RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

Date	18^{th} March 2024	Time	9:30 a.m. – 11:30 a.m.
Quiz & Test	I	Maximum Marks	10 + 50
Course Title	Linear Algebra, Probability and Queuing Theory	Course Code	MMA202T
Semester	I	Programs	MCE, MCN

Instructions: Answer all questions.

Sl. No.	Quiz	M	CO	BT
1	Find the value of k such that the vectors $2t^2+t+2, t^2-2t, kt^2-t+2$ are linearly dependent.	02	2	2
2	If $\mathbb{R}^+$, the set of all positive real numbers is a vector space over the field $\mathbb{R}$, defined under (i) $\alpha+\beta=\alpha\beta$ and (ii) $c\cdot\alpha=\alpha^c$, then the zero vector is ___ and the inverse vector of α is ___.	02	1	1
3	Show that the vector $(2,3)$ belongs to the column space of $A=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$.	02	2	2
4	Find the orthogonal projection of y onto u and the vector z orthogonal to u, where $y=(4,2)$ and $u=(1,1)$.	02	4	4
5	Suppose $Ax=b$ is inconsistent and $A^TA=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$ and $A^Tb=\begin{bmatrix}2\\ 2\end{bmatrix}$, then the least-squares solution is ___.	02	3	3

Sl. No.	Test	M	CO	BT
1a	Show that the set of vectors $V=\{a+b\sqrt{2}\,\vert\,a,b\in\mathbb{R}\}$ is a vector space over the field $\mathbb{R}$.	06	2	2
1b	Show that the subset $\mathbb{P}_2$, the set of all polynomials of degree at most 2, is a subspace of the vector space $\mathbb{P}_n$, the set of all polynomials of degree at most n.	04	1	1
2a	Show that the matrices $\begin{bmatrix}2 & 1\\ 2 & 4\end{bmatrix}, \begin{bmatrix}1 & -2\\ 0 & 2\end{bmatrix}, \begin{bmatrix}5 & -5\\ 2 & 10\end{bmatrix}, \begin{bmatrix}1 & 3\\ 2 & 2\end{bmatrix}$ are linearly dependent in $M_{2\times2}$. Extract a linearly independent subset. Also find the basis and dimension of the subspace spanned by them.	06	2	2
2b	Fit the line of best fit for the data points $(1,2),(2,3),(3,4),(4,3)$, by least-squares method.	04	4	4
3	Find the bases and dimension of the four fundamental subspace of the matrix $A=\begin{bmatrix}1 & 1 & 1 & 1\\ 3 & 1 & 1 & 3\\ 5 & 0 & 2 & 3\end{bmatrix}$.	10	3	3
4a	Show that the mapping $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(x+3y, 2x-2y)$ is a linear transformation.	04	1	1
4b	Find the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(1,1,1)=(2,2,0), T(1,2,1)=(4,3,1), T(2,1,0)=(4,1,3)$. Also determine the bases of the range space and null space.	06	4	4
5	Obtain the QR factorization of the matrix $A=\begin{bmatrix}-1 & 6 & 6\\ 3 & -8 & 3\\ 1 & -2 & 6\\ 1 & -4 & -3\end{bmatrix}$.	10	3	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Quiz Max Marks	2	4	2	2	2	4	2	2	-	-
	Test Max Marks	8	12	20	10	-	8	12	20	10	-

RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

Date	**29th April 2024**	**Time**	**9:30 a.m. – 11:30 a.m.**
Quiz & Test	**II**	**Maximum Marks**	**10 + 50**
Course Title	**Linear Algebra, Probability and Queuing Theory**	**Course Code**	**MMA202T**
Semester	**I**	**Programs**	**MCE, MCN**

Instructions: Answer all questions.

Sl. No.	**Quiz**	M	CO	BT
1	If the inner product of the vectors u and v is defined as $< u, v > = 2u_1v_1 + 3u_2v_2$, then the distance between $u = (3,2)$ and $v = (2,1)$ is ____.	02	2	2
2	The maximum value of the quadratic form $Q(x) = 10x_1x_2$ subject to the constraint $x^Tx = 1$ is ____.	02	2	2
3	If the sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} a & 2 \\ 0 & b \end{bmatrix}$ are 7 and 12 respectively, then the matrix A = ____.	02	1	1
4	The singular values of the matrix $A = [1 \quad -2 \quad 2]$ are ____.	02	4	4
5	Suppose the covariance matrix S has the eigenvector $[1 \quad 2]^T$ corresponding to the largest eigenvalue 84.36. Then a new variable y_1, such that y_1 has maximum possible variance over a given data is ____.	02	3	3
Sl. No.	**Test**	**M**	**CO**	**BT**
1a	Let V be $\mathbb{P}_3$ with the inner product defined by $< p, q > = p(t_0)q(t_0) + p(t_1)q(t_1) + p(t_2)q(t_2) + p(t_3)q(t_3)$, where $t_0 = -2, t_1 = -1, t_2 = 1, t_3 = 2$ and $\mathbb{P}_2$ be a subspace of V. Produce an orthogonal basis for $\mathbb{P}_2$ by applying Gram-Schmidt process to the polynomials $1, t, t^2$.	06	2	2
1b	Find the nth order Fourier approximation to the function $f(t) = 2$ on the interval $[0, 2\pi]$.	04	1	1
2a	Suppose the quadratic form is given by $Q(x) = 5x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$. Find (i) the maximum value of $Q(x)$ subject to the constraint $x^Tx = 1$, (ii) a unit vector u where this maximum is attained, (iii) the maximum of $Q(x)$ subject to the constrains $x^Tx = 1$ and $x^Tu = 0$.	06	2	2
2b	Make a change of variable, $x = Py$, that transforms the quadratic form $6x_1^2 + 4x_1x_2 + 3x_2^2$, into a quadratic form with no cross-product term. Give P and the new quadratic form.	04	1	1
3	Using the process of diagonalization decompose the matrix A as PDP^{-1}, where $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix}$.	10	3	3
4	Decompose the matrix A as $U\Sigma V^T$, using the singular value decomposition process, where $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	10	3	3
5	Given the matrix of observations as: $\begin{bmatrix} 20 & 16 & 14 & 18 & 15 & 19 \\ 9 & 7 & 8 & 6 & 5 & 7 \end{bmatrix}$, convert the matrix to mean deviation form, construct the covariance matrix and hence find its principal components. Also determine what percentage of the information is retrieved from the first principal component.	10	4	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Quiz Max Marks	2	4	2	2	2	4	2	2	-	-
	Test Max Marks	8	12	20	10	8	12	20	10	-	-

Linear Algebra, Probability and Queuing Theory (MMA202T)

CIE 1.

Quiz

1. $\langle u, v\rangle = 2u_1v_1 + 3u_2v_2$, $u = (3,2)$, $v = (2,1)$

   $u - v = (1,1)$ 1 $\|u-v\| = \sqrt{2\times1\times1 + 3\times1\times1} = \sqrt{5}$ 1

2. $Q(x) = 10x_1x_2$ $A = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} \Rightarrow \lambda^2 - 0\lambda + (-25) = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$ 1

   $\therefore$ max val' is 5 1

3. $A = \begin{bmatrix} a & 2 \\ 0 & b \end{bmatrix}$ $\left.\begin{matrix} a+b = 7 \\ ab = 12 \end{matrix}\right\} \Rightarrow a = 3, b = 4$ 1 $\therefore A = \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix}$ 1

4. $A = [1\ \ -2\ \ 2]$ $AA^T = [9]$ 1 $\therefore$ singular value is $\sqrt{9} = 3$ 1

5. $\lambda = 84.36$ $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \sim \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix} = \begin{bmatrix} 0.4472 \\ 0.8944 \end{bmatrix}$ 1 $y_1 = 0.45x_1 + 0.89x_2$ 1

Test

1a. $\langle p, q\rangle = p(t_0)q(t_0) + p(t_1)q(t_1) + p(t_2)q(t_2) + p(t_3)q(t_3)$, $t_0 = -2, t_1 = -1, t_2 = 1, t_3 = 2$ 1

$p_0(t) = 1$, $p_1(t) = t$, $p_2(t) = t^2$ $P_0(t) = 1$ 1; $P_1(t) = t - \frac{\langle t, P_0\rangle}{\langle P_0, P_0\rangle} P_0(t)$

$P_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $P_1 = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, $P_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 4 \end{bmatrix}$ 1 $P_1(t) = t - \frac{0}{4}\times 1 = t$ 1

$P_2(t) = t^2 - \frac{\langle t^2, P_0\rangle}{\langle P_0, P_0\rangle}\times 1 - \frac{\langle t^2, P_1\rangle}{\langle P_1, P_1\rangle}\times t = t^2 - \frac{10}{4}\times 1 - \frac{0}{14}\times t = t^2 - \frac{5}{2}$ 2

1b. $f(t) = 2$, $[0, 2\pi]$ $\frac{a_0}{2} = \frac{1}{2\pi}\int_0^{2\pi} 2\,dt = \frac{1}{2\pi}\left[2t\right]_0^{2\pi} = 2$ 1

$a_n = \frac{1}{2\pi}\int_0^{2\pi} 2\cos kt\,dt = \frac{1}{\pi}\left[\frac{\sin kt}{k}\right]_0^{2\pi} = \frac{1}{\pi}\times(0 - 0) = 0$ 1

$b_n = \frac{1}{2\pi}\int_0^{2\pi} 2\sin kt\,dt = \frac{1}{\pi}\left[\frac{\cos kt}{k}\right]_0^{2\pi} = \frac{1}{k\pi}\left[-\cos 2\pi + \cos 0\right] = 0$ 1

$\therefore f(t) = 2$ 1

2a. $Q(x) = 5x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$.

(i) max val' is 7 s.t $x^Tx = 1$. 1

$A = \begin{bmatrix} 5 & 1 & -1 \\ 1 & 4 & -2 \\ -1 & -2 & 4 \end{bmatrix} \Rightarrow \lambda^3 - 13\lambda^2 + 50\lambda - 56 = 0 \Rightarrow \lambda = 7, 4, 2$ 2

(ii) $A - 7I = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -3 & -2 \\ -1 & -2 & -3 \end{bmatrix} \Rightarrow u = \begin{bmatrix} 5 \\ 5 \\ -5 \end{bmatrix} \sim \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$ is unitvector 2

(iii) max val' is 4 s.t $x^Tx = 1$ & $x^Tu = 0$. 1

2b. $Q(x) = 6x_1^2 + 4x_1x_2 + 3x_2^2$ $A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \Rightarrow \lambda^2 - 9\lambda + 14 = 0 \Rightarrow \lambda = 7, 2$ 1

$\lambda = 7$: $A - 7I = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -4 \\ -2 \end{bmatrix} \sim \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\lambda = 2$: $A - 2I = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\therefore P = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & -2/\sqrt{5} \end{bmatrix}$ 2

$Q(y) = 7y_1^2 + 2y_2^2$ 1

3. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix} \Rightarrow \lambda^3 - 8\lambda^2 + 21\lambda - 18 = 0$ (1)

$\Rightarrow \lambda = 2, 3, 3$ (2)

$\lambda = 2$: $A - 2I = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ (1), $\lambda = 3$: $A - 3I \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \Rightarrow \begin{matrix} x_1 + x_2 + x_3 = 0 \\ x_1 = -x_2 - x_3 \end{matrix} \quad x = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ (1) (1)

$\therefore P = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (1), $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (1), $P^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ (2)

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4. $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$

$AA^T = \begin{bmatrix} 5 & 2 \\ 2 & 8 \end{bmatrix}$ (1) $\qquad A^TA = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$ (1)

$\Rightarrow \lambda = 9, 4$

$\lambda^2 - 13\lambda + 36 = 0$ (1)

$\lambda = 9, 4$

$\lambda = 9$: $A^TA - 9I = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -4 \\ -2 \end{bmatrix} \sim \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (1)

$\lambda = 9$: $AA^T - 9I = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}$

$\lambda = 4$: $A^TA - 4I = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (1)

$\Rightarrow x = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (1)

$V = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & -2/\sqrt{5} \end{bmatrix}$ (1)

$\lambda = 4$: $AA^T - 4I = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow x = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \sim \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (1) $\quad U = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$ (1) $\quad \Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (1)

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5. $X = \begin{bmatrix} 20 & 16 & 14 & 18 & 15 & 19 \\ 9 & 7 & 8 & 6 & 5 & 7 \end{bmatrix} \quad M = \begin{bmatrix} 17 \\ 7 \end{bmatrix}$ (2) $\quad B = \begin{bmatrix} 3 & -1 & -3 & 1 & -2 & 2 \\ 2 & 0 & 1 & -1 & -2 & 0 \end{bmatrix}$ (2)

$S = \frac{1}{5}\begin{bmatrix} 28 & 6 \\ 6 & 10 \end{bmatrix} = \begin{bmatrix} 5.6 & 1.2 \\ 1.2 & 2 \end{bmatrix}$ (2) $\Rightarrow \lambda^2 - 7.6\lambda + 9.76 = 0$ (1) $\Rightarrow \lambda = \begin{matrix} 5.9633 \\ 1.6367 \end{matrix}$ (1)

$\lambda = 5.9633$

$S - 5.96I = \begin{bmatrix} -0.36 & 1.2 \\ 1.2 & -3.96 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -3.96 \\ -1.2 \end{bmatrix}$ (1) $\sim \begin{bmatrix} 3.96/\sqrt{17.1216} \\ 1.2/\sqrt{17.1216} \end{bmatrix} \sim \begin{bmatrix} 0.9565 \\ 0.2899 \end{bmatrix}$ (1) first principal component

$\frac{5.9633}{7.6} = 78.46\%$ (1) of information is retrieved.

PART - B

1 The joint distribution of two random variables X and Y is given by the following table where X is scaled temperature and Y is difference in pressure (scaled) of a reactor. | 10 | 2 | 2

X \ Y	1	2
0	0.3	0.1
1	0.2	0.1
2	0.1	0.2

Determine (i) Covariance and Correlation matrix of (X, Y)

2 Let $X \sim N(\mu, \Sigma)$, $\mu = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 16 & -2 & 1 \\ -2 & 4 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Compute the following | 10 | 2 | 2

i) $P[X_1 < 7]$

ii) $P[-3X_1 + 3X_3 > 80]$

iii) $P[3X_1 + 4X_2 - 5X_3 < 70]$

3 Measurements in three characteristics are made on two individuals in a random sample from a population. The observation matrix is given as $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$. Using Singular Value Decomposition find the first singular value. | 10 | 4 | 4

4 The joint distribution of two random variables X and Y is given by the following table: | 10 | 3 | 3

X \ Y	0	1
0	0.1	0.2
1	0.4	0.2
2	0.1	0

(a) Find $P(X + Y > 1)$

(b) Determine the individual (marginal) probability distributions of X and Y and verify that X and Y are not independent.

(c) Compute $P(Y|X = 2)$, $P(X|Y = 1)$

5 The following table lists the weights and heights of five boys | 10 | 4 | 4

Boy	1	2	3	4	5
Weight (lb)	120	125	125	135	145
Height (m)	61	60	64	68	72

Compute the sample covariance matrix. Also find the principal component of the data and the percentage of information captured by the first and second principal components.