22MAT11CT

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RV COLLEGE OF ENGINEERING
(Autonomous Institution Affiliated to VTU)
DEPARTMENT OF MATHEMATICS
I Semester MTech. June - 2023 Examinations
MODEL QUESTION PAPER
LINEAR ALGEBRA AND PROBABILITY THEORY
Common to MDC, MIT, MSE
(2022 SCHEME)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit (1 to 5).

		UNIT I	
1	a	Give the row and column analysis for the system of equations $x+y=2$ and $2x-y=1$. Draw neat diagrams. What is the solution?	4
	b	Show that the set $V=\{\,a+b\sqrt{2}+c\sqrt{3}\ /\ a,b,c\in\mathbb{Q}\,\}$, over the field $\mathbb{Q}$ is a vector space under usual addition and scalar multiplication.	6
	c	Find the bases for the four fundamental subspaces of $A=\begin{bmatrix}1&2&0&1\\0&1&1&0\\1&2&0&1\end{bmatrix}$	10
		OR	
2	a	Let V be the vector space of function $f:R\to R$. Show that W is a subspace of V where $W=\{f(x):f(1)=0$, all function whose value at 1 is 0$\}$.	4
	b	Obtain the basis and dimension of the subspace spanned by the subset $S=\left\{\begin{pmatrix}2&-6\\1&-3\end{pmatrix},\begin{pmatrix}3&1\\1&4\end{pmatrix},\begin{pmatrix}1&-2\\3&0\end{pmatrix},\begin{pmatrix}2&5\\3&7\end{pmatrix}\right\}$ of the vector space of all 2×2 matrices over $\mathbb{R}$ of real numbers.	6
	c	Let $G:R^3\to R^3$ be given by $G(x,y,z)=(x+2y-z,y+z,x+y-2z)$. Find a basis and dimension of i) Range of G ii) Kernel of G. Verify Rank and Nullity theorem.	10

		UNIT - II	
3	a	Compute the orthogonal projection of $y=\begin{pmatrix}5\\-9\\5\end{pmatrix}$ onto span of $\{u_1,u_2\}$ where $u_1=\begin{pmatrix}-3\\-5\\1\end{pmatrix},u_2=\begin{pmatrix}-3\\2\\1\end{pmatrix}$. Also find the distance from y to the plane in $\mathbb{R}^3$ spanned by u_1 and u_2.	4

	b	Obtain the line of best fit for the equation $y = \beta_0 + \beta_1 x$ by least-squares for the given data points $(-2, 3), (-1,5), (0,5), (1,4), (2,3)$.	6
	c	Orthonormalize the vectors $a = \begin{bmatrix}1\\0\\1\end{bmatrix}, b = \begin{bmatrix}1\\0\\0\end{bmatrix}, c = \begin{bmatrix}2\\1\\0\end{bmatrix}$ using Gram-Schmidt procedure and hence give the QR factorization of the matrix $A = \begin{bmatrix}1 & 1 & 2\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$.	10
		OR	
4	a	The inner product on $\mathbb{P}_n$ for p and q is given by $\langle p, q\rangle = p(t_0)q(t_0) + p(t_1)q(t_1) + \_\_\_\_\_\_ + p(t_n)q(t_n)$. Compute $\langle p, q\rangle$, $\|p\|$, $\|q\|$ at -1, 0 and 1, given $p(t) = 4 + t,\ q(t) = 5 - 4t^2$.	4
	b	Find the third-order Fourier approximation to the square wave function, $f(t) = \begin{cases} 1, & 0 \le t < \pi \\ -1, & \pi \le t < 2\pi \end{cases}$.	6
	c	Compute the least square solution of the given inconsistent system of equations $Ax = b$ by i) Constructing the normal equations for $\hat{x}$. ii) Solving for $\hat{x}$ Given $A = \begin{bmatrix}1 & 2\\2 & -3\\1 & 3\end{bmatrix}$ and $b = \begin{bmatrix}4\\1\\2\end{bmatrix}$.	10

		UNIT - III	
5	a	Orthogonally diagonalize the matrix $A = \begin{bmatrix}3 & -2 & 4\\-2 & 6 & 2\\4 & 2 & 3\end{bmatrix}$.	10
	b	The matrix of observation of a certain process is given by $A = \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\end{bmatrix}$. Obtain the Singular value decomposition.	10
		OR	
6	a	Classify the quadratic forms $Q(x) = 3x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$. Hence find the maximum value subject to the constraint $x^T x = 1$, a unit vector u where this maximum is attained.	10
	b	Convert the matrix of observations $\begin{bmatrix}19 & 22 & 6 & 3 & 2 & 20\\12 & 6 & 9 & 15 & 13 & 5\end{bmatrix}$ to mean-deviation form and construct the sample covariance matrix and hence find the principal components of the data.	10

UNIT - IV

7	a	The joint distribution of two random variables X and Y is given by the following table where X denotes length and Y denotes width of CD covers	10

X \ Y	1	3	5
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine

(i) $E(X+Y)$

(ii) $E(X^2), E(Y^2)$

(iii) $Cov(X,Y)$, What is the relationship between X and Y.

(iv) $P(X/Y=5)$

(v) $P(X>1, Y>1)$

	b	Let $X \sim N(\mu, \Sigma)$, $\mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following i) $P[X_1 < 7]$ ii) $P[-3X_1 + 3X_3 > 80]$ iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$	10
		OR	
8	a	The joint density function of two continuous random variables X and Y is $f(x,y) = \begin{cases} c\,x\,y, & 0 < x < 4, \quad 1 < y < 4 \\ 0, & otherwise. \end{cases}$ Find (i) The value of c (ii) $E(2X+3Y)$ (iii) $P(1 < x < 2, 2 < y < 3)$ (iv) $P(x \geq 3,\ y \leq 2)$	10
	b	The joint probability distribution of two random variables X and Y is given by the following table	10

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

i) Determine the marginal PMF's of the random variable X and Y.

ii) Compute the covariance matrix and correlation matrix for

		the above data.	

Unit - V			
9	a	Show that the mean and variance of a random process $\{X(t)\}$ given by the probability law $\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ is identical.	10
	b	Compute the unique fixed probability vector of the stochastic matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$. Also show that A is regular.	10
		OR	
10	a	A man either goes for a walk or does yoga each day. He never goes for walking on 2 successive days, but if he does yoga, then the next day he is just likely to do yoga again as to go for a walk. (i) Write the transition matrix of the Markov chain (ii) Verify whether it is irreducible? (iii) If he goes for a walk on Tuesday what is the probability that he does yoga on immediate Saturday (iv) Find the stationary distribution of the Markov process.	10
	b	Show that the random process $x(t) = A\cos(w_0 t + \theta)$ is wide sense stationary if it is assumed that A and w_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.	10

Signature of Scrutinizer: Signature of Chairman

Name: Name: