

# Grading Scheme for LA Assignment 5

February 21, 2025

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## Question 1: Rank-Nullity Theorem (10 Marks)

### Grading Breakdown:

- Statement of Rank-Nullity Theorem (2 Marks)  
Correctly stating that  $\text{rank}(T) + \text{nullity}(T) = \dim(V)$ .
- Proof Setup (2 Marks)  
Clearly defining  $\ker(T)$  and  $\text{im}(T)$  and setting up the proof correctly.
- Logical Steps (4 Marks)  
Showing that  $\ker(T)$  and  $\text{im}(T)$  are subspaces, and their dimensions add up to  $\dim(V)$ .
- Conclusion (2 Marks)  
Correctly concluding that the theorem holds with proper justification.

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## Question 2: $L(V, W)$ as a Vector Space (10 Marks)

### Grading Breakdown:

- Definition of  $L(V, W)$  and Operations (3 Marks)  
Clearly defining  $L(V, W)$ , vector addition, and scalar multiplication.
- Proof of Closure Under Addition & Scalar Multiplication (3 Marks)  
Showing that these operations result in a transformation still within  $L(V, W)$ .
- Verification of Vector Space Axioms (3 Marks)  
Checking properties such as associativity, distributivity, and existence of identity elements.
- Conclusion (1 Mark)  
Concluding that  $L(V, W)$  is a vector space over  $F$ .

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## Question 3: Dimension of $L(V, W)$ (10 Marks)

### Grading Breakdown:

- Understanding the Problem (2 Marks)  
Correctly interpreting  $\dim(L(V,W))$  in terms of bases.
- Constructing a Basis for  $L(V,W)$  (4 Marks)  
Providing a basis for  $L(V,W)$  using basis vectors of  $V$  and  $W$ .
- Dimensionality Calculation (3 Marks)  
Correctly computing  $\dim(L(V,W)) = \dim(V) \times \dim(W)$ .
- Conclusion (1 Mark)  
Final statement confirming the result.

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Total: 30 Marks

This grading scheme ensures fair and structured assessment of student responses.