

- ▼ Ran by Sushant Gautam as a part of Assignemnt 5 for DL for VI course.

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```
1 # This mounts your Google Drive to the Colab VM.
2 from google.colab import drive
3 drive.mount('/content/drive')
4
5 # path of cs231n folder in google drive
6 FOLDERNAME = 'MS-DL-Assignemnt5'
7 assert FOLDERNAME is not None, "[!] Enter the foldername."
8
9 # Now that we've mounted your Drive, this ensures that
10 # the Python interpreter of the Colab VM can load
11 # python files from within it.
12 import sys
13 basefold = '/content/drive/My Drive/{}'.format(FOLDERNAME)
14 sys.path.append(basefold)
15
16 # This downloads the COCO dataset to your Drive
17 # if it doesn't already exist.
18 %cd /content/drive/My\ Drive/$FOLDERNAME/cs231n/datasets/
19 !bash get_datasets.sh
20 %cd /content/drive/My\ Drive/$FOLDERNAME
```

➤ Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).

/content/drive/My Drive/MS-DL-Assignemnt5/cs231n/datasets

/content/drive/My Drive/MS-DL-Assignemnt5

▼ Image Captioning with LSTMs

In the previous exercise, you implemented a vanilla RNN and applied it to image captioning. In this notebook, you will implement the LSTM update rule and use it for image captioning.

```
1 # Setup cell.
2 import time, os, json
3 import numpy as np
4 import matplotlib.pyplot as plt
5
6 from cs231n.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
7 from cs231n.rnn_layers import *
8 from cs231n.captioning_solver import CaptioningSolver
9 from cs231n.classifiers.rnn import CaptioningRNN
10 from cs231n.coco_utils import load_coco_data, sample_coco_minibatch, decode_captions
11 from cs231n.image_utils import image_from_url
12
13 %matplotlib inline
14 plt.rcParams['figure.figsize'] = (10.0, 8.0) # Set default size of plots.
15 plt.rcParams['image.interpolation'] = 'nearest'
16 plt.rcParams['image.cmap'] = 'gray'
17
18 %load_ext autoreload
19 %autoreload 2
20
21 def rel_error(x, y):
22     """ returns relative error """
23     return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

The autoreload extension is already loaded. To reload it, use:

%reload_ext autoreload

▼ COCO Dataset

As in the previous notebook, we will use the COCO dataset for captioning.

```
1 # Load COCO data from disk into a dictionary.
2 data = load_coco_data(pca_features=True)
3
4 # Print out all the keys and values from the data dictionary.
5 for k, v in data.items():
6     if type(v) == np.ndarray:
7         print(k, type(v), v.shape, v.dtype)
8     else:
9         print(k, type(v), len(v))
```

base dir /content/drive/My Drive/MS-DL-Assignemnt5/cs231n/datasets/coco_captioning

/content/drive/My Drive/MS-DL-Assignemnt5/cs231n/datasets/coco_captioning/train2014_vgg16_fc7_pca.h5

train_captions <class 'numpy.ndarray'> (400135, 17) int32

train_image_idxs <class 'numpy.ndarray'> (400135,) int32

val_captions <class 'numpy.ndarray'> (195954, 17) int32

val_image_idxs <class 'numpy.ndarray'> (195954,) int32

train_features <class 'numpy.ndarray'> (82783, 512) float32

val_features <class 'numpy.ndarray'> (40504, 512) float32

idx_to_word <class 'list'> 1004

word_to_idx <class 'dict'> 1004

train_urls <class 'numpy.ndarray'> (82783,) <U63

val_urls <class 'numpy.ndarray'> (40504,) <U63

LSTM

A common variant on the vanilla RNN is the Long-Short Term Memory (LSTM) RNN. Vanilla RNNs can be tough to train on long sequences due to vanishing and exploding gradients caused by repeated matrix multiplication. LSTMs solve this problem by replacing the simple update rule of the vanilla RNN with a gating mechanism as follows.

Similar to the vanilla RNN, at each timestep we receive an input $x_t \in \mathbb{R}^D$ and the previous hidden state $h_{t-1} \in \mathbb{R}^H$; the LSTM also maintains an H -dimensional *cell state*, so we also receive the previous cell state $c_{t-1} \in \mathbb{R}^H$. The learnable parameters of the LSTM are an *input-to-hidden* matrix $W_x \in \mathbb{R}^{4H \times D}$, a *hidden-to-hidden* matrix $W_h \in \mathbb{R}^{4H \times H}$ and a *bias vector* $b \in \mathbb{R}^{4H}$.

At each timestep we first compute an *activation vector* $a \in \mathbb{R}^{4H}$ as $a = W_x x_t + W_h h_{t-1} + b$. We then divide this into four vectors $a_i, a_f, a_o, a_g \in \mathbb{R}^H$ where a_i consists of the first H elements of a , a_f is the next H elements of a , etc. We then compute the *input gate* $g \in \mathbb{R}^H$, *forget gate* $f \in \mathbb{R}^H$, *output gate* $o \in \mathbb{R}^H$ and *block input* $g \in \mathbb{R}^H$ as

$$i = \sigma(a_i) \qquad f = \sigma(a_f) \qquad o = \sigma(a_o) \qquad g = \tanh(a_g)$$

where σ is the sigmoid function and \tanh is the hyperbolic tangent, both applied elementwise.

Finally we compute the next cell state c_t and next hidden state h_t as

$$c_t = f \odot c_{t-1} + i \odot g \qquad h_t = o \odot \tanh(c_t)$$

where \odot is the elementwise product of vectors.

In the rest of the notebook we will implement the LSTM update rule and apply it to the image captioning task.

In the code, we assume that data is stored in batches so that $X_t \in \mathbb{R}^{N \times D}$ and will work with *transposed* versions of the parameters:

$W_x \in \mathbb{R}^{D \times 4H}$, $W_h \in \mathbb{R}^{H \times 4H}$ so that activations $A \in \mathbb{R}^{N \times 4H}$ can be computed efficiently as $A = X W_x + H W_h + b$.

▼ LSTM: Step Forward

Implement the forward pass for a single timestep of an LSTM in the `lstm_step_forward` function in the file `cs231n/rnn_layers.py`. This should be similar to the `rnn_step_forward` function that you implemented above, but using the LSTM update rule instead.

Once you are done, run the following to perform a simple test of your implementation. You should see errors on the order of `e-8` or less.

```
1 N, D, H = 3, 4, 5
2 x = np.linspace(-0.4, 1.2, num=N*D).reshape(N, D)
3 prev_h = np.linspace(-0.3, 0.7, num=N*H).reshape(N, H)
4 prev_c = np.linspace(-0.4, 0.9, num=N*H).reshape(N, H)
5 Wx = np.linspace(-2.1, 1.3, num=4*D*H).reshape(D, 4 * H)
6 Wh = np.linspace(-0.7, 2.2, num=4*H*H).reshape(H, 4 * H)
7 b = np.linspace(0.3, 0.7, num=4*H)
8
9 next_h, next_c, cache = lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)
10
11 expected_next_h = np.asarray([
12     [ 0.24635157,  0.28610883,  0.32240467,  0.35525807,  0.38474904],
13     [ 0.49223563,  0.55611431,  0.61507696,  0.66844003,  0.7159181 ],
14     [ 0.56735664,  0.66310127,  0.74419266,  0.80889665,  0.858299  ]])
15 expected_next_c = np.asarray([
16     [ 0.32986176,  0.39145139,  0.451556,    0.51014116,  0.56717407],
17     [ 0.66382255,  0.76674007,  0.87195994,  0.97902709,  1.08751345],
18     [ 0.74192008,  0.90592151,  1.07717006,  1.25120233,  1.42395676]])
19
20 print('next_h error: ', rel_error(expected_next_h, next_h))
21 print('next_c error: ', rel_error(expected_next_c, next_c))

next_h error:  5.7054131967097955e-09
next_c error:  5.8143123088804145e-09
```

▼ LSTM: Step Backward

Implement the backward pass for a single LSTM timestep in the function `lstm_step_backward` in the file `cs231n/rnn_layers.py`. Once you are done, run the following to perform numeric gradient checking on your implementation. You should see errors on the order of `e-7` or less.

```
1 np.random.seed(231)
2
3 N, D, H = 4, 5, 6
4 x = np.random.randn(N, D)
5 prev_h = np.random.randn(N, H)
6 prev_c = np.random.randn(N, H)
7 Wx = np.random.randn(D, 4 * H)
8 Wh = np.random.randn(H, 4 * H)
9 b = np.random.randn(4 * H)
10
11 next_h, next_c, cache = lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)
12
13 dnext_h = np.random.randn(*next_h.shape)
14 dnext_c = np.random.randn(*next_c.shape)
15
16 fx_h = lambda x: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
17 fh_h = lambda h: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
18 fc_h = lambda c: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
19 fWx_h = lambda Wx: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
20 fWh_h = lambda Wh: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
21 fb_h = lambda b: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[0]
22
23 fx_c = lambda x: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
24 fh_c = lambda h: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
25 fc_c = lambda c: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
26 fWx_c = lambda Wx: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
27 fWh_c = lambda Wh: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
28 fb_c = lambda b: lstm_step_forward(x, prev_h, prev_c, Wx, Wh, b)[1]
29
30 num_grad = eval_numerical_gradient_array
31
32 dx_num = num_grad(fx_h, x, dnext_h) + num_grad(fx_c, x, dnext_c)
33 dh_num = num_grad(fh_h, prev_h, dnext_h) + num_grad(fh_c, prev_h, dnext_c)
34 dc_num = num_grad(fc_h, prev_c, dnext_h) + num_grad(fc_c, prev_c, dnext_c)
35 dWx_num = num_grad(fWx_h, Wx, dnext_h) + num_grad(fWx_c, Wx, dnext_c)
36 dWh_num = num_grad(fWh_h, Wh, dnext_h) + num_grad(fWh_c, Wh, dnext_c)
37 db_num = num_grad(fb_h, b, dnext_h) + num_grad(fb_c, b, dnext_c)
38
39 dx, dh, dc, dWx, dWh, db = lstm_step_backward(dnext_h, dnext_c, cache)
40
41 print('dx error: ', rel_error(dx_num, dx))
42 print('dh error: ', rel_error(dh_num, dh))
43 print('dc error: ', rel_error(dc_num, dc))
44 print('dWx error: ', rel_error(dWx_num, dWx))
45 print('dWh error: ', rel_error(dWh_num, dWh))
46 print('db error: ', rel_error(db_num, db))
```

```
dx error:  6.335032254429549e-10
dh error:  3.3963774090592634e-10
dc error:  1.5221723979041107e-10
dWx error:  2.1010960934639614e-09
dWh error:  9.712296109943072e-08
db error:  2.491522041931035e-10
```

▼ LSTM: Forward

In the function `lstm_forward` in the file `cs231n/rnn_layers.py`, implement the `lstm_forward` function to run an LSTM forward on an entire timeseries of data.

When you are done, run the following to check your implementation. You should see an error on the order of e^{-7} or less.

```
1 N, D, H, T = 2, 5, 4, 3
2 x = np.linspace(-0.4, 0.6, num=N*T*D).reshape(N, T, D)
3 h0 = np.linspace(-0.4, 0.8, num=N*H).reshape(N, H)
4 Wx = np.linspace(-0.2, 0.9, num=4*D*H).reshape(D, 4 * H)
5 Wh = np.linspace(-0.3, 0.6, num=4*H*H).reshape(H, 4 * H)
6 b = np.linspace(0.2, 0.7, num=4*H)
7
8 h, cache = lstm_forward(x, h0, Wx, Wh, b)
9
10 expected_h = np.asarray([
11     [[ 0.01764008,  0.01823233,  0.01882671,  0.0194232 ],
12      [ 0.11287491,  0.12146228,  0.13018446,  0.13902939],
13      [ 0.31358768,  0.33338627,  0.35304453,  0.37250975]],
14     [[ 0.45767879,  0.4761092,   0.4936887,   0.51041945],
15      [ 0.6704845,   0.69350089,  0.71486014,  0.7346449 ],
16      [ 0.81733511,  0.83677871,  0.85403753,  0.86935314]]])
17
18 print('h error: ', rel_error(expected_h, h))

h error:  8.610537452106624e-08
```

▼ LSTM: Backward

Implement the backward pass for an LSTM over an entire timeseries of data in the function `lstm_backward` in the file `cs231n/rnn_layers.py`.

When you are done, run the following to perform numeric gradient checking on your implementation. You should see errors on the order of e^{-8} or less. (For `dWh`, it's fine if your error is on the order of e^{-6} or less).

```
1 from cs231n.rnn_layers import lstm_forward, lstm_backward
2 np.random.seed(231)
3
4 N, D, T, H = 2, 3, 10, 6
5
6 x = np.random.randn(N, T, D)
7 h0 = np.random.randn(N, H)
8 Wx = np.random.randn(D, 4 * H)
9 Wh = np.random.randn(H, 4 * H)
10 b = np.random.randn(4 * H)
11
12 out, cache = lstm_forward(x, h0, Wx, Wh, b)
13
14 dout = np.random.randn(*out.shape)
15
16 dx, dh0, dWx, dWh, db = lstm_backward(dout, cache)
17
18 fx = lambda x: lstm_forward(x, h0, Wx, Wh, b)[0]
19 fh0 = lambda h0: lstm_forward(x, h0, Wx, Wh, b)[0]
20 fWx = lambda Wx: lstm_forward(x, h0, Wx, Wh, b)[0]
21 fWh = lambda Wh: lstm_forward(x, h0, Wx, Wh, b)[0]
22 fb = lambda b: lstm_forward(x, h0, Wx, Wh, b)[0]
23
24 dx_num = eval_numerical_gradient_array(fx, x, dout)
25 dh0_num = eval_numerical_gradient_array(fh0, h0, dout)
26 dWx_num = eval_numerical_gradient_array(fWx, Wx, dout)
27 dWh_num = eval_numerical_gradient_array(fWh, Wh, dout)
28 db_num = eval_numerical_gradient_array(fb, b, dout)
29
30 print('dx error: ', rel_error(dx_num, dx))
31 print('dh0 error: ', rel_error(dh0_num, dh0))
32 print('dWx error: ', rel_error(dWx_num, dWx))
33 print('dWh error: ', rel_error(dWh_num, dWh))
34 print('db error: ', rel_error(db_num, db))

dx error:  6.9939005453315376e-09
dh0 error:  1.5042746972106784e-09
dWx error:  3.2262956411424662e-09
dWh error:  2.6984652580094597e-06
db error:  8.236633698313836e-10
```

▼ LSTM Captioning Model

Now that you have implemented an LSTM, update the implementation of the `loss` method of the `CaptioningRNN` class in the file `cs231n/classifiers/rnn.py` to handle the case where `self.cell_type` is `lstm`. This should require adding less than 10 lines of code.

Once you have done so, run the following to check your implementation. You should see a difference on the order of e^{-10} or less.

```
1 N, D, W, H = 10, 20, 30, 40
2 word_to_idx = {'<NULL>': 0, 'cat': 2, 'dog': 3}
3 V = len(word_to_idx)
4 T = 13
5
6 model = CaptioningRNN(
7     word_to_idx,
8     input_dim=D,
9     wordvec_dim=W,
10    hidden_dim=H,
11    cell_type='lstm',
12    dtype=np.float64
13 )
14
15 # Set all model parameters to fixed values
16 for k, v in model.params.items():
17     model.params[k] = np.linspace(-1.4, 1.3, num=v.size).reshape(*v.shape)
18
19 features = np.linspace(-0.5, 1.7, num=N*D).reshape(N, D)
20 captions = (np.arange(N * T) % V).reshape(N, T)
21
22 loss, grads = model.loss(features, captions)
23 expected_loss = 9.82445935443
```

```
24
25 print('loss: ', loss)
26 print('expected loss: ', expected_loss)
27 print('difference: ', abs(loss - expected_loss))

    loss:   9.824459354432264
    expected loss:   9.82445935443
    difference:   2.2648549702353193e-12
```

▼ Overfit LSTM Captioning Model on Small Data

Run the following to overfit an LSTM captioning model on the same small dataset as we used for the RNN previously. You should see a final loss less than 0.5.

```
1 np.random.seed(231)
2
3 small_data = load_coco_data(max_train=50)
4
5 small_lstm_model = CaptioningRNN(
6     cell_type='lstm',
7     word_to_idx=data['word_to_idx'],
8     input_dim=data['train_features'].shape[1],
9     hidden_dim=512,
10    wordvec_dim=256,
11    dtype=np.float32,
12 )
13
14 small_lstm_solver = CaptioningSolver(
15     small_lstm_model, small_data,
16     update_rule='adam',
17     num_epochs=50,
18     batch_size=25,
19     optim_config={
20         'learning_rate': 5e-3,
21     },
22     lr_decay=0.995,
23     verbose=True, print_every=10,
24 )
25
26 small_lstm_solver.train()
27
28 # Plot the training losses
29 plt.plot(small_lstm_solver.loss_history)
30 plt.xlabel('Iteration')
31 plt.ylabel('Loss')
32 plt.title('Training loss history')
33 plt.show()
```

Print final training loss. You should see a final loss of less than 0.5.

```
1 print('Final loss: ', small_lstm_solver.loss_history[-1])

    Final loss:   0.07901461745592495
```

▼ LSTM Sampling at Test Time

Modify the `sample` method of the `CaptioningRNN` class to handle the case where `self.cell_type` is `lstm`. This should take fewer than 10 lines of code.

When you are done run the following to sample from your overfit LSTM model on some training and validation set samples. As with the RNN, training results should be very good, and validation results probably won't make a lot of sense (because we're overfitting).

```
1 # If you get an error, the URL just no longer exists, so don't worry!
2 # You can re-sample as many times as you want.
3 for split in ['train', 'val']:
4     minibatch = sample_coco_minibatch(small_data, split=split, batch_size=2)
5     gt_captions, features, urls = minibatch
6     gt_captions = decode_captions(gt_captions, data['idx_to_word'])
```

```
7
8     sample_captions = small_lstm_model.sample(features)
9     sample_captions = decode_captions(sample_captions, data['idx_to_word'])
10
11 for gt_caption, sample_caption, url in zip(gt_captions, sample_captions, urls):
12     img = image_from_url(url)
13     # Skip missing URLs.
14     if img is None: continue
15     plt.imshow(img)
16     plt.title('%s\n%s\nGT:%s' % (split, sample_caption, gt_caption))
17     plt.axis('off')
18     plt.show()
```

In addition to qualitatively evaluating your model by inspecting its results, you can also quantitatively evaluate your model using the BLEU unigram precision metric. In order to achieve full credit you should train a model that achieves a BLEU unigram score of >0.3. BLEU scores range from 0 to 1; the closer to 1, the better. Here's a reference to the [paper](#) that introduces BLEU if you're interested in learning more about how it works.

```
1 import nltk
2 def BLEU_score(gt_caption, sample_caption):
3     """
4     gt_caption: string, ground-truth caption
5     sample_caption: string, your model's predicted caption
6     Returns unigram BLEU score.
7     """
8     reference = [x for x in gt_caption.split(' ')
9                   if ('<END>' not in x and '<START>' not in x and '<UNK>' not in x)]
10    hypothesis = [x for x in sample_caption.split(' ')
11                  if ('<END>' not in x and '<START>' not in x and '<UNK>' not in x)]
12    BLEUScore = nltk.translate.bleu_score.sentence_bleu([reference], hypothesis, weights = [1])
13    return BLEUScore
14
15 def evaluate_model(model):
16     """
17     model: CaptioningRNN model
18     Prints unigram BLEU score averaged over 1000 training and val examples.
19     """
20     BLEUScores = {}
21     for split in ['train', 'val']:
22         minibatch = sample_coco_minibatch(data, split=split, batch_size=1000)
23         gt_captions, features, urls = minibatch
24         gt_captions = decode_captions(gt_captions, data['idx_to_word'])
25
26         sample_captions = model.sample(features)
27         sample_captions = decode_captions(sample_captions, data['idx_to_word'])
28
29
30         total_score = 0.0
31         for gt_caption, sample_caption, url in zip(gt_captions, sample_captions, urls):
32             total_score += BLEU_score(gt_caption, sample_caption)
33
34         BLEUScores[split] = total_score / len(sample_captions)
35
36     for split in BLEUScores:
37         print('Average BLEU score for %s: %f' % (split, BLEUScores[split]))
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