEBYZERO',
'SHIFT_INVALID',
'SHIFT_OVERFLOW',
'SHIFT_UNDERFLOW',
'ScalarType',
'Tester',
'TooHardError',
'True_',
'UFUNC_BUFSIZE_DEFAULT',
'UFUNC_PYVALS_NAME',
'VisibleDeprecationWarning',
'WRAP',
'_CopyMode',
'_NoValue',
'_UFUNC_API',
'__NUMPY_SETUP__',
'__all__',
'__builtins__',
'__cached__',
'__config__',
'__deprecated_attrs__',
'__dir__',
'__doc__',
'__expired_functions__',
'__file__',
'__former_attrs__',
'__future_scalars__',
'__getattr__',
'__git_version__',
'__loader__',
'__mkl_version__',
'__name__',
'__package__',
'__path__',
'__spec__',
'__version__',
'_add_newdoc_ufunc',
'_builtins',
```

```
'_distributor_init',
'_financial_names',
'_get_promotion_state',
'_globals',
'_int_extended_msg',
'_mat',
'_no_nep50_warning',
'_pyinstaller_hooks_dir',
'_pytesttester',
'_set_promotion_state',
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'add_newdoc',
'add_newdoc_ufunc',
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'alltrue',
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'apply_over_axes',
'arange',
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'arccosh',
'arcsin',
'arcsinh',
'arctan',
'arctan2',
'arctanh',
'argmax',
'argmin',
'argpartition',
'argsort',
'argwhere',
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'array_equiv',
'array_repr',
```

```
'array_split',
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'asfortranarray',
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'atleast_2d',
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'bitwise_xor',
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'broadcast_to',
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'busday_offset',
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'byte',
'byte_bounds',
'bytes_',
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'can_cast',
'cast',
'cbrt',
'cdouble',
'ceil',
'cfloat',
'char',
'character',
'chararray',
'choose',
'clip',
```

```
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'compat',
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'complex256',
'complex64',
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'compress',
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'cov',
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'cumsum',
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'datetime_data',
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'degrees',
'delete',
'deprecate',
'deprecate_with_doc',
'diag',
'diag_indices',
'diag_indices_from',
'diagflat',
'diagonal',
'diff',
'digitize',
'disp',
```

```
'divide',
'divmod',
'dot',
'double',
'dsplit',
'dstack',
'dtype',
'e',
'ediff1d',
'einsum',
'einsum_path',
'emath',
'empty',
'empty_like',
'equal',
'error_message',
'errstate',
'euler_gamma',
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'exp2',
'expand_dims',
'expm1',
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'eye',
'fabs',
'fastCopyAndTranspose',
'fft',
'fill_diagonal',
'find_common_type',
'finfo',
'fix',
'flatiter',
'flatnonzero',
'flexible',
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'fliplr',
'flipud',
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'float16',
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'float64',
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'float_power',
'floating',
'floor',
'floor_divide',
'fmax',
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```
'fmin',
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'get_printoptions',
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'geterr',
'geterrcall',
'geterrobj',
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'hanning',
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'histogramdd',
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'hstack',
'hypot',
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'iinfo',
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'index_exp',
```

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'indices',
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'insert',
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'int64',
'int8',
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'intc',
'integer',
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'isclose',
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'isfortran',
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'isinf',
'isnan',
'isnat',
'isneginf',
'isposinf',
'isreal',
'isrealobj',
'isscalar',
'issctype',
'issubclass_',
'issubdtype',
'issubsctype',
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'piecewise',
'place',
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'poly1d',
'polyadd',
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'polyfit',
'polyint',
'polymul',
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'polyval',
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'record',
'remainder',
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'require',
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'right_shift',
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'savez_compressed',
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'select',
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'set_printoptions',
'set_string_function',
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'sometrue',
'sort',
'sort_complex',
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'str_',
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'transpose',
'trapz',
'tri',
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'tril_indices_from',
'trim_zeros',
'triu',
'triu_indices',
'triu_indices_from',
'true_divide',
'trunc',
'typecodes',
'typename',
'ubyte',
'ufunc',
'uint',
'uint16',
'uint32',
'uint64',
'uint8',
'uintc',
'uintp',
'ulonglong',
'unicode_',
'union1d',
'unique',
'unpackbits',
'unravel_index',
'unsignedinteger',
'unwrap',
'use_hugepage',
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 'ushort',
 'vander',
 'var',
 'vdot',
 'vectorize',
 'version',
 'void',
 'vsplit',
 'vstack',
 'w',
 'where',
 'who',
 'zeros',
 'zeros_like']
```

3.0.1 Creating a Numpy Array

- create an array from a regular Python list or tuple using the `array` function.

`np.array(list/tuple)`

- functions from Numpy to create special arrays
  - `np.arange()`: create evenly spaced values within a given interval.
  - `np.linspace(start, stop, num=50)`: create evenly spaced numbers over a specified interval.
  - `np.ones(shape)`: create new array of given shape and type, filled with ones.
  - `np.zeros(shape)`: create a new array of given shape and type, filled with zeros.
  - `np.eye(N)`: create a 2-D array with ones on the diagonal and zeros elsewhere.

[13]:
```python
a1 = np.array([1,2])
a1
```

[13]:
```
array([1, 2])
```

[14]:
```python
type(a1)
```

[14]:
```
numpy.ndarray
```

[15]:
```python
a1.size
```

[15]:
```
2
```

`array.size` gives the number of items in the array.

[16]:
```python
len(a1)
```

[16]:
```
2
```

`len(array)` gives the same result to `array.size`

[17]:
```
a1.ndim
```

[17]:
```
1
```

`array.ndim` gives the number of axes (dimensions) of the array.

[18]:
```
a1.shape
```

[18]:
```
(2,)
```

[19]:
```
a1
```

[19]:
```
array([1, 2])
```

`array.shape` gives the dimensions of the array. This is a tuple of integers indicating the **size** of the array in each dimension. For a matrix with n rows and m columns, shape will be (n,m). The length of the shape tuple is therefore the number of axes, ndim.

[20]:
```
a1.dtype
```

[20]:
```
dtype('int64')
```

`array.dtype` returns an object describing the type of the elements in the array

[21]:
```
a_str = np.array([1.0,2,"1"])
a_str
```

[21]:
```
array(['1.0', '2', '1'], dtype='<U32')
```

[22]:
```
a_str.dtype #32-character  string
```

[22]:
```
dtype('<U32')
```

[23]:
```
a2 = np.array([[1,2], [3,4]])
a2
```

[23]:
```
array([[1, 2],
       [3, 4]])
```

[24]:
```
a2.ndim
```

[24]:
```
2
```

[25]:
```
a2.size
```

[25]:
```
4
```

[26]:
```
len(a2)
```

[26]:
```
2
```

`len(array)` gives the number of rows or the size of the first dimension when encountering a 2-dimensional array

[27]:
```
a2.shape
```

[27]: (2, 2)

[28]:
```
a2.dtype
```

[28]: dtype('int64')

[29]:
```
a3 = np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])
a3
```

[29]:
```
array([[ 1,  2],
       [ 3,  4],
       [ 5,  6],
       [ 7,  8],
       [ 9, 10]])
```

[30]:
```
a3.ndim
```

[30]: 2

[31]:
```
len(a3)
```

[31]: 5

[32]:
```
a3.size
```

[32]: 10

[33]:
```
a3
```

[33]:
```
array([[ 1,  2],
       [ 3,  4],
       [ 5,  6],
       [ 7,  8],
       [ 9, 10]])
```

[34]:
```
a3.shape
```

[34]: (5, 2)

[35]:
```
a3.dtype
```

[35]: dtype('int64')

We can create a 3-dimensional array

[36]:
```python
import numpy as np
```

[37]:
```python
c = np.array([[[1,1], [2,2]],
              [[3,23], [4,5]],
              [[5,3], [9,10]]])
c
```

[37]:
```
array([[[ 1,  1],
        [ 2,  2]],

       [[ 3, 23],
        [ 4,  5]],

       [[ 5,  3],
        [ 9, 10]]])
```

[38]:
```python
c.ndim
```

[38]: 3

[39]:
```python
c.shape
```

[39]: (3, 2, 2)

[40]:
```python
c.size
```

[40]: 12

[41]:
```python
len(c)
```

[41]: 3

[42]:
```python
a = np.array(1, 2, 3, 4)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[42], line 1
----> 1 a = np.array(1, 2, 3, 4)

TypeError: array() takes from 1 to 2 positional arguments but 4 were given
```

The input needs to be an ordered sequence data type: list or tuples

[43]:
```python
a = np.array([1, 2, 3, 4])
a
```

[43]: array([1, 2, 3, 4])

[44]:
```python
a = np.array((1, 2, 3, 4))
a
```

[44]:
```
array([1, 2, 3, 4])
```

[45]:
```python
a = np.array((1, 2, 3, 4), (1, 2, 3, 4))
a
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[45], line 1
----> 1 a = np.array((1, 2, 3, 4), (1, 2, 3, 4))
      2 a

TypeError: Tuple must have size 2, but has size 4
```

[46]:
```python
a = np.array(((1, 2, 3, 4), (1, 2, 3, 4)))
a
```

[46]:
```
array([[1, 2, 3, 4],
       [1, 2, 3, 4]])
```

[47]:
```python
a = np.array([[1, 2, 3, 4], [1, 2, 3, 4]])
a
```

[47]:
```
array([[1, 2, 3, 4],
       [1, 2, 3, 4]])
```

3.1 Numpy functions to generate special arrays

3.1.1 `numpy.arange()`

`numpy.arange()` gives an array of evenly spaced values in a defined interval. Similar to `range()`

Syntax:

`numpy.arange(start, stop, step)`

where `start` by default is zero, `stop` is not inclusive, and the default for `step` is one.

[48]:
```python
list(range(3))
```

[48]:
```
[0, 1, 2]
```

[49]:
```python
np.arange(3) # 0 .. n-1 (!)
```

[49]:
```
array([0, 1, 2])
```

[50]:
```python
np.arange(2, 6)
```

[50]: array([2, 3, 4, 5])

```
[51]: np.arange(2, 6, 2) # start, end (exclusive), step
```

[51]: array([2, 4])

```
[52]: np.arange(2, 6, 0.5) # start, end (exclusive), step
```

[52]: array([2. , 2.5, 3. , 3.5, 4. , 4.5, 5. , 5.5])

3.1.2 numpy.linspace()

`numpy.linspace()` is similar to `numpy.arange()`, but uses number of samples instead of a step size. It returns an array with evenly spaced numbers over the specified interval.

Syntax:

`numpy.linspace(start, stop, num)`

`stop` is included by default (it can be removed, read the docs), and `num` by default is 50.

```
[53]: np.linspace(2.0, 3.0)
```

```
[53]: array([2.        , 2.02040816, 2.04081633, 2.06122449, 2.08163265,
             2.10204082, 2.12244898, 2.14285714, 2.16326531, 2.18367347,
             2.20408163, 2.2244898 , 2.24489796, 2.26530612, 2.28571429,
             2.30612245, 2.32653061, 2.34693878, 2.36734694, 2.3877551 ,
             2.40816327, 2.42857143, 2.44897959, 2.46938776, 2.48979592,
             2.51020408, 2.53061224, 2.55102041, 2.57142857, 2.59183673,
             2.6122449 , 2.63265306, 2.65306122, 2.67346939, 2.69387755,
             2.71428571, 2.73469388, 2.75510204, 2.7755102 , 2.79591837,
             2.81632653, 2.83673469, 2.85714286, 2.87755102, 2.89795918,
             2.91836735, 2.93877551, 2.95918367, 2.97959184, 3.        ])
```

```
[54]: len(np.linspace(2.0, 3.0))
```

[54]: 50

```
[55]: np.linspace(0, 1, 6) # start, end, num of points
```

[55]: array([0. , 0.2, 0.4, 0.6, 0.8, 1.])

```
[56]: np.linspace(-1, 1, 9)
```

```
[56]: array([-1.  , -0.75, -0.5 , -0.25,  0.  ,  0.25,  0.5 ,  0.75,  1.  ])
```

We can also create special arrays using Numpy functions

```
[57]: a = np.ones(3) # creating a 1-D array full of 1s
```

```
[58]: a
```

[58]: array([1., 1., 1.])

```
[59]: a = np.ones((3, 2)) # (3,2) is the shape of the array we want to create, which
      ↪needs to be a tuple
```

```
[60]: a
```

```
[60]: array([[1., 1.],
             [1., 1.],
             [1., 1.]])
```

```
[61]: b = np.zeros(10) # creating a 1-D array full of 0s
```

```
[62]: b
```

```
[62]: array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

```
[63]: b = np.zeros((3, 3))
```

```
[64]: b
```

```
[64]: array([[0., 0., 0.],
             [0., 0., 0.],
             [0., 0., 0.]])
```

```
[65]: np.eye(5)
```

```
[65]: array([[1., 0., 0., 0., 0.],
             [0., 1., 0., 0., 0.],
             [0., 0., 1., 0., 0.],
             [0., 0., 0., 1., 0.],
             [0., 0., 0., 0., 1.]])
```

```
[66]: np.eye(2)
```

```
[66]: array([[1., 0.],
             [0., 1.]])
```

```
[67]: np.empty((2, 3))
```

```
[67]: array([[1., 1., 1.],
             [1., 1., 1.]])
```

3.2 Arithmetic operations on arrays

- Arithmetic operators on arrays apply elementwise
- Different from the application of Arithmetic operators to lists

[68]:
```
a = np.array([20, 30, 40, 50])
b = np.array([1,2,3,4])
```

[69]:
```
c = a + b
c
```

[69]:
```
array([21, 32, 43, 54])
```

[70]:
```
d = np.array([1,2,3])
```

[71]:
```
a + d
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[71], line 1
----> 1 a + d

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

[72]:
```
list_a = list(a)
list_b = list(b)
```

[73]:
```
type(list_a)
```

[73]:
```
list
```

[74]:
```
list_a
```

[74]:
```
[20, 30, 40, 50]
```

[75]:
```
list_b
```

[75]:
```
[1, 2, 3, 4]
```

[76]:
```
list_a + list_b
```

[76]:
```
[20, 30, 40, 50, 1, 2, 3, 4]
```

[77]:
```
a
```

[77]:
```
array([20, 30, 40, 50])
```

[78]:
```
b
```

[78]:
```
array([1, 2, 3, 4])
```

[79]:
```python
d = a - b
d
```

[79]: array([19, 28, 37, 46])

[80]:
```python
list_d = list_a - list_b
list_d
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[80], line 1
----> 1 list_d = list_a - list_b
      2 list_d

TypeError: unsupported operand type(s) for -: 'list' and 'list'
```

[81]:
```python
a
```

[81]: array([20, 30, 40, 50])

[82]:
```python
a ** 2
```

[82]: array([400, 900, 1600, 2500])

[83]:
```python
list_a ** 2
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[83], line 1
----> 1 list_a ** 2

TypeError: unsupported operand type(s) for ** or pow(): 'list' and 'int'
```

[84]:
```python
a
```

[84]: array([20, 30, 40, 50])

[85]:
```python
a * 2
```

[85]: array([40, 60, 80, 100])

[86]:
```python
list_a
```

[86]: [20, 30, 40, 50]

[87]:
```python
list_a * 2
```

[87]: [20, 30, 40, 50, 20, 30, 40, 50]

[88]:
```
a
```

[88]: array([20, 30, 40, 50])

[89]:
```
b
```

[89]: array([1, 2, 3, 4])

[90]:
```
a < b
```

[90]: array([False, False, False, False])

[91]:
```
list_a
```

[91]: [20, 30, 40, 50]

[92]:
```
list_b
```

[92]: [1, 2, 3, 4]

[93]:
```
list_a < list_b
```

[93]: False

[94]:
```
a
```

[94]: array([20, 30, 40, 50])

[95]:
```
b
```

[95]: array([1, 2, 3, 4])

[96]:
```
a/b
```

[96]: array([20.   , 15.   , 13.33333333, 12.5   ])

[97]:
```
list_a/list_b
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[97], line 1
----> 1 list_a/list_b

TypeError: unsupported operand type(s) for /: 'list' and 'list'
```

```
[98]: a.shape
```

```
[98]: (4,)
```

```
[99]: a
```

```
[99]: array([20, 30, 40, 50])
```

```
[100]: a + 1
```

```
[100]: array([21, 31, 41, 51])
```

Broadcasting with scalar numerical data type

```
[101]: list_a + 1
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[101], line 1
----> 1 list_a + 1

TypeError: can only concatenate list (not "int") to list
```

```
[102]: a
```

```
[102]: array([20, 30, 40, 50])
```

```
[103]: a < 30
```

```
[103]: array([ True, False, False, False])
```

```
[104]: list_a < 30
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[104], line 1
----> 1 list_a < 30

TypeError: '<' not supported between instances of 'list' and 'int'
```

```
[105]: c = np.array([10,15,20])
```

```
[106]: a
```

```
[106]: array([20, 30, 40, 50])
```

```
[107]: a + c
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[107], line 1
----> 1 a + c

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

Shape mismatches

Arithmetic operation on 2-D arrays

[108]:
```
X = np.array([[1, 2], [3, 4]])
print(X)
```

```
[[1 2]
 [3 4]]
```

[109]:
```
Y = np.array([[1, -1], [0, 1]])
print(Y)
```

```
[[ 1 -1]
 [ 0  1]]
```

[110]:
```
X + Y
```

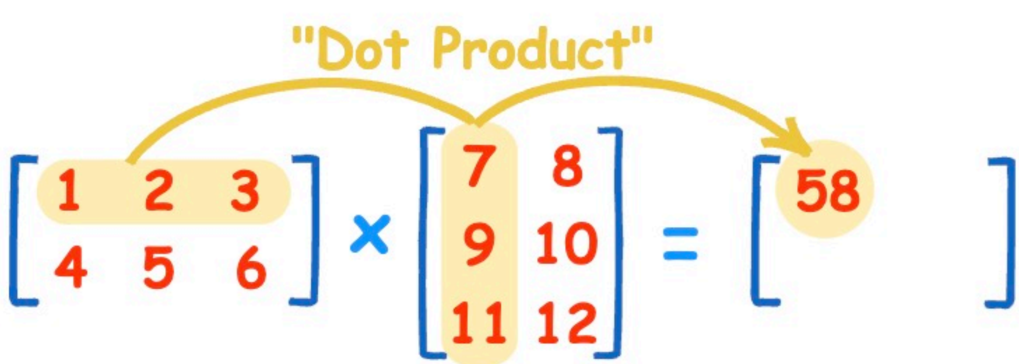
[110]:
```
array([[2, 1],
       [3, 5]])
```

[111]:
```
X * Y
```

[111]:
```
array([[ 1, -2],
       [ 0,  4]])
```

The multiplication using the '*' operator is element-wise.

What if we want to do matrix multiplication? Using the '@' operator:



[112]:
```python
X * Y
```

[112]:
```
array([[ 1, -2],
       [ 0,  4]])
```

[113]:
```python
X @ Y
```

[113]:
```
array([[1, 1],
       [3, 1]])
```

Or equivalently we can use `np.dot()`:

[114]:
```python
np.dot(X, Y)
```

[114]:
```
array([[1, 1],
       [3, 1]])
```

[115]:
```python
Z = np.arange(12)
Z
```

[115]:
```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])
```

[116]:
```python
Z.reshape(3,4) #reshape() change the shape of an array
```

[116]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[117]:
```python
Z.reshape((3,4)) #reshape() change the shape of an array
```

[117]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[118]:
```python
Z = Z.reshape(3,4)
Z
```

[118]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[119]:
```python
Z.sum()
```

[119]:
```
66
```

[120]:
```python
Z.max()
```

[120]:
```
11
```

[121]:
```
Z.min()
```

[121]:
```
0
```

[122]:
```
Z
```

[122]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[123]:
```
Z.sum(axis=0) # sum of each column
```

[123]:
```
array([12, 15, 18, 21])
```

[124]:
```
Z.sum(axis=1) # sum of each row
```

[124]:
```
array([ 6, 22, 38])
```

[125]:
```
Z.mean(axis=1) # average of each row
```

[125]:
```
array([1.5, 5.5, 9.5])
```

[126]:
```
Z.mean(axis=0) # average of each column
```

[126]:
```
array([4., 5., 6., 7.])
```

3.3 Indexing, Slicing and Iterating

- 1-dimensional arrays can be indexed, sliced and iterated over, much like lists and other Python sequences.
- Multidimensional arrays have one index per axis. These indices are given in a tuple separated by commas

[127]:
```
a = np.arange(10)**3
a
```

[127]:
```
array([  0,   1,   8,  27,  64, 125, 216, 343, 512, 729])
```

[128]:
```
a[0]
```

[128]:
```
0
```

[129]:
```
a[3]
```

[129]:
```
27
```

[130]:
```
a[2:5]
```

[130]:
```
array([ 8, 27, 64])
```

[131]:
```
for i in a:
    print(i)
```

```
0
1
8
27
64
125
216
343
512
729
```

[132]:
```
b = np.arange(12).reshape(3,4)
b
```

[132]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[133]:
```
list_b = []
for i in b:
    list_b.append(list(i))
list_b
```

[133]:
```
[[0, 1, 2, 3], [4, 5, 6, 7], [8, 9, 10, 11]]
```

[134]:
```
list_b[0][0]
```

[134]:
```
0
```

[135]:
```
b
```

[135]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[136]:
```
b[0,0]
```

[136]:
```
0
```

[137]:
```
b
```

[137]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[138]:
```
b[2,3]
```

[138]: 11

[139]:
```
b[:2,0]
```

[139]:
```
array([0, 4])
```

[140]:
```
b[:,0]
```

[140]:
```
array([0, 4, 8])
```

[141]:
```
b
```

[141]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[142]:
```
b[0]
```

[142]:
```
array([0, 1, 2, 3])
```

[143]:
```
b[0, :]
```

[143]:
```
array([0, 1, 2, 3])
```

The missing indices are considered complete slices: `b[0]` is equivalent to `b[0,:]`

Exercise:

```
b = np.arange(12).reshape(3,4)
```

- Obtain each column in the second and third row of b
- Obtain the first three rows and columns of b

[144]:
```
b = np.arange(12).reshape(3,4)
b
```

[144]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[145]:
```
b[1:3,]
```

[145]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[146]:
```
b[1:3,:]
```

[146]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[147]:
```
b[1:3]
```

[147]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[148]:
```
b[0:3,0:3]
```

[148]:
```
array([[ 0,  1,  2],
       [ 4,  5,  6],
       [ 8,  9, 10]])
```

[149]:
```
b[:3,:3]
```

[149]:
```
array([[ 0,  1,  2],
       [ 4,  5,  6],
       [ 8,  9, 10]])
```

[150]:
```
b
```

[150]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[151]:
```
for i in b:
    print(i)
```

```
[0 1 2 3]
[4 5 6 7]
[ 8  9 10 11]
```

Iterating over multidimensional arrays is done with respect to the first axis: row by row

More flexible indexing - fancy indexing

- Indexing with Arrays of Indices
- Indexing with Boolean Arrays

[152]:
```
a = np.arange(12)**2  # the first 12 square numbers
a
```

[152]:
```
array([  0,   1,   4,   9,  16,  25,  36,  49,  64,  81, 100, 121])
```

[153]:
```
np.__version__
```

[153]:
```
'1.24.3'
```

[154]:
```
i = np.array([1, 1, 3, 8, 5])  # an array of indices
i
```

[154]:
```
array([1, 1, 3, 8, 5])
```

[155]:
```
a[i]  # the elements of `a` at the positions `i`
```

[155]:
```
array([ 1,  1,  9, 64, 25])
```

[156]:
```
j = np.array([[3, 4], [9, 7]])  # a bidimensional array of indices
j
```

[156]:
```
array([[3, 4],
       [9, 7]])
```

[157]:
```
a
```

[157]:
```
array([  0,   1,   4,   9,  16,  25,  36,  49,  64,  81, 100, 121])
```

[158]:
```
a[j] # the same shape as `j`
```

[158]:
```
array([[ 9, 16],
       [81, 49]])
```

What if a is multidimensional?

[159]:
```
a = a.reshape(4,3)
a
```

[159]:
```
array([[  0,   1,   4],
       [  9,  16,  25],
       [ 36,  49,  64],
       [ 81, 100, 121]])
```

[160]:
```
i = np.array([[2, 1],  # indices for the first dim of `a`
              [3, 3]])
```

[161]:
```
j = np.array([[0, 1],  # indices for the second dim of `a`
              [1, 2]])
```

[162]:
```
a[i,j]
```

[162]:
```
array([[ 36,  16],
       [100, 121]])
```

[163]:
```
a
```

[163]:
```
array([[  0,   1,   4],
       [  9,  16,  25],
       [ 36,  49,  64],
       [ 81, 100, 121]])
```

[164]:
```
b = a> 14
b
```

[164]:
```
array([[False, False, False],
       [False,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]])
```

[165]:
```
a[b]   # 1d array with the selected elements
```

[165]:
```
array([ 16,  25,  36,  49,  64,  81, 100, 121])
```

use boolean arrays that have the same shape as the original array

[166]:
```
a[b] = 0   # All elements of `a` higher than 14 become 0
a
```

[166]:
```
array([[0, 1, 4],
       [9, 0, 0],
       [0, 0, 0],
       [0, 0, 0]])
```

3.4 More on Shape manipulation

[167]:
```
a = np.arange(20)
a
```

[167]:
```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16,
       17, 18, 19])
```

[168]:
```
a.shape
```

[168]:
```
(20,)
```

[169]:
```
a.shape = (4,5)
```

[170]:
```
a
```

[170]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

[171]:
```
a.shape = (2,4)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[171], line 1
----> 1 a.shape = (2,4)

ValueError: cannot reshape array of size 20 into shape (2,4)
```

[172]:
```python
a.shape = (2,10)
```

[173]:
```python
a
```

[173]:
```
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]])
```

[174]:
```python
a.transpose() # Transpose of the array
```

[174]:
```
array([[ 0, 10],
       [ 1, 11],
       [ 2, 12],
       [ 3, 13],
       [ 4, 14],
       [ 5, 15],
       [ 6, 16],
       [ 7, 17],
       [ 8, 18],
       [ 9, 19]])
```

[175]:
```python
a.T # Transpose of the array
```

[175]:
```
array([[ 0, 10],
       [ 1, 11],
       [ 2, 12],
       [ 3, 13],
       [ 4, 14],
       [ 5, 15],
       [ 6, 16],
       [ 7, 17],
       [ 8, 18],
       [ 9, 19]])
```

[176]:
```python
a.reshape(4,5)
```

[176]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

[177]:
```python
a.reshape(4,-1)
```

[177]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

If in a reshaping operation a dimension is given as -1, it is automatically calculated to correspond to the other dimensions.

[178]:
```
a.reshape(7,-1)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[178], line 1
----> 1 a.reshape(7,-1)

ValueError: cannot reshape array of size 20 into shape (7,newaxis)
```

[179]:
```
mean_row = a.mean(axis=1)
mean_row
```

[179]: array([4.5, 14.5])

[180]:
```
mean_row.shape
```

[180]: (2,)

[181]:
```
mean_row + a
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[181], line 1
----> 1 mean_row + a

ValueError: operands could not be broadcast together with shapes (2,) (2,10)
```

[182]:
```
a.shape
```

[182]: (2, 10)

[183]:
```
mean_row = mean_row.reshape(2, -1)
mean_row
```

[183]:
```
array([[ 4.5],
       [14.5]])
```

[184]:
```
mean_row.shape
```

[184]: (2, 1)

[185]:
```
mean_row + a
```

[185]:
```
array([[ 4.5,  5.5,  6.5,  7.5,  8.5,  9.5, 10.5, 11.5, 12.5, 13.5],
       [24.5, 25.5, 26.5, 27.5, 28.5, 29.5, 30.5, 31.5, 32.5, 33.5]])
```

3.4.1 Broadcasting for 2-d arrays

How NumPy treats arrays with different shapes during arithmetic operations * One dimension has the same size * The other dimension is of size 1

[186]:
```
mean_row = a.mean(axis=1)
mean_row
```

[186]:
```
array([ 4.5, 14.5])
```

[187]:
```
mean_row.shape
```

[187]:
```
(2,)
```

[188]:
```
a
```

[188]:
```
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]])
```

[189]:
```
a - mean_row
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[189], line 1
----> 1 a - mean_row

ValueError: operands could not be broadcast together with shapes (2,10) (2,)
```

[190]:
```
mean_row = mean_row.reshape(2, -1)
mean_row
```

[190]:
```
array([[ 4.5],
       [14.5]])
```

[191]:
```
mean_row.shape
```

[191]:
```
(2, 1)
```

[192]:
```
a - mean_row
```

[192]:
```
array([[-4.5, -3.5, -2.5, -1.5, -0.5,  0.5,  1.5,  2.5,  3.5,  4.5],
       [-4.5, -3.5, -2.5, -1.5, -0.5,  0.5,  1.5,  2.5,  3.5,  4.5]])
```

3.4.2 Concatenating two numpy arrays

[193]:
```
a.flatten() # turn the array into 1-d
```

[193]: array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
       17, 18, 19])

Stacking arrays together

[194]:
```
a
```

[194]: array([[0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
       [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]])

[195]:
```
a.shape
```

[195]: (2, 10)

[196]:
```
b=np.arange(200).reshape(-1,10)
b
```

[196]:
```
array([[  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [ 20,  21,  22,  23,  24,  25,  26,  27,  28,  29],
       [ 30,  31,  32,  33,  34,  35,  36,  37,  38,  39],
       [ 40,  41,  42,  43,  44,  45,  46,  47,  48,  49],
       [ 50,  51,  52,  53,  54,  55,  56,  57,  58,  59],
       [ 60,  61,  62,  63,  64,  65,  66,  67,  68,  69],
       [ 70,  71,  72,  73,  74,  75,  76,  77,  78,  79],
       [ 80,  81,  82,  83,  84,  85,  86,  87,  88,  89],
       [ 90,  91,  92,  93,  94,  95,  96,  97,  98,  99],
       [100, 101, 102, 103, 104, 105, 106, 107, 108, 109],
       [110, 111, 112, 113, 114, 115, 116, 117, 118, 119],
       [120, 121, 122, 123, 124, 125, 126, 127, 128, 129],
       [130, 131, 132, 133, 134, 135, 136, 137, 138, 139],
       [140, 141, 142, 143, 144, 145, 146, 147, 148, 149],
       [150, 151, 152, 153, 154, 155, 156, 157, 158, 159],
       [160, 161, 162, 163, 164, 165, 166, 167, 168, 169],
       [170, 171, 172, 173, 174, 175, 176, 177, 178, 179],
       [180, 181, 182, 183, 184, 185, 186, 187, 188, 189],
       [190, 191, 192, 193, 194, 195, 196, 197, 198, 199]])
```

[197]:
```
b.shape
```

[197]: (20, 10)

[198]:
```
np.vstack((a,b)) #Stack arrays in sequence vertically (row wise): number of␣
↪columns have to match
```

[198]:
```
array([[  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
```

```
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [ 20,  21,  22,  23,  24,  25,  26,  27,  28,  29],
       [ 30,  31,  32,  33,  34,  35,  36,  37,  38,  39],
       [ 40,  41,  42,  43,  44,  45,  46,  47,  48,  49],
       [ 50,  51,  52,  53,  54,  55,  56,  57,  58,  59],
       [ 60,  61,  62,  63,  64,  65,  66,  67,  68,  69],
       [ 70,  71,  72,  73,  74,  75,  76,  77,  78,  79],
       [ 80,  81,  82,  83,  84,  85,  86,  87,  88,  89],
       [ 90,  91,  92,  93,  94,  95,  96,  97,  98,  99],
       [100, 101, 102, 103, 104, 105, 106, 107, 108, 109],
       [110, 111, 112, 113, 114, 115, 116, 117, 118, 119],
       [120, 121, 122, 123, 124, 125, 126, 127, 128, 129],
       [130, 131, 132, 133, 134, 135, 136, 137, 138, 139],
       [140, 141, 142, 143, 144, 145, 146, 147, 148, 149],
       [150, 151, 152, 153, 154, 155, 156, 157, 158, 159],
       [160, 161, 162, 163, 164, 165, 166, 167, 168, 169],
       [170, 171, 172, 173, 174, 175, 176, 177, 178, 179],
       [180, 181, 182, 183, 184, 185, 186, 187, 188, 189],
       [190, 191, 192, 193, 194, 195, 196, 197, 198, 199]])
```

[199]:
```
np.hstack((a,b)) #Stack arrays in sequence horizontally (column wise): : number␣
↪of rows have to match
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[199], line 1
----> 1 np.hstack((a,b))

File <__array_function__ internals>:200, in hstack(*args, **kwargs)

File ~/opt/anaconda3/lib/python3.9/site-packages/numpy/core/shape_base.py:370,␣
↪in hstack(tup, dtype, casting)
    368     return _nx.concatenate(arrs, 0, dtype=dtype, casting=casting)
    369 else:
--> 370     return _nx.concatenate(arrs, 1, dtype=dtype, casting=casting)

File <__array_function__ internals>:200, in concatenate(*args, **kwargs)

ValueError: all the input array dimensions except for the concatenation axis␣
↪must match exactly, but along dimension 0, the array at index 0 has size 2 an␣
↪the array at index 1 has size 20
```

3.5 Other array functions

[200]:
```
a=np.array( [[ 7,2, 1], [4,3, 8] ])
a
```

[200]:
```
array([[7, 2, 1],
       [4, 3, 8]])
```

[201]:
```
np.sort?
```

[202]:
```
np.sort(a)
```

[202]:
```
array([[1, 2, 7],
       [3, 4, 8]])
```

[203]:
```
np.sort(a,axis=1)
```

[203]:
```
array([[1, 2, 7],
       [3, 4, 8]])
```

[204]:
```
np.sort(a,axis=0)
```

[204]:
```
array([[4, 2, 1],
       [7, 3, 8]])
```

3.6 Further reading

- read Numpy tutorial to learn more about numpy functionalities