BRITTLE FRACTURE IN CHARPY IMPACT TEST

INFLUENCE OF MATERIAL PARAMETERS

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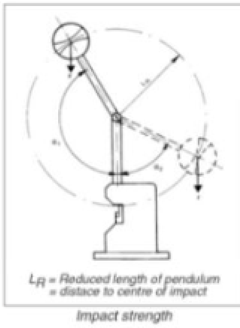
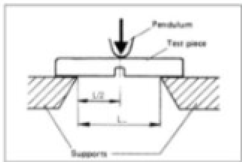
1 INTRODUCTION

Goal of this work is to analyze the conditions under which fractures and crack propagate in materials. It relies on the theory of fracture mechanics. It has many important applications, especially in the industry field, in which built mechanical components often have flaws. The probability of the failure of the material during its operation must be assessed. To that end, engineers do a damage tolerance analysis.

The Charpy impact test is commonly used to study fractures. Its functioning is quite simple: a striker is dropped and hits a notched tensile, the difference between the final and initial heights correspond to the energy dissipated in the specimen to create a fracture.

In this work, Finite Element Element simulations of this experiment are done to analyze the influence of some parameters of the material on its resistance to fracture.





2 PRELIMINARY FINITE ELEMENT MODEL

We present here a FEM model of the Charpy specimen.

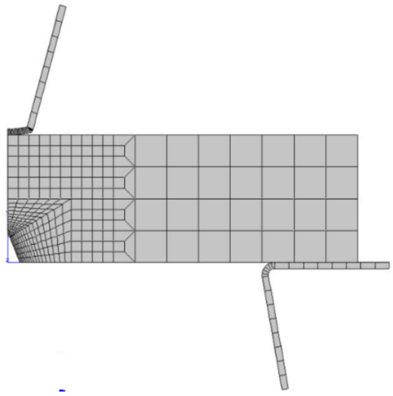
It has an isotropic elastic behavior under low strain or stress, with a Young Modulus $E = 208\text{GPa}$ which is standard for steel. It then has a isotropic non linear plastic behavior ruled by: $\sigma = R_0 + Q(1 - \exp^{-bp})$ where p is the cumulative plastic deformation $p = \int_{t=0}^{t=t*} \varepsilon_p(t)dt$.

To model the transition in plastic regime, the *Von Mises* criterion is used. It relies on the *Von Mises* equivalent stress:

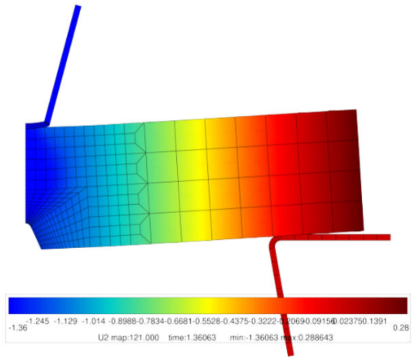
$$\sigma_{eq} = \sqrt{\frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)} \tag{1}$$

Lastly, we use the Beremin model to predict the evolution of the crack. Concerning the boundary conditions, anvils on which the specimen lies are immobile, the striker has a prescribed vertical downward movement at constant speed $\dot{u}_2 = -1\text{mm.s}^{-1}$. The ligament, ie the nodes in the vertical plane cutting the specimen in half, are also immobile along the horizontal axis. While this assumption is not physically justified, it is useful to shrink the problem size to only the right half of all aforementioned objects, because the problem can be considered symmetric along the ligament.

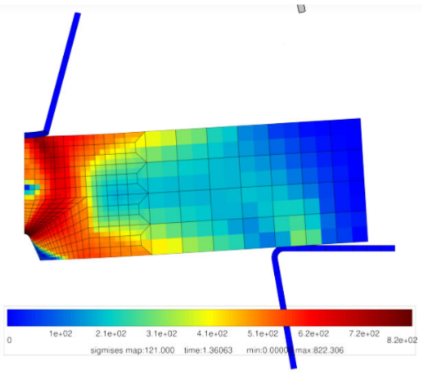
The first simulation returned an error during the last iterations.



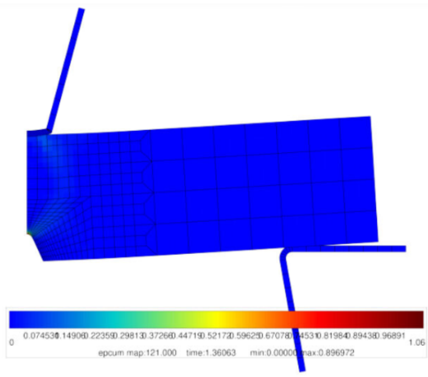
(a) Initial mesh



(b) Vertical displacement



(c) *Von Mises* stress



(d) cumulative plastic deformation

Figure 1: Results of experiment 1

We observe a concentration of high *Von Mises* stress around the notch and near the striker, $\sigma_{eq} > 700$MPa, and also a bit of plastic deformation. A possible explanation of the error could be a lacking mesh precision around those zones and also near the anvils where it reaches ~ 310MPa.

3 LOCAL MESH REFINEMENT

Thanks to this analysis, we choose a finer mesh in those areas :

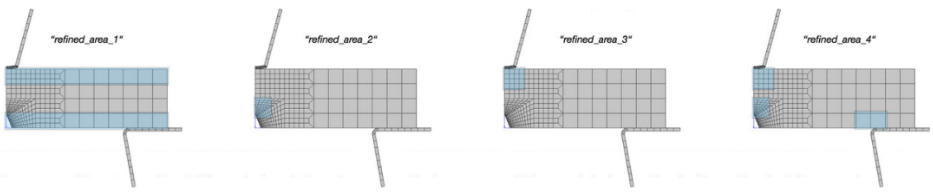
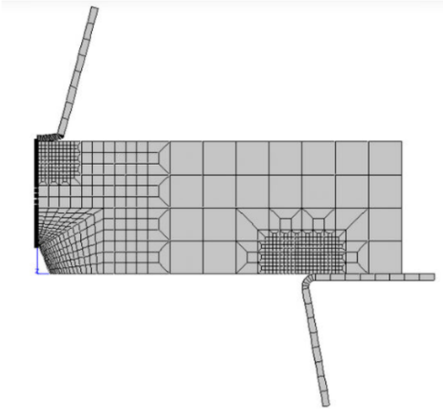


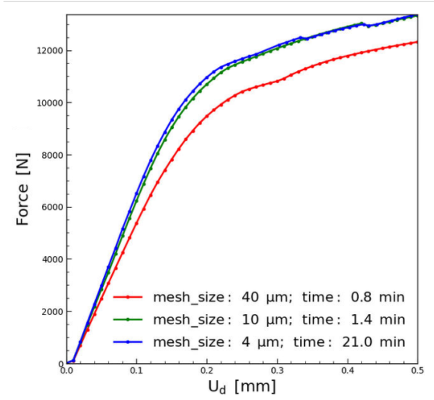
Figure 2: Propositions of mesh refinement.

The 4-th mesh fulfills our desires of refinement, so we select it. In order to select the size of the smallest mesh elements, we rerun the previous experiment with $40\mu m, 10\mu m, 4\mu m$:

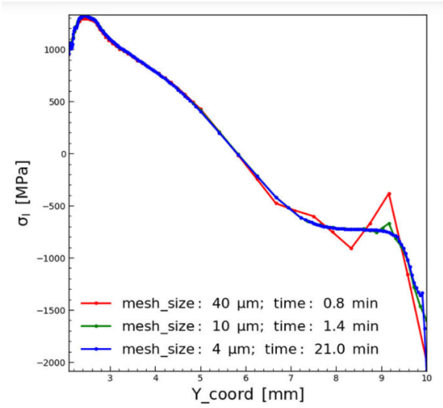
With these finer mesh, the previous simulation now converges. Results on Figures 3b and 3c quite differ between $40\mu m$ and the 2 smaller sizes, meaning we gain some model accuracy with refinement. However, $10\mu m$ and $4\mu m$ have similar



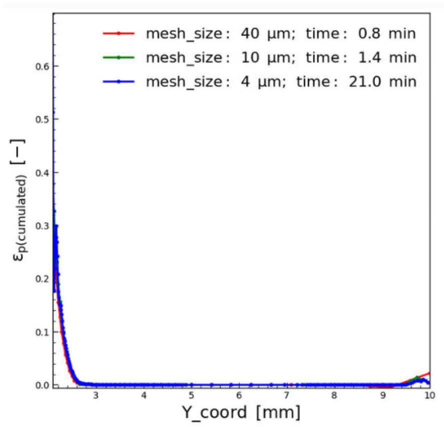
(a) Refined mesh



(b) Relation force on striker - displacement



(c) Relation first principal stress - y coordinate



(d) Relation cumulative plastic deformation - y coordinate

Figure 3: Comparison of mesh refinements

results for our precision, but the 10μm simulation was 15 times faster, therefore it is more interesting for us to choose it instead of the smallest size. We see again on Figures 3c and 3d the concentration of stress and plastic deformation near the notch ie in the low y zones.

4 GEOMETRIC PARAMETER SENSITIVITY

Now that we have a more adapted mesh and converging simulations, we wish to assess the dependency of the geometry of the Charpy specimen in its probability of failure. Numerical simulation is here useful to run multiple Charpy tests, that in reality are hard to set up. The considered parameters are the notch radius N_R, the height H and half of the width H_W:

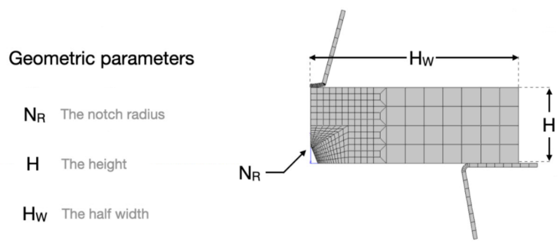
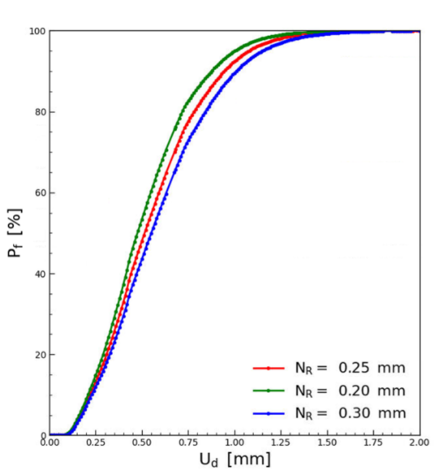
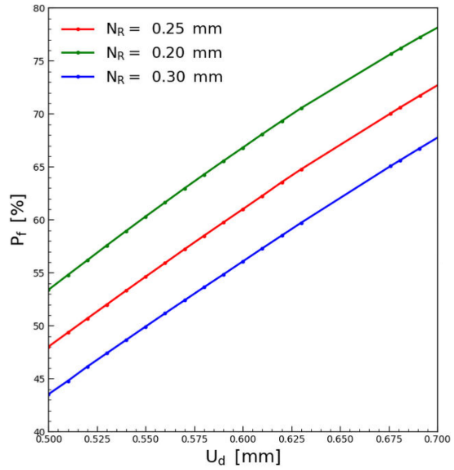


Figure 4: Considered geometric parameters.

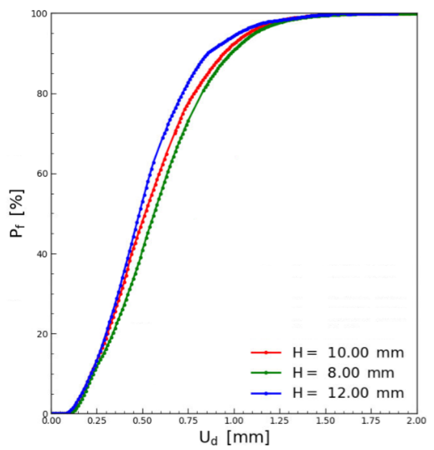
The strategy is here to run 3 simulations for each parameter, in each of them we add a certain variation of the parameter: −20%, 0% and +20%, while keeping every other at its default value.



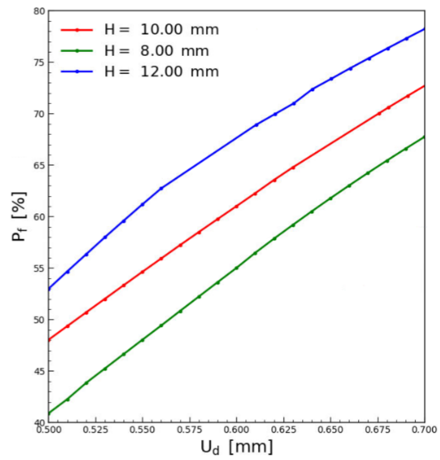
(a) Probability of failure for various N_R



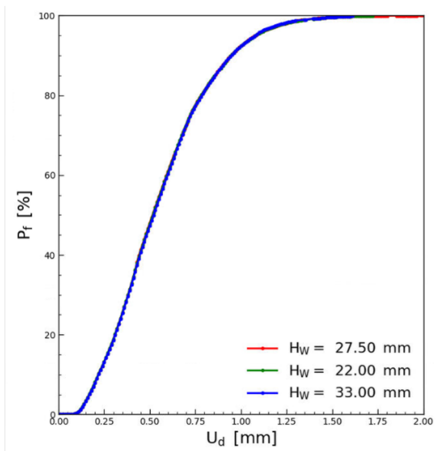
(b) Zoom



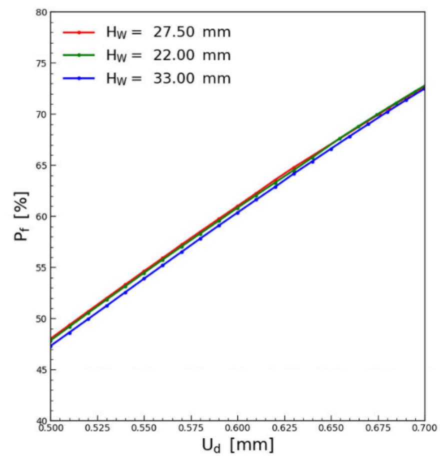
(c) Probability of failure for various H



(d) Zoom



(e) Probability of failure for various H_W



(f) Zoom

Figure 5: Probability of failure with respect to the striker displacement for various values of N_r, H and H_W.

On Figure 5, we see that the probability of failure curve has a similar shape for each parameter. The plots of the right column are zooms on the zones of highest variance between curves, with $U_d \in [0.5\text{mm}, 0.7\text{mm}]$. On these zooms, all P_f curves are approximately linear, with same slope $\frac{\Delta P_f}{\Delta U_d} \sim \frac{72-48}{0.7-0.5} = 120\%.\text{mm}^{-1}$. Only the offsets at a given displacement u_d differ between parameters variations $\nu_1, \nu_2 \in$

$\{\pm 20\%, 0\%\}$ ie $|P_{f,\nu_1}(u_d) - P_{f,\nu_2}(u_d)|$. Variating H_W by $\pm 20\%$ induces the smallest variation of P_f, of $\sim 1\%$, where for N_R and H, $\Delta P_f \sim 5\%$. Therefore H and N_R are the 2 most impactful parameters on the failure probability. The lower the notch radius and the higher the height, the higher the failure probability. But if one industrial operator should choose to focus only on one parameter, I would advice to focus on H that is simpler to adjust than N_R, eventhough the latter seem more dangerous, because the failure probability increases as N_R diminishes, which makes the notch harder to spot.

5 THE BEREMIN MODEL

A difficulty link to estimating the failure probability is that failure sometimes appear eventhough the applied stress is below the yield value of the material, because there were defects and cracks in it. The Beremin model focus on those local micromechanisms.

The idea of the Beremin model relies on 2 main assumption. The first states that microcraks appear dut to inhomogeneous plastic deformation in grains, the second models the propagation of those micracks with the stress normal to their planes exceeding a critical stress σ_c. This value is often approximated for a microcrak of length l_0 by:

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi(1-\nu^2)l_0}} \tag{2}$$

The local approach is to discretize the stressed volume V_p that becomes plastic into n smaller ones of volume V_0, mutually independant and each containing a notch. It is assumed the general stressed region fails if and only if one of its subregion fails, and the failure will appear at the longest microcrak. Given the probability distribution of the crack within one small volume :$\mathbb{P}_{crack}$, the probability of failure under stress σ in this volume is the probability to find a crack longer than a critical length l_c linked to a critical stress:

$$\mathbb{P}_{V_0,fail}(\sigma) = \int_{l_c}^{+\infty} \mathbb{P}_{crack}(l_0)d0 = \left(\frac{\sigma}{\sigma_u}\right)^m \tag{3}$$

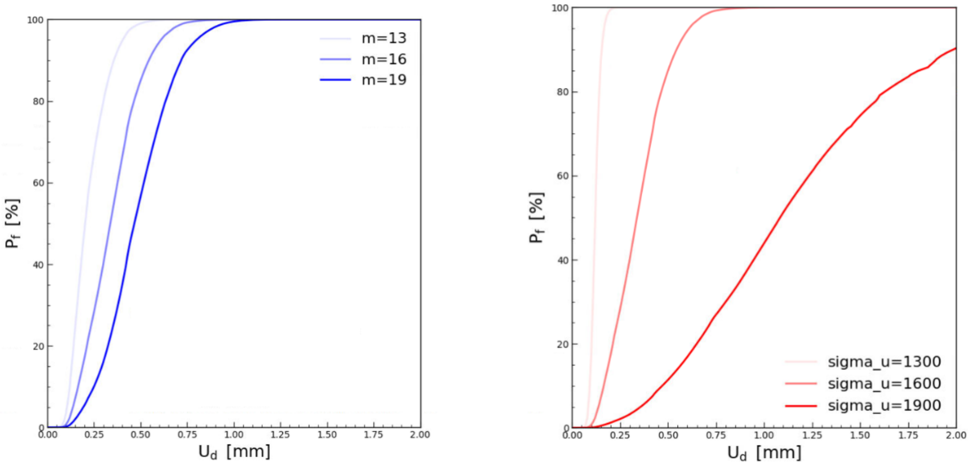
Where m and σ_u are constant. we then find the overall probability of failure:

$$P_f = 1 - \exp\left(-\left(\frac{\sigma_w}{\sigma_u}\right)^m\right) \tag{4}$$

Where $\sigma_w = \left(\frac{1}{V_0}\int_{V_p} \sigma_{I,p}^m\right)^{\frac{1}{m}}$ represents a mean local stress among all subvolumes.

The 2 Beremin parameters m and σ_u have specific influence on P_f: m is linked to the distribution of length of microcraks. Beremin originally used $\mathbb{P}_{crac}(l_0)dl_0 = \frac{\alpha}{l_0^\beta}$ with $\beta = \frac{m}{2} + 1$. To assess its impact on P_f, we can assume σ_w is approximately constant when m varies, then an increase of m increases P_f if $\sigma_w > \sigma_u$ and decreases it if $\sigma_w < \sigma_u$. For σ_u, its increase induces a decrease of P_f. Those relations are intuitive if we interpret m as the number of subzones of defects in the stressed zone, and σ_u a stress linked to the critical σ_c: increasing this threshold is equivalent to say the material is more resistant to fracture, where increasing the number of defects increases its instability, and the probability of failure when the local stress exceeds the critical threshold.

When we look at the results of simulations on Figure ??, we see the trend that m shifts the curve of P_f to the right, without noticeably changing the slope, probably because the variation of σ_w w.r.t m must be considered. As expected, a σ_u reduces the slope in the linear part.



(a) Probability of failure for various m.

(b) Probability of failure for various σ_u.

We wish now to find m and σ_u for a given material, for which we experimentally statistics of failure P_f for load u_d. Because the FEM model has long computation time, a regression approach will be very time consuming. In fact, one simulation that computes u_d, P_f given m, σ_u takes ~ 1 minute. Therefore it is preferable to adjust manually those parametres, knowing their influence as explained before. Pretty quickly a satisfying estimation has been found on Figure 6.

The mean square error of the estimation interpolated on the experimental values of u_d is $\sum\limits_{u_{d,exp}} \left(P_{f,exp}(u) - P_{f,sim}(u)\right)^2 = 10.14$.

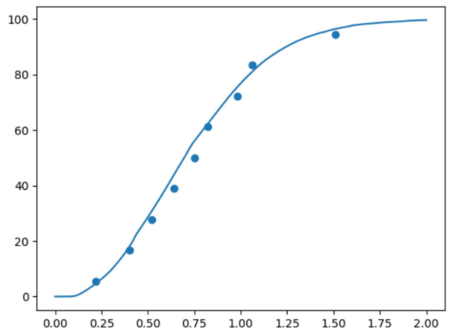


Figure 6: Observed $P_f(u_d)$ (blue dots). Estimated $P_f(u_d)$ with FEM method (blue curve) and
$m = 15, \sigma_u = 1830$.

6 METHODS

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6.1 Paragraphs

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DIFFERENT PARAGRAPH DESCRIPTION Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

6.2 Math

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$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{5}$$

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Definition 1 (Gauss)**.** To a mathematician it is obvious that $\int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}$.

Theorem 1 (Pythagoras)**.** *The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.*

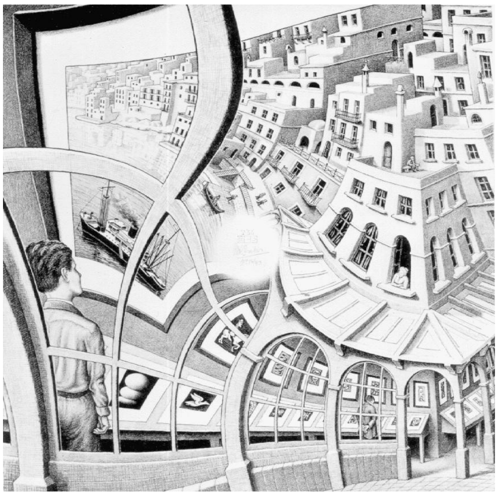

Figure 7: An example of a floating figure (a reproduction from the *Gallery of prints*, M. Escher, from `http://www.mcescher.com/`).

Proof. We have that $\log(1)^2 = 2\log(1)$. But we also have that $\log(-1)^2 = \log(1) = 0$. Then $2\log(-1) = 0$, from which the proof. $\square$

7 RESULTS AND DISCUSSION

Reference to Figure 7.

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7.1 Subsection

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7.1.1 Subsubsection

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WORD Definition

CONCEPT Explanation

IDEA Text

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7.1.2 *Table*

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Table 1: Table of Grades

Name		
First name	Last Name	Grade
John	Doe	7.5
Richard	Miles	2

Reference to Table 1.

7.2 Figure Composed of Subfigures

Reference the figure composed of multiple subfigures as Figure 8 on the following page. Reference one of the subfigures as Figure 8b on the next page.

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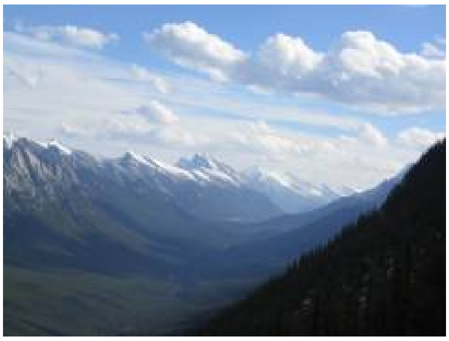
(a) A city market.

(b) Forest landscape.



(c) Mountain landscape.

(d) A tile decoration.

Figure 8: A number of pictures with no common theme.

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REFERENCES