Control of Linear Multivariable Systems
Lab 1

Submitted by

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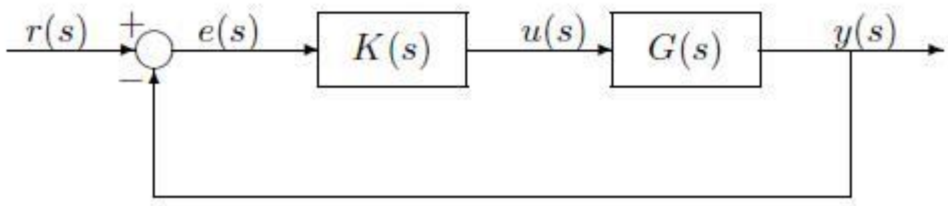
The aim of the lab is to study different PID type controllers, $K(s)$, with the code called ASTA for the system

G(s) = 1/(1 + s)^2/(1 + 10s)

in the following closed loop architecture



Now the
G(s) can be simplified as

$$G(s) = \frac{1}{(10s^3 + 21s^2 + 12s + 1)}$$

Editing the transfer function of the system and use the input box EDIT dedicated to the “System” with the numerator as “1.0” and denominator as “[10 21 12 1]”. Then time domain response and frequency domain response is plotted.

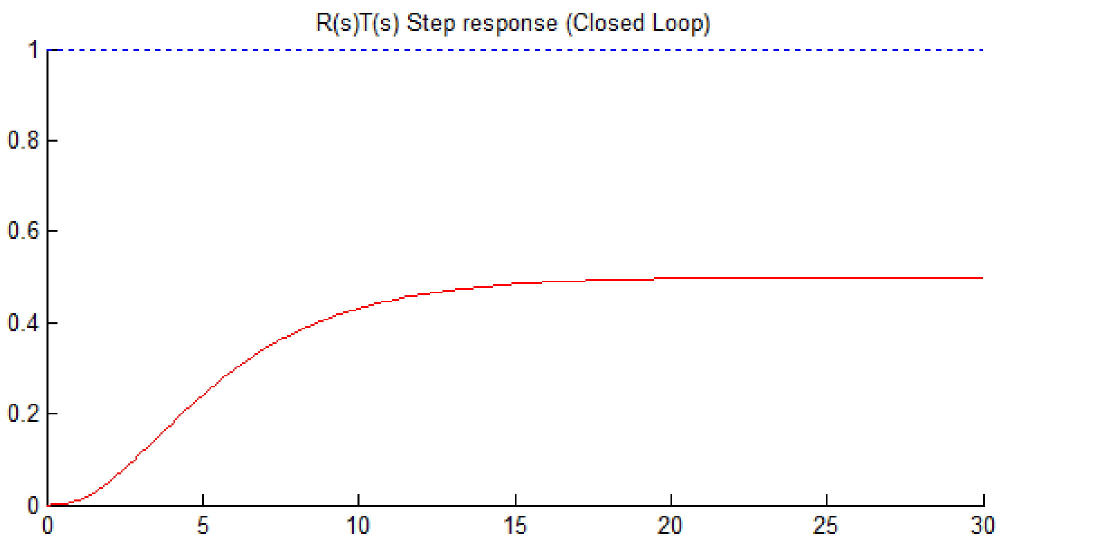


Fig.1 Time domain response (closed loop) – R(s)T(s)

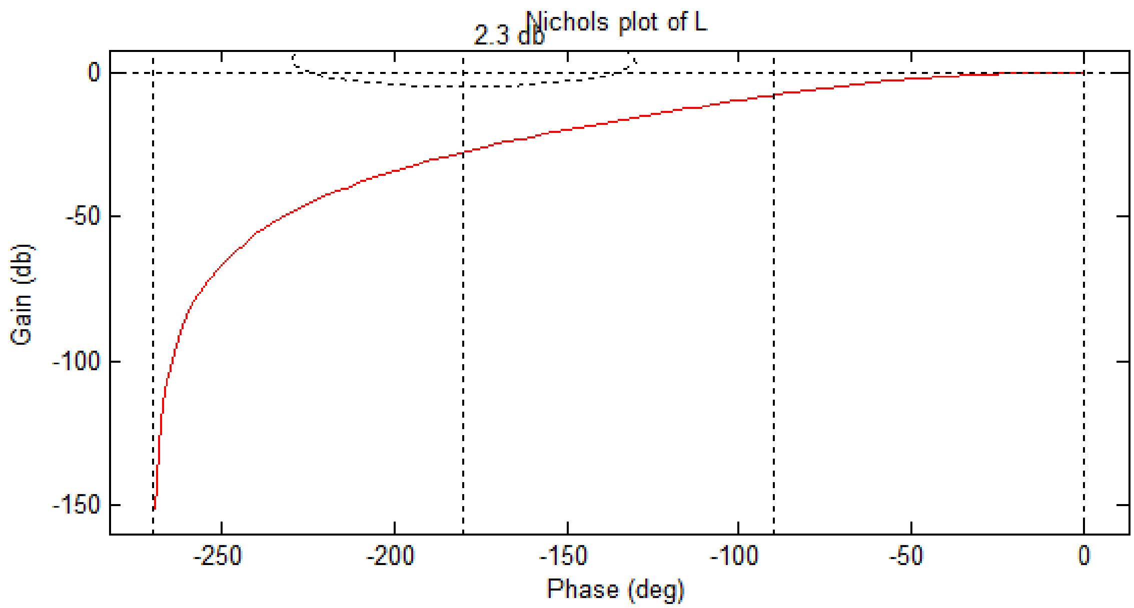


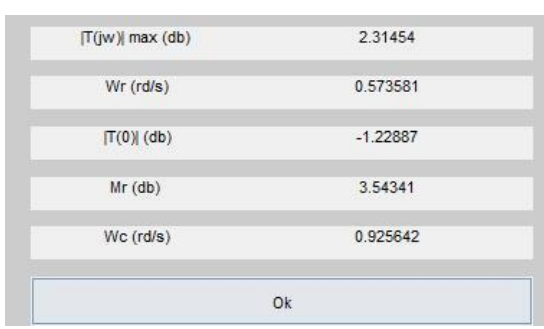
Fig.2 Frequency domain response - Nichols plot of L

1. Proportional Controller:

Tune a proportional control $K(s) = P$ such that the maximum of the complementary sensitivity function is 2.3 *db*. In order to compare the performances of the different closed loop that will be designed, note the characteristics of the step response (*Mp*, *tp*, *tr*5%, *tr*2%, _*r*) and of the frequency response (*Mr*, !*r*). Note also the stability margins. (Note that *Mr* 6= 2.3 *db* and that *Mp* is far from 23%. What is the value of *P* which gives *Mr* = 2.3 *db* ?)

Proportional control is a type of linear feedback control system in which a correction is applied to the controlled variable which is proportional to the difference between the desired value and the measured value. Now tuning the proportional controller is given below:

Using the proportional controller to meet the curve at tangent to the M-circle using the trial and error method. This method obtains the curve to be meet at tangent to the M-circle y giving the value of P = 6.58 when defining the proportional controller, the time domain and frequency domain response are plotted below for P controller with P = 6.58. From the bode magnitude plot of T(s), the complementary sensitivity function is equal to 2.3 db. Therefore the characteristics are plotted below and the performance is analyzed and noted down.



|T(jw)| max (db)	2.31454
Wr (rd/s)	0.573581
|T(0)| (db)	-1.22887
Mr (db)	3.54341
Wc (rd/s)	0.925642
Ok	

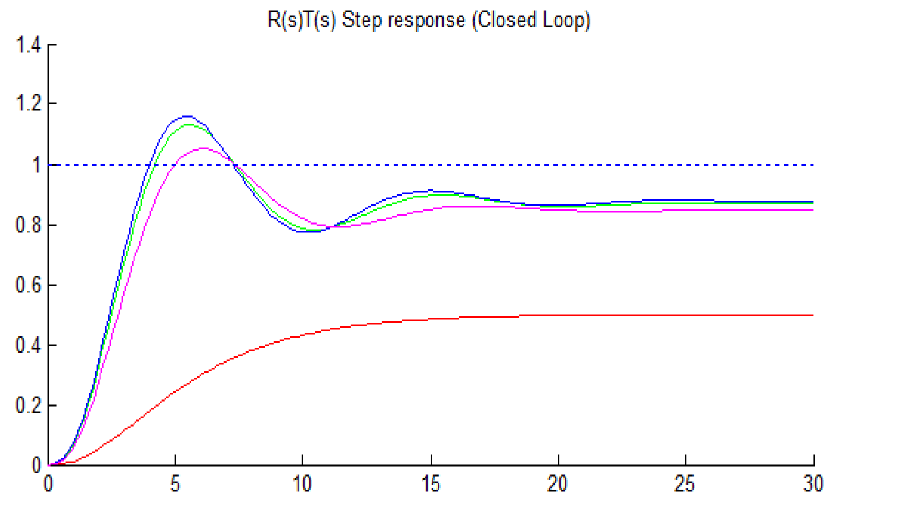


Fig.3 Time domain response (closed loop) of Proportional controller with P = 6.58

(The red color indicates the plot of with P = 1.0 and the green plot indicates the controller with P = 6.58)

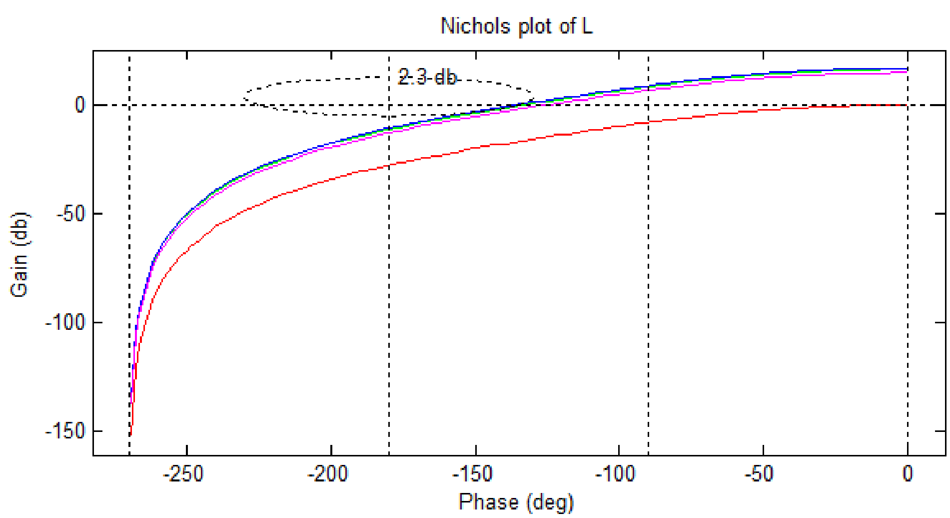


Fig.4 Frequency domain response of Proportional Controller with P = 6.58

(The red color indicates the plot of with P = 1.0 and the green plot indicates the controller with P = 6.58)

The performance of the controller is given below using the parameters. The value of P which gives M_r= 2.3 db is 5.36.

Controller P	M_p	t_p	t_{s5}	t_{s2}	ε_r	ω_r	M_r	$T(j\omega)$	$T(s)$
6.58	30.24	5.57	12.26	16.94	13.21	0.57	3.542	2.31	-1.228
5.36	23.41	6.129	12.53	13.95	15.72	0.51	2.3	0.818	-1.485

2. Proportional – Integral (PI) Controller:

The PI controller has the equation of $K(s) = P(1 + \frac{1}{sT_i})$. Initially the P controller to tuned and the body gain and the body phase is evaluated.
The P value is given for the frequency response will go out of M-circle (P=5.6) and the static error of all the controllers are reduced to 0% than the previous controller. The T_i value is chosen between the $\frac{1}{T_i} < \omega_R < \frac{10}{T_i}$ where $\omega_R$is taken from the bode plot of P controller. Now to tune the exact value the $T_i = \frac{10}{\omega_R}$is chosen. Now the ω_r= 0.57 is taken from the bode plot of P controller. The T_i value will be 17.54.

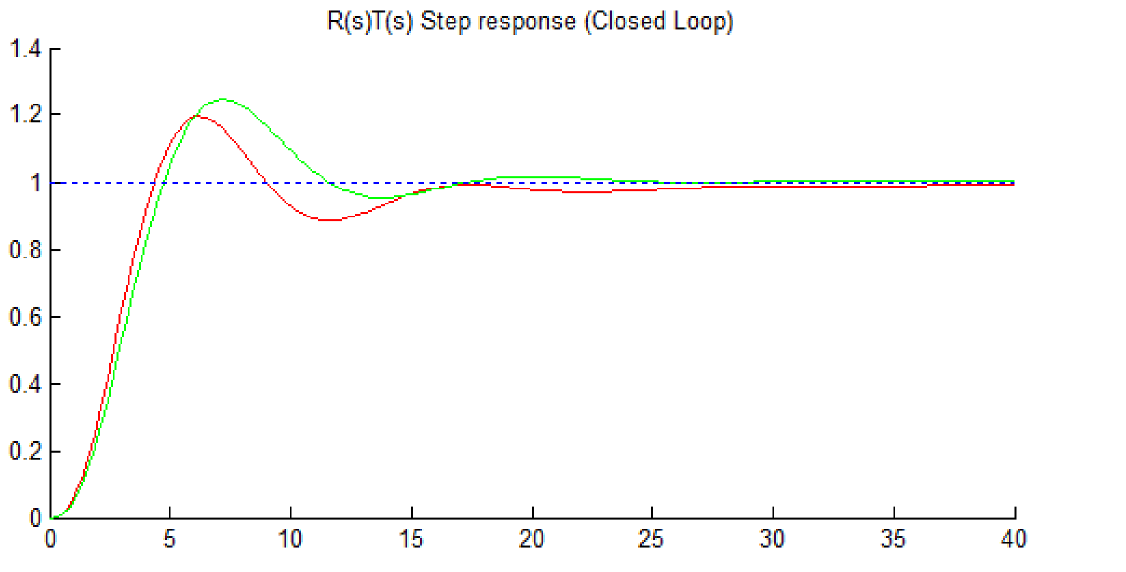


Fig.5 Step response (closed loop) of PI controller

(The red color indicates the plot of with P = 5.62 and T_i = 17.543* and *the green color indicates the plot of with P = 4.5 and T_i = 8.77)

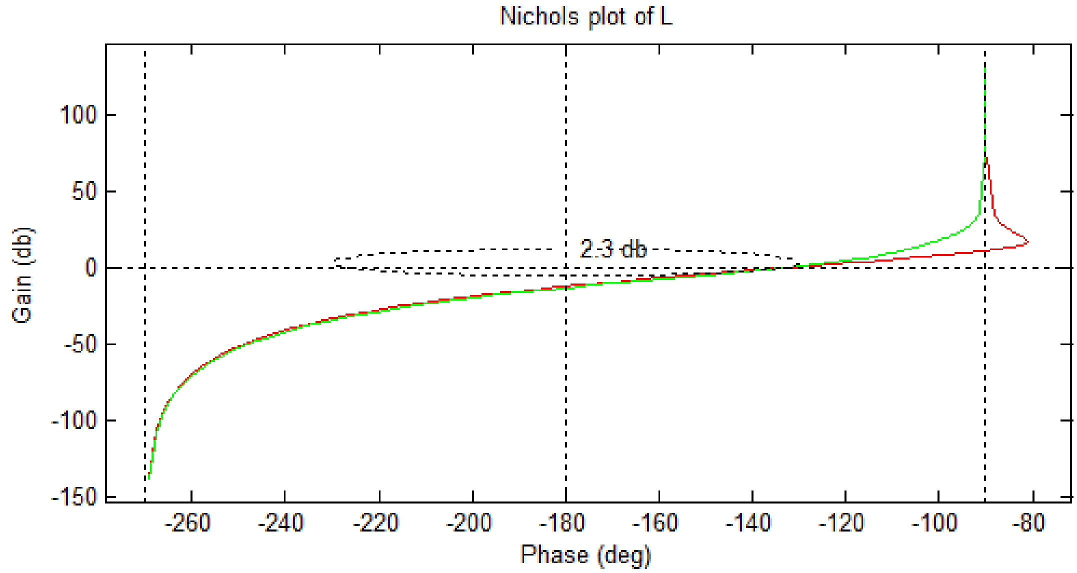


Fig.6 Frequency domain response of PI controller

(The red color indicates the plot of with P = 5.62 and T_i = 17.543 and the green color indicates the plot of with P = 4.5 and T_i = 8.77)

Controller P, T_i	M_p	t_p	t_{s5}	t_{s2}	ε_r	ω_r	M_r	$T(j\omega)$	$T(s)$	ω_c
P = 5.62, T_i = 17.54	19.56	6.219	14.459	25.771	0	0.51	2.33	2.33	0	0.8
P = 4.5, T_i = 8.77	24.5719	7.173	10.641	16.098	0	0.408756	2.32874	2.328	0	0.7

3. Lag compensator:

Now for the lag compensator the equation is

$$K(s) = P\left(\frac{1+sT}{1+sTb}\right)$$

For the maximum complementary sensitivity function (T(s)=2.3 db), the static error equals to the 5% for the step response.

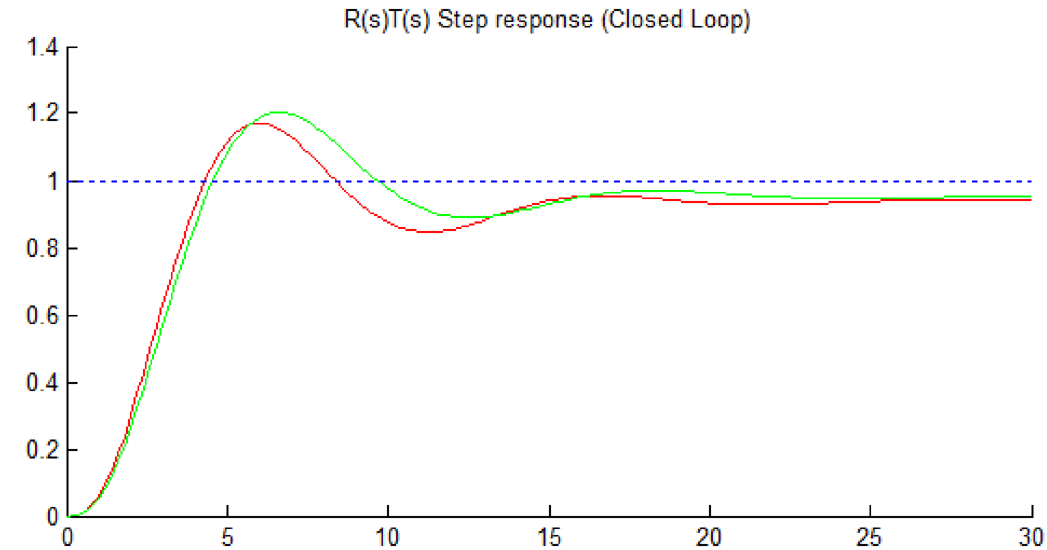


Fig.7 Step response (closed loop) of Lag Compensator

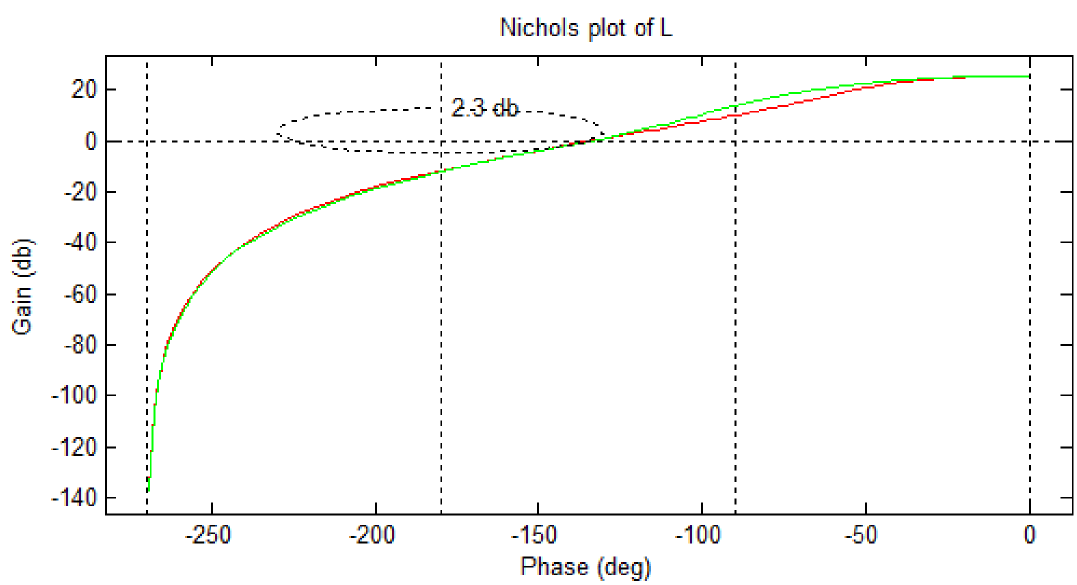


Fig.8 Frequency domain response of Lag Compensator

Controller P, T, b	M_p	t_p	t_{s5}	t_{s2}	ε_r	ω_r	M_r	$T(j\omega)$	$T(s)$	ω_c
P = 19, T = 17.54, b = 3.22	23.2317	5.997	13.611	22.983	5	0.56	2.27	2.3	-0.44	0.84
P = 19, T = 8.77, b = 3.75	26.6567	6.578	13.743	19.047	5	0.45618	2.728	2.28286	-0.44553	0.768