January-2020 PG Examinations

USN | | | | | | | | | |

18MAT11B

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU, Belagavi)

I Semester Master of Technology

Common to MCS / MCE / MCN / MDC / MRM / MSE / MIT

PROBABILITYTHEORY AND LINEAR ALGEBRA

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit.
3. Statistical table permitted

UNIT-1

1	a	Let W be a subspace of R^4 spanned by the vectors $u_1 = (1, -2,5, -3)$, $u_2 = (2,3,1, -4)$ and $u_3 = (3,8, -3, -5)$. Find a basis and dimension of W.	10
	b	Prove that the set of all polynomial of degree 2 or less (P_2) is vector space.	10
		OR	
2	a	Determine the dimension and basis for the four fundamental subspace for $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.	10
	b	Examine: i) The set of vectors $\{(a, b, c) \mid 2a + 3b + 4c = 0 \; and \; a, b, c \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$. ii) The transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x, y, 0)$ is linear.	10

UNIT-2

3	a	Compute the QR factorization of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	10
	b	Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. Hence evaluate A^4	10
		OR	
4	a	Compute the least squares solution of the inconsistent system $Ax = b$ for $\begin{bmatrix} 2 & -1 & 1 \\ 1 & -5 & 2 \\ -3 & 1 & -4 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 5 \\ -1 \end{bmatrix}$.	10
	b	Construct a singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.	10

UNIT-3

5	a	A box contains 12 items of which 4 are defective. A sample of three items is selected from the box. Let X denotes the number of defective items in the sample. Find the probability distribution of X. Determine the mean and standard deviation of the distribution.	10
	b	A random signal can have any voltage value defined by the following function $f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, elsewhere \end{cases}$ i) Does $f(x)$ form a pdf? ii) Compute CDF.	10
		OR	
6	a	The daily consumption of electric power 'c' in millions of kW-hours is a random variable having $PDF f(x) = \begin{cases} \frac{1}{9}x\left(e^{-\frac{x}{3}}\right) & , \quad x > 0 \\ 0 & , \quad x \leq 0 \end{cases}$ If the total production is 12 million kW-hours, determine: i) The probability that there is power cut (shortage) on given day. ii) Average daily consumption of electric power.	10
	b	For the pdf $f(x) = \frac{1}{2b} e^{\frac{\lvert x\rvert}{b}}$ find: i) Characteristic function $\emptyset_x(w)$ ii) Taylor's series expansion of $\emptyset_x(w)$ iii) A general expression for the k^{th} moment of X.	10

UNIT-4

7	a	Two persons A and B play a game in which their chances of winning are in the ratio $3:2$. If six games are played, then find A's chance of winning at least three games.	06
	b	At a certain city bus stop, three buses arrive per hour, on an average. Assuming that the time between successive arrivals is exponentially distributed, find the probability that the time between the arrival of successive buses is i) less than 10 minutes ii) At least 30 minutes.	07
	c	The joint probability distribution of 2 discrete random variables X&Y is given by the following table: (see below) Determine: i) Marginal distribution of X&Y ii) $P(X \geq 1, Y < 4)$ iii) $P(X + Y > 0)$.	07
		OR	

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

8	a	Derive mean and variance of Poisson distribution.	06
	b	The lifetime of a certain brand of an electric bulb may be considered as a normal random variable with mean 1200 hrs and standard deviation 250 hrs. Find the probability that: i) The lifetime of a bulb is more than 1250 hrs ii) Life time of a bulb is less than 1100 hrs Given $A(0.4) = 0.15542,\ A(0.2) = 0.07926$, where $A(z) = \frac{1}{\sqrt{2\pi}}\int_0^z \left(e^{-\frac{z^2}{2}}\right) dz$	07
	c	Find the constant k so that: $P(x,y) = \begin{cases} k(x+1)e^{-y} & , \quad 0 < x < 1, y > 0 \\ 0 & , \quad elsewhere \end{cases}$ Is a joint probability density function.Are x and y independent.	07

UNIT-5

9	a	If$\{X(t)\} = P + Qt$, where P and Q are independent random varaibles with $E(P) = p, E(Q) = q, Var(P) = \sigma_1^2, Var(Q) = \sigma_2^2$. Find $E\{X(t)\}, R(t_1, t_2)$ and $C(t_1, t_2)$. Examine the process$\{X(t)\}$ is stationary.	10
	b	Compute the unique fixed probability vector of the stochastic matrix $A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$. Also show that A is regular.	10
		OR	
10	a	A random process $\{X(t)\}$is characterized by four sample functions $X(t, S_1) = -1, X(t, S_2) = -2, X(t, S_3) = 3$ and $X(t, S_4) = t$.Assuming that the sample functions are equally likely, find the auto correlation function, examine if $\{X(t)\}$is WSS.	10
	b	A software engineer goes to his work place everyday by a bike or car. He never goes by bike on two consecutive days, but if he goes by car on a day then he is equally likely to go by car or bike on the next day. Find the transition matrix for the chain of the mode of transport he uses, if car is used on first day of a week, find the probability that: i) Bike is used on the fifth day ii) Car is used on the fifth day.	10