June-2024 PG Examinations

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MMA202T

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU, Belagavi)

I Semester Master of Technology (Common to MCE and MCN)

LINEAR ALGEBRA, PROBABILITY AND QUEUING THEORY

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Each unit consists of two questions of 20 marks each.
2. Answer FIVE full questions selecting one from each unit.

UNIT-1

1	a	Let $\mathbb{R}$ be the field of real numbers and let $V = \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$, which is closed under usual vector addition and scalar multiplication. Prove that V is a vector space over the field $\mathbb{R}$.	06
	b	Show that the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (2x-4y, y-3x)$ is a linear transformation.	04
	c	Find the bases and dimension of the four fundamental sub spaces of the matrix, $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$.	10
		OR	
2	a	Let V be a vector space of real valued derivable functions on $(0,\infty)$. Then show that the set $S = \{x^2+2x, 3x+2, x^2+4x-3\}$ is linearly independent. Show that $u = x^2+x-5$ belongs to the span of S and express u as a linear combination of the vectors in S.	06
	b	Show that the set W= the set of 2×2 upper triangular matrices is a subspace of the set $M_{2\times2}$= the set of 2×2 matrices.	04
	c	Find the Linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T(1,1,0) = (3,2,-2)$, $T(1,1,1) = (3,-1,1)$, $T(1,0,1) = (1,-1,1)$. Also find the range space and null space of the Linear transformation.	10

UNIT-2

3	a	Let V be a vector space with the inner product defined by $< f,g >= \int_0^1 f(t)g(t)dt$. Find the lengths of the vectors $f(t) = t^2$ and $g(t) = t$.	04
	b	Find a least-squares solution of the inconsistent system $Ax = b$, where $A = \begin{bmatrix} 2 & 2 \\ 1 & -1 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 8 \\ 1 \\ 6 \\ 4 \end{bmatrix}$.	06
	c	Obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \\ 1 & 4 & -3 \end{bmatrix}$	10
		OR	
4	a	Suppose $\mathbb{P}_2$ is a vector space having the inner product defined by $< p,q > = p(t_0)q(t_0) + p(t_1)q(t_1)$, where $t_0 = -2, t_1 = 2$. Compute the distance between the vectors $p(t) = t+2$ and $q(t) = 5+t$.	04

	b	Using Gram-Schmidt process, convert the linearly independent vectors $(1,1,1,1,1), (2,-1,1,1,2), (2,-1,2,0,2)$ to an orthogonal basis of a subspace of $\mathbb{R}^5$.	06
	c	A certain experiment produces the data $(1,3), (2,5), (3,15), (4,24)$. Fit a least-squares curve of the form $y = \beta_0 + \beta_1 x + \beta_2 x^2$ to the above data.	10

UNIT-3

5	a	Suppose the quadratic form is given by $Q(x) = 9x_1^2 + 7x_2^2 + 11x_3^2 - 8x_1x_2 + 8x_1x_3$. Find: i) the maximum value of $Q(x)$ subject to the constraint $x^T x = 1$, ii) a unit vector u where this maximum is attained, iii) the maximum of $Q(x)$ subject to the constraint $x^T x = 1$ and $x^T u = 0$, iv) a unit vector v where the second maximum is obtained, v) The new quadratic form.	10
	b	Obtain the singular value decomposition of the matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$.	10

OR

6	a	Given the matrix of observations as: $\begin{bmatrix} 5 & 4 & 6 & 5 & 3 & 7 \\ 12 & 13 & 11 & 16 & 17 & 15 \end{bmatrix}$, convert the matrix to mean deviation form, construct the covariance matrix and hence find its principal components. Also determine what percentage of the information is retrieved from the first principal component.	10
	b	Decompose the matrix $A = \begin{bmatrix} 5 & -1 & 3 \\ -3 & 5 & 3 \\ 1 & -1 & 7 \end{bmatrix}$ as $A = PDP^{-1}$, by using the diagonalization process.	10

UNIT-4

7	a	Let X and Y be the number of hardware failures in two computer labs in a given month. The joint distribution of X and Y is given by $P(X,Y)$, where $P(0,0) = 0.52$, $P(0,1) = 0.20$, $P(0,2) = 0.04$, $P(1,0) = 0.14$, $P(1,1) = 0.02$, $P(1,2) = 0.01$, $P(2,0) = 0.06$, $P(2,1) = 0.01$, $P(2,2) = 0$. Construct the joint distribution of the total number of hardware failures in two computer labs. Find: i) the marginal distributions of X and Y, ii) the expectation of X, Y and XY, covariance of X and Y, iii) the conditional probability of X given $Y = 1$.	10
	b	Determine the value of c such that the function $f(x,y) = cxy$ for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function. Find: i) $P(X < 2)$, ii) $P(Y > 1)$.	06
	c	Obtain the axes of the constant probability density contours for a bivariate normal distribution with the covariance matrix $\Sigma = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ and whose eigen values are 4, 2.	04

OR

8	a	Let the random variable X denote the time until a computer server connects to your machine (in milliseconds), and let Y denote the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. Assume that the joint probability density function for X and Y is $f(x,y) = 2e^{-x-y}$ $for\ 0 < x < y < \infty$. i) Verify that $f(x,y)$ is a valid joint density function, ii) Compute the probability that Y exceeds 2 milliseconds, the marginal density function of X, the conditional probability density function for Y, given that $X = x$.	10
	b	Determine the value of c that makes the function $f(x,y) = c(x + y)$ a joint probability mass function over the nine points with $x = 1,2,3$ and $y = 1,2,3$. Find (i) $P(x \geq 2)$, $P(y < 3)$.	06
	c	Find the correlation matrix R_X for the two random variables X_1 and X_2 whose joint and marginal probabilities are represented as follows:	04

$X_1 \downarrow X_2$	1	2
0	0.20	0.06
1	0.20	0.14
2	0.15	0.25

X_1	0	1	2
$P(X_1)$	0.26	0.34	0.40

X_2	1	2
$P(X_2)$	0.55	0.45

UNIT-5

9	a	Customers arrive at a one man counter of a coffee shop according to a Poisson process with a mean interarrival time of 4 minutes. Customers spend an average of 2 minutes at the counter. i) What is the expected number of customers in the queue and in the shop? ii) What is the average length of the queue that forms from time to time? iii) How much time can a customer expect to spend in the coffee shop? iv) What is the probability that a customer arriving at the shop will have to wait in the queue? v) What is the probability that there are more than 3 customers in the shop? (Hint: Use $(M\|M\|1)$: $(\infty\|FIFO)$ model)	10
	b	A Commercial bank has 3 cash paying assistants. Customers are found to arrive in a Poisson fashion at an average rate of 6 per hour for business transaction. The service time is found to have an exponential distribution with a mean of 20 mins. Calculate i) P_0 ii) Average number of customers in the system iii) Average time a customer spends in the system, iv) The probability that a customer would have to wait in the queue. (Hint: Use $(M\|M\|s)$: $(\infty\|FIFO)$ model)	10
		OR	

10	a	In a single server queuing system with Poisson input and exponential service times, if the mean arrival rate is 4 calling units per hour, the expected service time is 0.2 hours and the maximum possible number of calling units in the system is 3, Find i) $P_n (n \geq 0)$ ii) P_0, iii) the average number of calling units in the system iv) Average waiting time in the system. (Hint: Use $(M\|M\|1):(k\|FIFO)$ model)	10
	b	A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 15 cars per day. The service time in both the bays is exponentially distributed with 10 cars per day per bay. Find i) P_0, ii) P_1 iii) The average number of cars waiting in the queue. (Hint: Use $(M\|M\|s):(k\|FIFO)$ model)	10