June 2022 PG Examinations

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18MAT11B

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU, Belagavi)

I Semester Master of Technology

PROBABILITY THEORY AND LINEAR ALGEBRA

(Common to MCS, MCE, MCM, MDC, MRM, MSC, MIT)

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit.
3. Use of statistical table permitted.

UNIT-1

1	a	Show that the set of vectors $P_1 = \{a_0 + a_1x /\ a_0, a_1 \in R\}$ is closed under vector addition and scalar multiplication. Also find the zero vector and the inverse element.	04
	b	Obtain the basis and dimension of the null-space and row-space of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 \\ 2 & 4 & 3 & 8 & 3 \\ 1 & 2 & 2 & 5 & 3 \\ 4 & 8 & 6 & 16 & 7 \end{bmatrix}$.	08
	c	Show that the mapping $T: R^3 \to R^3$ defined by $T(x, y, z) = (3x + y - 2z, 2x - y + 2z, -x + 2y + 2z)$ is a linear transformation. Also verify the rank-nullity theorem for the transformation.	08
		OR	
2	a	Show that the subset $W = \left\{ \begin{bmatrix} 2x & -y \\ y & x \end{bmatrix} / x, y, z \in R \right\}$ is a subspace of the vector space $M_{\{2\times2\}}$, the set of all 2×2 matrices.	04
	b	Obtain the basis and dimension of the Left Null space and Column space of the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 5 & 4 & 5 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 6 & 7 & 7 & 9 \end{bmatrix}$.	08
	c	Obtain the linear transformation $T: R^3 \to R^4$, defined by $T(1,2,1) = (1,4,5,2)$, $T(2,0,4) = (2,4,4,8)$, $T(2,-1,2) = (2,3,0,4)$. Also find the range-space and null-space of the linear transformation.	08

UNIT-2

3	a	A certain experiment produces the data $(2,3), (3,2), (5,1), (6,2)$. Fit an equation of the form $y = \beta_0 + \beta_1 x$ to the given data by the method of least-squares.	04
	b	Diagonalise the matrix $A = \begin{bmatrix} -2 & -1 & 4 \\ 0 & -3 & 4 \\ 3 & -3 & 1 \end{bmatrix}$.	08
	c	The matrix $A = \begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 0 & 2 \end{bmatrix}$ can be decomposed as $A = U \sum V^T$ by SVD process. Construct the matrices U and $\sum$ by the said process.	08
		OR	

4	a	Apply Gram-Schmidt orthogonalization process to the basis vectors $x_1 = (-1,1,0), x_2 = (1,0,1), x_3 = (0,-1,1)$ to convert to an orthogonal basis.	04
	b	The electric current and voltage in electronic devices is given by the symmetric matrix $A = \begin{bmatrix} 4 & -5 & -2 \\ -5 & 4 & 2 \\ -2 & 2 & 2 \end{bmatrix}$. Factorize the given matric in the form PDP^{-1}.	08
	c	Obtain the singular value decomposition of the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$.	08

UNIT-3

5	a	The probability mass function of a random variable X is defined as $P(X=0) = 2c^2, P(X=1) = 5c - 9c^2, P(X=2) = 4c - 1$ where $c > 0$. Find the value of c and also the mean and variance of X.	06
	b	The following table represents a frequency distribution. Calculate the first four simple moments (i.e., about $a = 0$) and hence the first four central moments. x: 1, 2, 3, 4, 5, 6, 7, 8 f: 8, 3, 14, 15, 21, 20, 14, 5	08
	c	For a random variable X whose pdf is given by $f(x) = \frac{1}{6} e^{-\frac{\lvert x \rvert}{3}}$, $(-\infty, \infty)$, Find i. The characteristic function $\phi(\omega)$ ii. A general expression for the k^{th} moment of X.	06
		OR	
6	a	The diameter of an electric cable X is a continuous random variable with pdf $f(x) = kx(4-x), 0 \le x \le 4$. Find i. The value of k ii. The probability that the diameter of the electric cable is • Anywhere between 1 and 3 $units$ • Less than 2 $units$.	06
	b	If the density function of a random variable X is given by $f(x) = \frac{3}{10} x(2x-1), 0 \le x \le 2$, find the first four central moments.	08
	c	If X is a discrete random variable defined by $p(x) = \frac{1}{3}$ for $x = 0,1,2$, find the characteristic function $\phi(\omega)$ and hence the mean and variance (using the characteristic function).	06

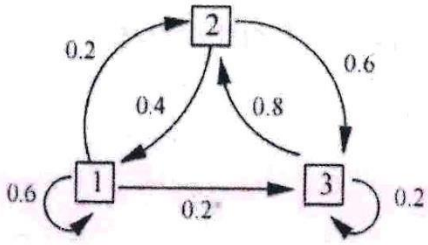
UNIT-4

7	a	The probability that an applicant for a driver's license will pass the road test on any given trial is 0.75. Using Binomial distribution, find the probability that he will finally pass the test in less than 3 trials out of 10 such trials.	04
	b	The local authorities in a certain city install 1000 solar lamps in the streets of the city. If these lamps have an average life of 1500 $burning\ hours$ with a standard deviation of 150 $hours$, how many lamps might be expected to	

		i. Fail in the first 1000 $burning\ hours$? ii. Burn for 50 more hours provided it has already burned for 1200 $hours$?	06
	c	A joint probability mass function of (X, Y) is given by $p(x, y) = c(x + 2y), 1 \leq x \leq 3,\ 1 \leq y \leq 3$. Find i. The value of c ii. The marginal distributions of $X \& Y$ iii. Covariance of $X \& Y$ iv. Are $X \& Y$ independent?	10
		OR	
8	a	In an industrial complex, the average number of fatal accidents per month is 1/2. The number of accidents per month is adequately described by a Poisson distribution. What is the probability that 4 months will pass without a fatal accident?	04
	b	The length of shower on a tropical island during rainy season has an exponential distribution with parameter mean $= 2/3$, time being measured in minutes. What is the probability that a shower will last for less than 2 $minutes$ on i. Atleast 3 of the next 7 $days$ ii. Atmost 5 of the next 7 $days$?	06
	c	If the joint pdf of (X, Y) is $f(x, y) = kxy,\ 0 \leq y \leq 4,\ 0 \leq x \leq y$, find i. The value of k ii. The marginal density functions of $X \& Y$ iii. $P(y \geq 2x)$ iv. $P(x + y \leq 4)$	10

UNIT-5

9	a	A random process is defined by $X(t) = T + (1 - t)$ where T is uniformly distributed on (0,1). Find i. Mean of the process $E[X(t)]$ ii. Autocorrelation of the process $R(t_1, t_2)$ iii. Auto-covariance of the process $C(t_1, t_2)$.	10
	b	The transition probability matrix of a Markov chain $\{X_n\}$ having 3 states 1,2,3 is $P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$. Compute the unique fixed probability vector of the stochastic matrix. Also show that P is regular. Hence find $P\{X_2 = 3\ \ /X_0 = 2\}$ and $P\{X_2 = 2\ \ /X_0 = 1\}$	10
		OR	
10	a	A random process is defined by $X(t) = A\cos 2\pi t$ where A is uniformly distributed on $(0, 2\pi)$. Find i. Mean of the process $E[X(t)]$ ii. Autocorrelation of the process $R(t_1, t_2)$ iii. Auto-covariance of the process $C(t_1, t_2)$. iv. Is the process $X(t)$ stationary?	10

b	A flea zooms around the vertices of the transition diagram as shown in Fig 10b. Find i. The transition matrix P ii. $P\{X_2 = 3 \;\; / X_0 = 1\}$ iii. Suppose that the flea is equally likely to start at any vertex at time 0, find the probability distribution of X_1. iv. Suppose that the flea begins at vertex 1 at time 0, find the probability distribution of X_2.  Fig 10b	10