Improving Generalization of Encoder-Decoder Neural Network with Stochastic Latent Space

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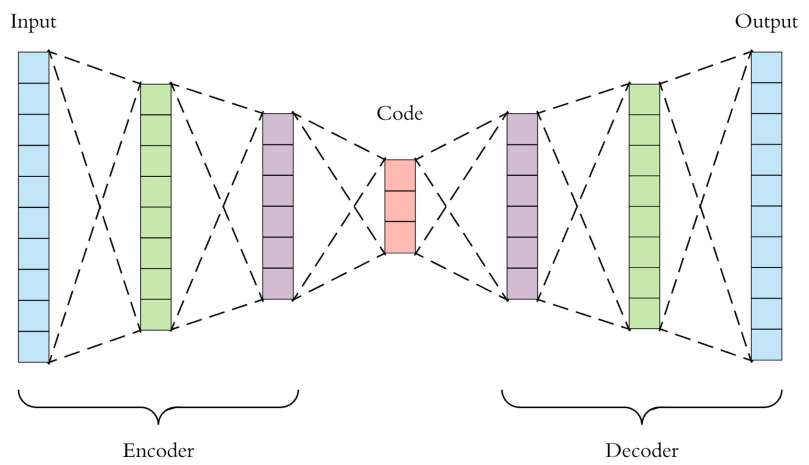
Introduction

Encoder-Decoder Neural Network

- Autoencoder
- Encoder-Decoder



Key Ideas

- How to improve generalization ability of an encoder decoder network?

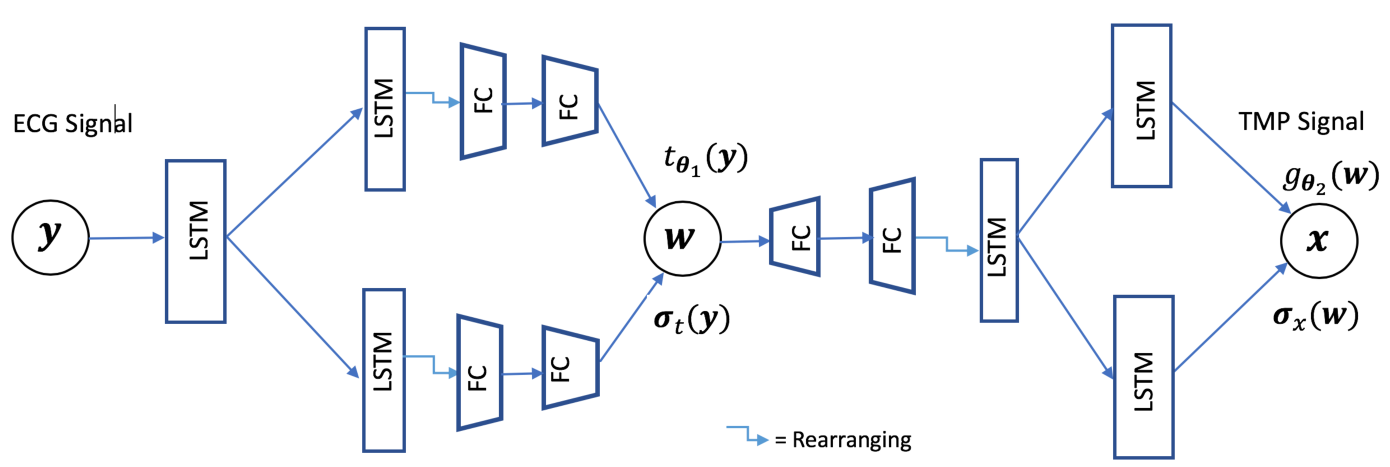
→ With constrained stochasticity of latent space

- Why does constrained stochasticity help?

→ Because it renders decoder less sensitive to variation in latent space

Application to ECGi

- Inverse reconstruction of TMP from ECG signal
- Ill posed inverse problem
- Estimating sequence from sequence



- LSTM and FC layer in both encoder and decoder
- Conditional distributions as Gaussian

Contribution

- Theoretically show that stochastic latent space helps in improving generalization
- Experimentally support the theory with application to ECGi

Theory

Generalization Gap

$\mathcal{T}$ = Encoder

f = loss composite decoder

Theorem 1 ((Kawaguchi and Bengio 2018)). *For any ℓ, let $(\mathcal{T}, f)$ be a pair such that $\mathcal{T} : (\mathcal{Z}, \mathcal{S}) \to ([0,1]^d, \mathcal{B}([0,1]^d))$ is a measurable function, $f : ([0,1]^d, \mathcal{B}([0,1]^d)) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ is of bounded variation as $V[f] < \infty$, and $\ell(\boldsymbol{x}, h(\boldsymbol{y})) = (f \circ \mathcal{T})(\boldsymbol{z}) \forall \boldsymbol{z} \in \mathcal{Z}$, where $\mathcal{B}(A)$ indicates the Borel σ- algebra on A. Then for any dataset pair (D_m, Z_m) and any $\ell(\boldsymbol{x}, h_{\mathcal{A}(D_n)}(\boldsymbol{y}))$,*

$$E_\mu \ell(\boldsymbol{x}, h_{\mathcal{A}(D_m)}(\boldsymbol{y})) - E_{Z_m} \ell(\boldsymbol{x}, h_{\mathcal{A}(D_n)}(\boldsymbol{y})) \leq V[f]\mathcal{D}^*[\mathcal{T}_*\mu, \mathcal{T}(Z_m)]$$

where $\mathcal{T}_*\mu$ is pushforward measure of μ under the map $\mathcal{T}$.

Implication:
Generalization gap can be reduced by decreasing either variation or discrepancy

Proposition 1 ((Kawaguchi and Bengio 2018)). *Suppose that $f_{j_1,..j_k}$ is a function for which $\partial_{1,\dots k} f_{j_1,..j_k}$ exists on $[0,1]^k$. Then,*

$$V^k[f_{j_1 \dots j_k}] \leq \sup_{\boldsymbol{t}_{j_1},..,\boldsymbol{t}_{j_k} \in [0,1]^k} |\partial_{1,\dots k} f_{j_1,..j_k}(\boldsymbol{t}_{j_1},..,\boldsymbol{t}_{j_k})|.$$

If $\partial_{1,\dots k} f_{j_1,..j_k}$ is also continuous on $[0,1]^k$,

$$V^k[f_{j_1 \dots j_k}] = \int_{[0,1]^k} |\partial_{1,\dots k} f_{j_1,..j_k}(\boldsymbol{t}_{j_1},..,\boldsymbol{t}_{j_k})| dt_{j_1}..dt_{j_k}.$$

$$\partial_{1,\dots k} f_{j_1,..j_k}(\boldsymbol{t}_{j_1},..,\boldsymbol{t}_{j_k}) = \frac{\partial^k \ell}{\partial \boldsymbol{t}_{j_1},..,\partial \boldsymbol{t}_{j_k}}$$

Learning with Stochastic Latent Space

$$p_{\boldsymbol{\theta}_1}(\boldsymbol{w}|\boldsymbol{y}) = \mathcal{N}(\boldsymbol{w}|\boldsymbol{t}_{\theta_1}(y), \boldsymbol{\sigma_t}^2(\boldsymbol{y}))$$
$$p_{\boldsymbol{\theta}_2}(\boldsymbol{x}|\boldsymbol{w}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{g}_{\theta_2}(\boldsymbol{w}), \boldsymbol{\sigma_x}^2(\boldsymbol{w}))$$

$$\text{minimize} - E_{P(x,y)} \log \int p_{\boldsymbol{\theta}_2}(\boldsymbol{x}|\boldsymbol{w}) p_{\boldsymbol{\theta}_1}(\boldsymbol{w}|\boldsymbol{y}) d\boldsymbol{w}$$
$$\text{such that } D_{KL}(p_{\boldsymbol{\theta}_1}(\boldsymbol{w}|\boldsymbol{y}) || \mathcal{N}(\boldsymbol{w}|\boldsymbol{0}, \boldsymbol{I})) < \delta$$

Loss Function

$$\mathcal{L} \leq E_{P(\boldsymbol{x},\boldsymbol{y})}\Big[E_{\boldsymbol{\epsilon}\sim\mathcal{N}(\boldsymbol{0},\boldsymbol{I})}\Big(\sum_i \overbrace{\frac{1}{\boldsymbol{\sigma_{x}}_i^2}(x_i - g_i(\boldsymbol{t} + \boldsymbol{\sigma_t} \odot \boldsymbol{\epsilon}))^2}^{T_1} + \log \boldsymbol{\sigma_{x}}_i^2\Big) + \lambda . D_{KL}(p_{\boldsymbol{\theta}_1}(\boldsymbol{w}|\boldsymbol{y}) || \mathcal{N}(\boldsymbol{w}|\boldsymbol{0}, \boldsymbol{I}))\Big] + constant$$

Result I
(Using Taylor series)

Deterministic loss

$$T_1 = \sum_i \frac{1}{\boldsymbol{\sigma}_{xi}^2}\Big[\ell_i(\boldsymbol{x}_i, \boldsymbol{t}) + \langle \boldsymbol{\sigma}_t \odot E_\epsilon[\boldsymbol{\epsilon}], \frac{\partial}{\partial t}\ell_i(\boldsymbol{x}_i, \boldsymbol{t})\rangle$$
$$+ \frac{1}{2}\langle [\boldsymbol{\sigma}_t \otimes \boldsymbol{\sigma}_t] \odot E_\epsilon[\boldsymbol{\epsilon} \otimes \boldsymbol{\epsilon}], \Big[\frac{\partial^2}{\partial \boldsymbol{t}_{j_1}, \partial \boldsymbol{t}_{j_2}}\ell_i(\boldsymbol{x}_i, \boldsymbol{t})\Big]\rangle + ..$$
$$+ \frac{1}{k!}\langle [\boldsymbol{\sigma}_t \otimes^k \boldsymbol{\sigma}_t] \odot E_\epsilon[\boldsymbol{\epsilon} \otimes^k \boldsymbol{\epsilon}], \Big[\frac{\partial^k}{\partial \boldsymbol{t}_{j_1}, .., \partial \boldsymbol{t}_{j_k}}\ell_i(\boldsymbol{x}_i, \boldsymbol{t})\Big]\rangle + ..\Big]$$

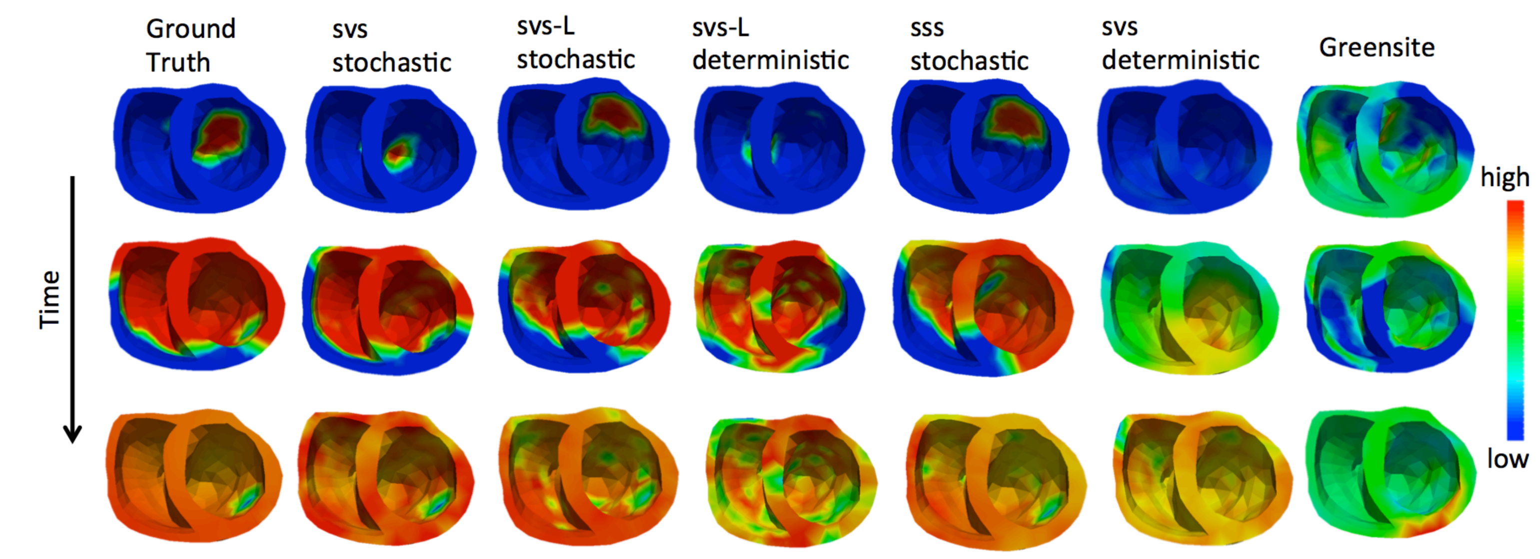
Weight for partial derivative
Tensor modulating noise
Noise tensor
Partial derivative tensor

Experiment

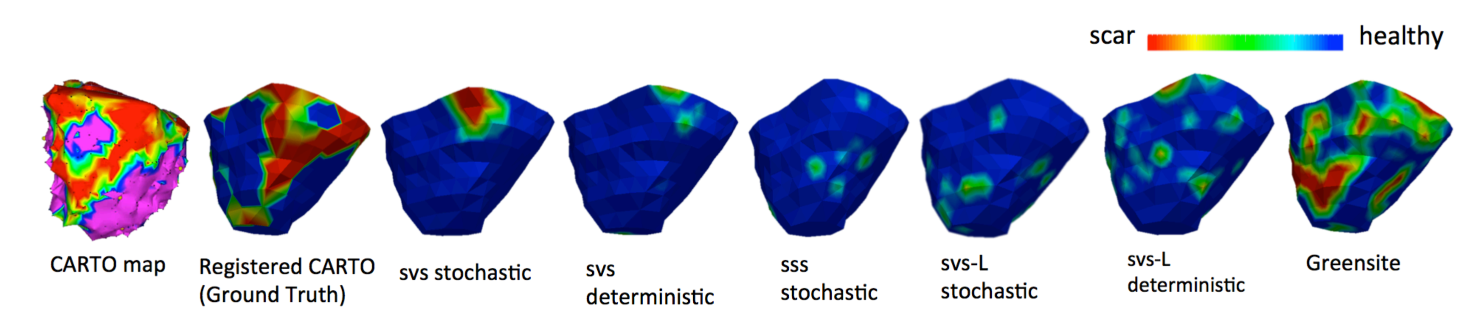
Settings:

- TMP simulated using Aliev Panfilov model on 3 human-torso geometry models.
- Simulations varying origin of excitation and tissue property
- Test data simulated with new origin of excitation and tissue property

Experiment on Simulated data



Experiment on Real data



Stochastic latent space generalizes well in both svs and svs-L architectures qualitatively and quantitatively

Metric	MSE	TMP Corr.	AT Corr.	Dice Coeff.
svs stochastic	**0.037± 0.021**	**0.885± 0.061**	**0.885± 0.072**	**0.645± 0.181**
svs deterministic	0.075 ± 0.013	0.77 ± 0.038	0.12 ± 0.13	0.01 ± 0.006
svs-L stochastic	0.068 ± 0.023	0.838 ± 0.053	0.601 ± 0.074	0.28 ± 0.154
svs-L deterministic	0.067 ± 0.02	0.84 ± 0.053	0.57 ± 0.052	0.165 ± 0.092
Greensite	–	–	0.514 ± 0.006	0.138 ± 0.005

Conclusion

By drawing from analytical learning theory, we have shown that constrained stochasticity of latent variable improves generalization ability and then supported the theory with experiments on electrophysiological imaging.

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