

ANOVA-Problems- One Way

2. Set up an analysis of variance table for the following per acre production data for three Varieties of wheat, each grown on four plots. Consider variety differences to be significant

	Per Area Production data variety of Wheat		
Plot of Land	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

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Solutions

A	B	C		A ²	B ²	C ²
6	5	5		36	25	25
7	5	4		49	25	16
3	3	3		9	9	9
8	7	4		64	49	16
$\sum A = 24$	$\sum B = 20$	$\sum C = 16$		$\sum A^2 = 24$	$\sum B^2 = 20$	$\sum C^2 = 16$
$\sum X = \sum A + \sum B + \sum C = 60$				$\sum X^2 = \sum A^2 + \sum B^2 + \sum C^2 = 332$		

Step 5

DoF for sum of squares = $N-1 = 12-1 = 11$

DoF for sum of squares between SSC = $K-1 = 3-1 = 2$

DoF for squares with the groups = $N-K = 9$

Step 1: Null Hypothesis (H₀): let us take the hypothesis that there is no significant difference in production of three varieties

Step 2: Correction Factor (CF)

$$CF = (\sum X)^2 / N = 60^2 / 12 = 300$$

Step 3: Sum of Squares Total = $\sum X^2 - CF = 332 - 300$
SST = 30

Step 4: Sum of squares between SSB

$$SSB = (\sum A)^2 / 4 + (\sum B)^2 / 4 + (\sum C)^2 / 4 - CF = 8$$

Step 5: SSW = SST - SSB = 30 - 8 = 24

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Solutions

Sum of variance	Sum of Squares	DoF	Mean of Squares	F Ratio	F critical for 5% level
Between Sample	SSB = 8	K-1 = 2	$8/2 = 4$	$4 / 2.67 = 1.5$	F(2,9) =19.45 Ho accepted
With in sample	SSW = 24	N-K = 9	$24/9 = 2.67$		
Total	32	N-1 =11			

ANOVA-Problems- One Way

2. Is there a statistically significant difference in the mean weight loss among the four diets? We will run the ANOVA using the five-step approach

Per Area Production data variety of Wheat			
Low Calorie(A)	Low Fat(B)	Low carbohydrate(C)	Control (D)
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3

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Solutions

A	B	C	D		A ²	B ²	C ²	D ²
8	2	3	2		64	4	9	4
9	4	5	2		81	16	25	4
6	3	4	-1		36	9	16	1
7	5	2	0		49	25	4	0
3	1	3	3		9	1	9	9
$\sum A$ =33	$\sum B$ =15	$\sum C$ =17	$\sum D$ =6		$\sum A^2$ =239	$\sum B^2$ =55	$\sum C^2$ =63	$\sum D^2$ =18
$\sum A + \sum B + \sum C + \sum D = 71$					$\sum X^2 = \sum A^2 + \sum B^2 + \sum C^2 + \sum D^2 = 375$			

Step 5

DoF for sum of squares = $N-1 = 20-1 = 19$

DoF for sum of squares between SSB = $K-1 = 4-1 = 3$

DoF for squares with the groups = $N-K = 20-4 = 16$

Step 1: Set up hypotheses and determine level of significance. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ H_1 : Means are not all equal $\alpha=0.05$

Step 2: Correction Factor (CF)

$$CF = (\sum X)^2 / N = 71^2 / 20 = 252.5$$

Step 3: Sum of Squares Total = $\sum X^2 - CF = 375 - 252.5$ SST = 119.95

Step 4: Sum of squares between SSB

$$SSB = (\sum A)^2 / 5 + (\sum B)^2 / 5 + (\sum C)^2 / 5 + (\sum D)^2 / 5 - CV = 75.75$$

Step 5: Sum of squares with the groups

$$SSW = SST - SSB = 119.95 - 75.75 = 44.2$$

ANOVA-Problems- One Way

Solutions

Sum of variance	Sum of Squares	DoF	Mean of Squares	F Ratio	F critical for 5% level
Between Sample	SSB = 75.75	K-1 = 3	$75.75/3 = 25.25$	$25.25 / 2.95 = 8.56$	$F(3,16) = 3.25$ H ₀ Rejected
With in sample	SSW = 47.2	N-K = 16	$47.2/16 = 2.95$		
Total	122.95	N-1 =19			

We reject H₀ because $8.43 \geq 3.24$. We have statistically significant evidence at $\alpha=0.05$ to show that there is a difference in mean weight loss among the four diets.



ANOVA-Problems- One Way

3. Is there a statistically significant difference in mean calcium intake in patients with normal bone density as compared to patients with osteopenia and osteoporosis? We will run the ANOVA using the five-step approach.

Normal Bone Density (A)	Osteopenia (B)	Osteoporosis (C)
1200	1000	890
1000	1100	650
980	700	1100
900	700	1100
780	500	400
800	700	350

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Solutions

A	B	C		A ²	B ²	C ²
1200	1000	890		1440000	1000000	792100
1000	1100	650		1000000		
				0	1210000	422500
980	700	1100		960400	490000	1210000
900	700	1100		810000	490000	1210000
780	500	400		608400	250000	160000
800	700	350		640000	490000	122500
5660	4700	4490		5458800	3930000	3917100
$\sum X = \sum A + \sum B + \sum C = 14850$				$\sum X^2 = \sum A^2 + \sum B^2 + \sum C^2 = 13305900$		

Step 5

DoF for sum of squares = $N-1 = 18-1 = 17$

DoF for sum of squares between SSC = $K-1 = 6-1 = 5$

DoF for squares with the groups = $N-K = 13$

Step 1: Set up hypotheses and determine level of significance $H_0: \mu_1 = \mu_2 = \mu_3$ H_1 : Means are not all equal
 $\alpha=0.05$

Step 2: Correction Factor (CF)
 $CF = (\sum X)^2 / N = 12251250$

Step 3: Sum of Squares Total $SST = \sum X^2 - CF = 1054650$

Step 4: Sum of squares between SSB
 $SSB = (\sum A)^2 / 4 + (\sum B)^2 / 4 + (\sum C)^2 / 4 - CV = 129700$

Step 5: $SSW = SST - SSB = 924950$

ANOVA-Problems- One Way

Solutions

Sum of variance	Sum of Squares	DoF	Mean of Squares	F Ratio	F critical for 5% level
Between Sample	SSB = 129700	K-1 = 5	$129700/5 = 25940$	$4 / 2.67 = 1.5$	$F(5,13) = 3.68$ H ₀ accepted
With in sample	SSW = 924950	N-K = 13	$924950/13 = 71150$	$71150/25940 = 2.74$	
Total	1054650	N-1 = 17			

we do not reject H₀ because $1.395 < 3.68$. We do not have statistically significant evidence at $\alpha = 0.05$ to show that there is a difference in mean calcium intake in patients with normal bone density as compared to osteopenia and osteoporosis.

ANOVA-Problems- Two Way

1. A farmer applied three types of fertilizers on Four separate plots. Two Figure on yield per acre are tabulated bellow

	Yield			
Plots Fertilizers	A	B	C	D
Nitrogen	6	4	8	6
Potash	7	6	6	9
Phosphates	8	5	10	9

Find out if the plots are materially different in fertility as also, if three fertilizers any material difference in yield

ANOVA-Problems- Two Way

Step 1: Null hypothesis: Let us take the hypothesis that

- All plots are not significantly differ in fertility (Column wise Analysis)
- All the fertilizers are not significantly differ in the yields (Row wise analysis)

Step 2:

A	B	C	D	Total	A ²	B ²	C ²	D ²
6	4	8	6	24	36	16	64	36
7	6	6	9	28	49	36	36	81
8	5	10	9	32	64	25	100	81
21	15	24	24	T=84	149	77	200	198

Step 3: Correlation Factor (CF) = $(\sum x)^2/N = 88^2/12 = 588$

Step 4: Sum of Square Total (SST) = $\sum x^2 - C = 149 + 77 + 200 + 198 - 588 = 36$

Step 5: Sum of square Between Columns = SSC = $(\sum A)^2/n + (\sum B)^2/n + (\sum C)^2/n + (\sum D)^2/n - C = 21^2/3 + 15^2/3 + 24^2/3 + 24^2/4 - 588 = 18$



ANOVA-Problems- Two Way

Step 6: Sum of square between Rows = SSR = $24^2/4 + 28^2/4 + 32^2/4 - 588 = 8$

Step 7: Sum of square with the Group = SSW = SST – SSC – SSR = $36 - 18 - 8 = 10$

Step 8:

DoF for total sum of square = $N-1 = 12-1 = 11$

DoF for sum of squares between the column = $C - 1 = 4-1 = 3$

Dof for sum of squares between the Rows = $R-1 = 3-1 = 2$

DoF for sum of square with the Group = $(C-1)(R-1) = 6$

Step 9: Anova construction

Source of variance	Sum of Squares	Degree of Freedom	Mean of sum of squares	F Ratio	Table value
Between Column	SSC = 18	$C-1 = 4-1 = 3$	$18/3 = 6$	$6/1.667 = 3.6$	$F(3,6) = 8.94$ Ho accepted
Between Rows	SSR = 8	$R-1 = 3-1 = 2$	$8/2 = 4$	$4/1.667 = 2.4$	$F(2,6) = 19.33$ H) Accepted
With the Group	SSW = 10	$(R-1)(C-1) = 6$	$10/6 = 1.667$		
Total	SST = 36	$N-1 = 11$			



ANOVA-Problems- Two Way

Step 10: Interpretation

Columns wise analysis: The computed values of $F = 3.6$ is less than the Table value 8.94, hence the null hypothesis is accepted, it means the plots are not significantly different in fertility

Row wise Analysis: The calculated value of $F = 2.4$ is less than the Table Value 19.33, hence the null hypothesis is accepted, it means the fertilizers are alike so far as productivity concern

ANOVA-Problems- Two Way

2. A Manger applied three five type of operator and four type of machines. Two Figure on production are tabulated bellow

Operataor (j)	Machne (i)			
	1	2	3	4
1	46	56	35	47
2	54	55	51	56
3	48	56	50	58
4	46	60	51	59
5	51	53	53	55

Find out if the operator are materially different in Machine as also, if four machines any material difference in yield

ANOVA-Problems- Two Way

Step 1: Null hypothesis: Let us take the hypothesis that

- All operator are not significantly differ in productivity (Column wise Analysis)
- All the Machines are not significantly differ in the productivity (Row wise analysis)

Step 2:

	0	1	2	3	4	R-sum						
1	46	56	35	47	184		2116	3136	1225	2209		
2	54	55	51	56	216		2916	3025	2601	3136		
3	48	56	50	58	212		2304	3136	2500	3364		
4	46	60	51	59	216		2116	3600	2601	3481		
5	51	53	53	55	212		2601	2809	2809	3025		
C-Sum	245	280	240	275	T=1040		12053	15706	11736	15215	$\sum X^2 =$ 54710	

Step 3: Correlation Factor (CF) = $(\sum x)^2/N = 1040^2/12 = 54080$

Step 4: Sum of Square Total (SST) = $\sum x^2 - C = 57710 - 54080 = 3630$

Step 5: Sum of square Between Columns = SSC = $(\sum 1)^2/n + (\sum 2)^2/n + (\sum 3)^2/n + (\sum 4)^2/n - C =$
250



ANOVA-Problems- Two Way

Step 6: Sum of square between Rows = SSR = $54264 - 54080 = 184$

Step 7: Sum of square with the Group = SSW = $SST - SSC - SSR = 196$

Step 8:

DoF for total sum of square = $N-1 = 20-1 = 19$

DoF for sum of squares between the column = $4 - 1 = 4 - 1 = 3$

DoF for sum of squares between the Rows = $R-1 = 5-1 = 4$

DoF for sum of square with the Group = $(C-1)(R-1) = 12$

Step 9: Anova construction

Source of variance	Sum of Squares	Degree of Freedom	Mean of sum of squares	F Ratio	Table value
Between Column	SSC = 250	$C-1 = 4-1 = 3$	$250/3 = 83.33$	$83.33/16.33 = 5.1$	$F(3,12) = 3.49$ No rejected
Between Rows	SSR = 184	$R-1 = 5-1 = 4$	$184/4 = 46$	$46/16.33 = 2.81$	$F(4,12) = 3.26$ H) Accepted
With the Group	SSW = 196	$(R-1)(C-1) = 12$	$196/12 = 16.33$		
Total	SST = 36	$N-1 = 19$			

ANOVA-Problems- Two Way

Step 10: Interpretation

From the F-tables $F(4,12) = 3.26$ and $F(3,12) = 3.49$.

- Since $6.81 > 3.49$ we conclude that we do have sufficient evidence at the 5% level of significance to reject the null hypothesis that there is difference between the machines.
- Since $2.81 < 3.26$ we conclude that we do not have sufficient evidence to reject the null hypothesis that there is no difference between the operators.