22MAT11B

USN

RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
DEPARTMENT OF MATHEMATICS
I Semester M.Tech (MCE, MCN)
June-2023 Examinations
LINEAR ALGEBRA, PROBABILITY AND QUEUEING THEORY
(2022 SCHEME)
MODEL QUESTION PAPER

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit (1 to 5).

UNIT -1

1a	If $f: V_2(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ is defined by $f(x,y,z) = (\cos\theta\, x - \sin\theta\, y, \sin\theta\, x + \cos\theta\, y)$, show that f is a linear transformation.	04
1b	Let $\mathbb{R}$ be the field of real numbers and let $\mathbb{P}_3$be the set of all polynomials of degree at most 3, over the field $\mathbb{R}$. Prove that $\mathbb{P}_3$ is a vector space over the field $\mathbb{R}$.	06
1c	Find the bases and dimension of the four fundamental sub spaces of the matrix $A = \begin{bmatrix} 2 & 5 & 2 & 6 \\ 3 & 4 & 2 & 3 \\ 5 & 9 & 4 & 9 \\ 4 & -1 & 2 & -3 \end{bmatrix}.$	10
	OR	
2a	Let $H = \{(a-2b, b+2a, 3a, 2b)\}$ where a and b are arbitrary scalars. Show that H is a subspace of $\mathbb{R}^4$.	04
2b	Show that the matrices $\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -4 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$ are linearly dependent in $M_{2\times 2}$. Extract a linearly independent subset. Also find the basis and dimension of the subspace spanned by them.	06
2c	Find the Linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$, such that $T(1,1,0) = (-1,4,0,1), T(0,1,1) = (-2,0,3,2), T(1,1,1) = (-1,3,2,2)$. Also find the range space and null space of the Linear transformation.	10

UNIT -2

3a	Suppose $\mathbb{P}_2$is a vector space having the inner product defined by $< p, q > = p(t_0)q(t_0) + p(t_1)q(t_1) + p(t_2)q(t_2)$, where $t_0 = -1, t_1 = 0, t_2 = 1$. Compute the lengths of the vectors $p(t) = 4 + t$ and $q(t) = 5 - 4t^2$.	04
3b	Find a least-squares solution of the inconsistent system $Ax = b$, where $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$.	06
3c	Obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & -1 \\ 0 & 2 & 2 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$.	10
	OR	
4a	Find the n^{th} order Fourier approximation to the function $f(t) = t$ on the interval $[0, 2\pi]$.	04
4b	Obtain an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$.	06
4c	A simple curve that often makes a good model for the variable costs of a company, as a function of the sales level x, has the form $y = \beta_0 x + \beta_1 x^2 + \beta_3 x^3$. There is no constant term because fixed costs are not included. Find the least-squares curve of the form above to fit the data $(1, 1.8), (2, 2.7), (3, 3.4), (4, 3.8), (5, 3.9)$.	10

UNIT -3

5a	(i) Make a change of variable, $x = Py$, that transforms the quadratic form $x_1^2 + 10x_1x_2 + x_2^2$, into a quadratic form with no cross-product term. Give P and the new quadratic form. (ii) Suppose the quadratic form is given by $Q(x) = 5x_1^2 + 6x_2^2 + 7x_3^2 + 4x_1x_2 - 4x_2x_3$. Find the maximum value of $Q(x)$ subject to the constraint $x^T x = 1$, a unit vector u where this maximum is attained, the maximum of $Q(x)$ subject to the constrains $x^T x = 1$ and $x^T u = 0$.	10
5b	Obtain an SVD of the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	10
	OR	
6a	Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 4 & 8 & 4 \end{bmatrix}$.	10
6b	Given the matrix of observations as: $\begin{bmatrix} 12 & 6 & 9 & 15 & 13 & 5 \\ 19 & 22 & 6 & 3 & 2 & 20 \end{bmatrix}$, convert the matrix to mean deviation form, construct the covariance matrix and hence find its principal components. Also determine what percentage of the information is retrieved from the first principal component.	10

UNIT -4

<table>
<tr><td>7a</td><td>The joint probability distribution of two random variables X and Y is defined by the function $P(X,Y)=\frac{1}{27}(2X+Y)$ where X and Y assume the integer values $0,1,2$. Find (i) the joint distribution of X and Y, (ii) the marginal distributions of X and Y, (iii) the expected values of X, Y and XY, (iv) $Cov(X,Y)$ and $\rho(X,Y)$.</td><td>10</td></tr>
<tr><td>7b</td><td>Random variables X and Y have the joint PDF $f_{X,Y}(x,y)=\begin{cases}2, & 0\le y\le x\le 1\\ 0, otherwise\end{cases}$.
(i) For $0\le x\le 1$, find the conditional PDF $f_{Y|X}(y|x)$,
(ii) For $0\le y\le 1$, find the conditional PDF $f_{X|Y}(x|y)$.</td><td>06</td></tr>
<tr><td>7c</td><td>A random variable X has a mean of 4 and variance 2. (i) Using Markov inequality, find $P[X\ge 4]$, (ii) using Chebyshev inequality find $P[|X-4|\ge 2]$.</td><td>04</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>8a</td><td>If the joint probability density function for (x,y) is
$f(x,y)=\begin{cases}c(x^2+y^2), & 0\le x\le 1, 0\le y\le 1, c\ge 0\\ & 0\,, otherwise\end{cases}$,
determine (i) the value of c, (ii) $P\left(x<\frac{1}{2}, y>\frac{1}{2}\right)$, (iii) $P\left(\frac{1}{4}<x<\frac{3}{4}\right)$, (iv) $P\left(y<\frac{1}{2}\right)$.</td><td>10</td></tr>
<tr><td>8b</td><td>Find the covariance matrix for the two random variables X_1 and X_2 whose joint probability is represented as follows:
<table>
<tr><td>x₂
x₁</td><td>0</td><td>1</td></tr>
<tr><td>-1</td><td>0.24</td><td>0.06</td></tr>
<tr><td>0</td><td>0.16</td><td>0.14</td></tr>
<tr><td>1</td><td>0.40</td><td>0.00</td></tr>
</table>
</td><td>06</td></tr>
<tr><td>8c</td><td>Obtain the axes of the constant probability density contours for a bivariate normal distribution with the covariance matrix $\Sigma=\begin{bmatrix}0.5 & 0.25\\ 0.25 & 0.5\end{bmatrix}$.</td><td>04</td></tr>
</table>

UNIT -5

9a	Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes. (i) Find an average number of persons waiting in the system. (ii) What is the probability that a person arriving at the booth will have to wait in the queue? (iii) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call? (iv) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 minutes for phone. By how much the flow of arrivals should increase in order to justify a second booth?	10

9b	There are three typists in an office. Each typist can type an average of 5 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, (i) What fraction of the time all the typists will be busy? (ii) What is the average numbers of letters waiting to be typed? (iii) What is the average time a letter has to spend for waiting and for being typed? (iv) What is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed?	10
	OR	
10a	Customers arrive at a one man barber shop according to a Poisson process with a mean interarrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. (i) What is the expected number of customers in the barber shop in the queue? (ii) What is the probability that a customer will not have to wait for a hair cut? (iii) How much can a customer expect to spend in the barber shop? (iv) Management will put another chair and hire another barber when a customer's average waiting time in the shop exceeds 1.25 hours. How much must the average rate of arrivals increase to warrant a second barber?	10
10b	A petrol pump station has 4 pumps. The service time follow the exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. (i) What is the probability that an arrival would have to wait in line? (ii) Find the average waiting time, average time spent in the system and the average number of cars in the system. (iv) For what percentage of time would a pump be idle on an average?	10