

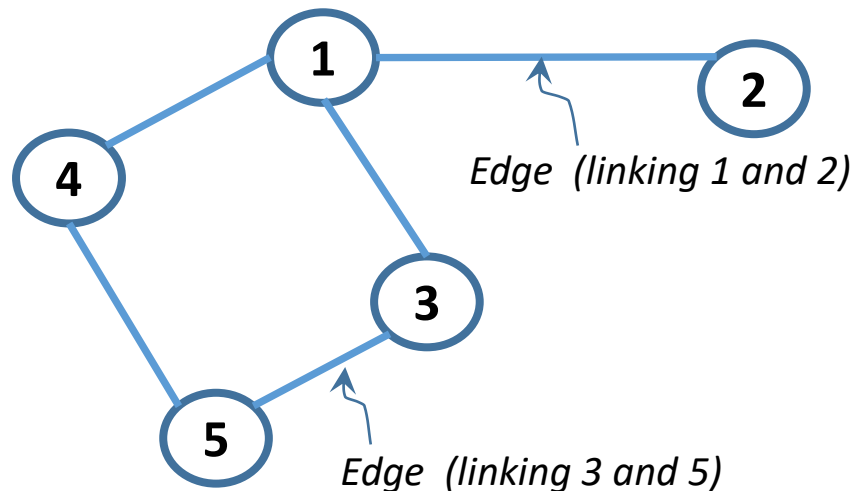
ET0736

Lesson 9

Graph

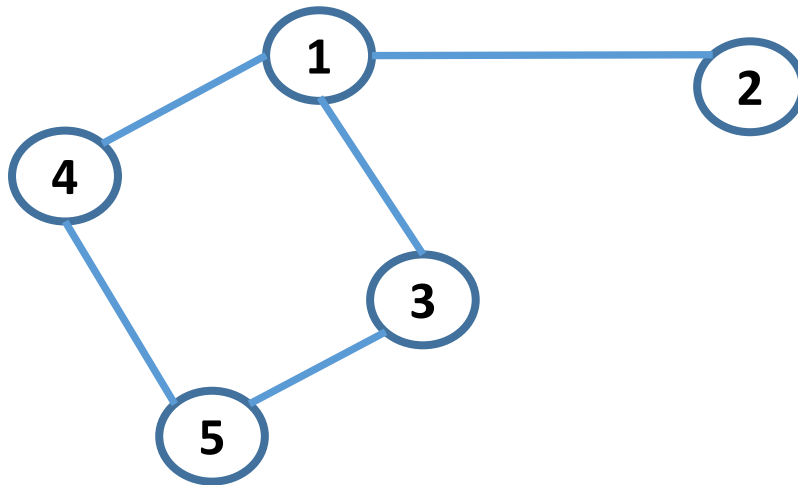
Graph

- is a non-linear data structure consisting of
 - vertices
 - edges
- vertices are connected by edges.
- used to represent relationships between different entities.
- Example: There are 5 vertices in the following graph: 1, 2, 3, 4 and 5



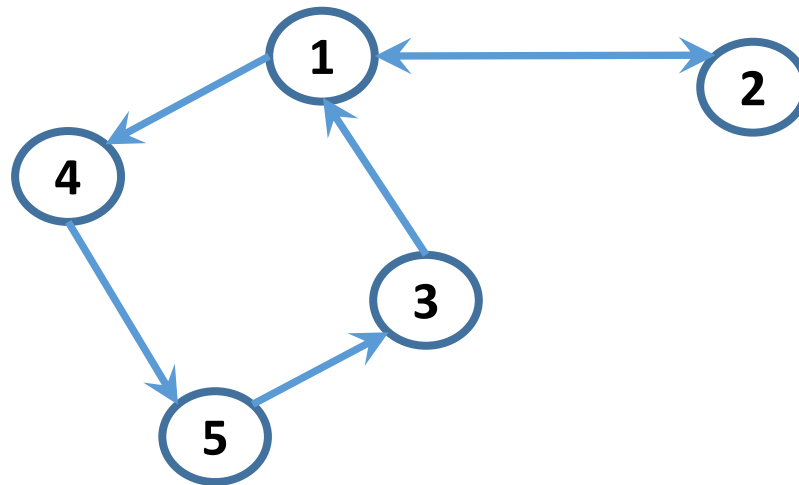
Graph

- A simple undirected graph has the edges as bidirectional (i.e. vertex 2 can go to vertex 1 and vis versa)
- A simple unweighted graph has no value (such as distance) attached to its edge



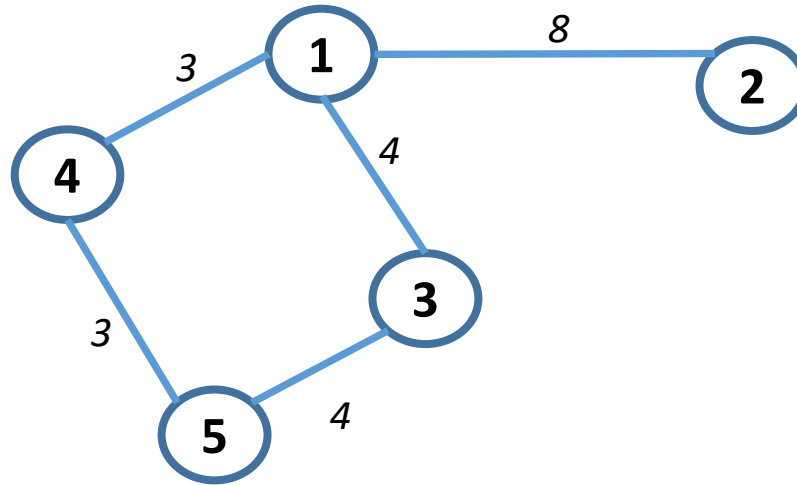
Graph

- Example of directed graph



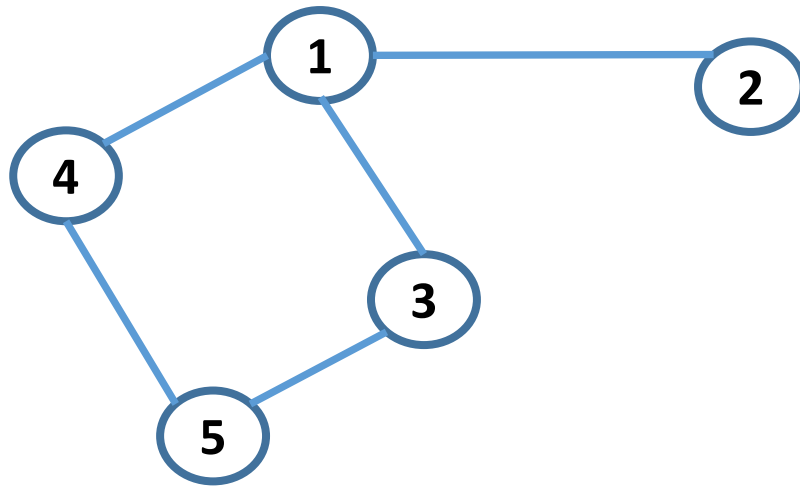
Graph

- Example of weighted graph



Representation of Simple Graph

- Use a simple **ArrayList** to house all the vertices 1, 2, 3, 4 and 5
- Use another **ArrayList** for each vertex to house all its neighboring vertices, aka adjacent list (i.e. edges)



1	2	3	4
2	1		
3	1	5	
4	1	5	
5	3	4	

Traversal of Graph

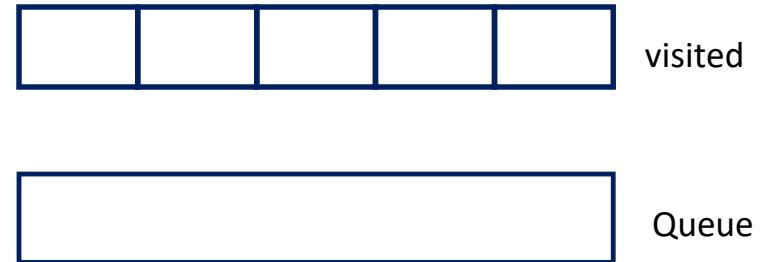
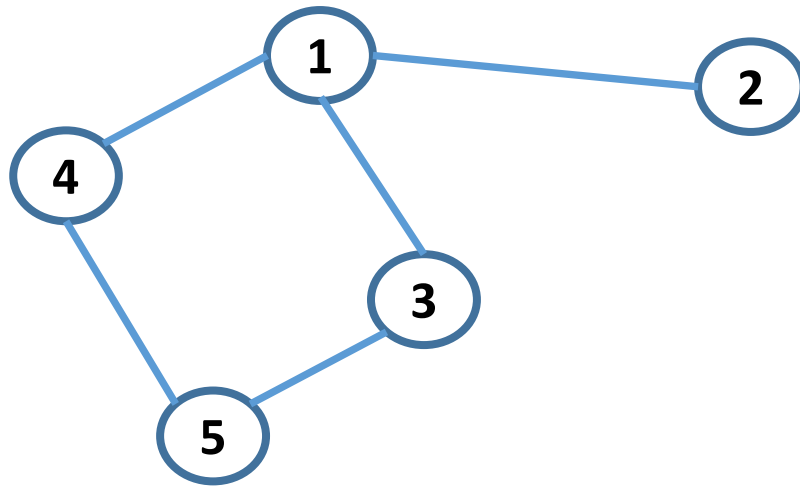
There are 2 fundamental methods in traversing a graph:

- Breadth First Search (BFS)
- Depth First Search (DFS)

The aim is to visit all the vertices and every vertex can only be visited once.

Breath First Search

- Prepare a visited List (initially empty)
- Prepare a Queue (initially empty)



Breath First Search

Steps:

put the root (starting) vertex in queue

while (queue is not empty)

 poll() a vertex from queue

 if (vertex has not been visit before)

 process the vertex (whatever process is needed)

 push all the neighbours (from adjacent list) into queue

repeat till queue is empty



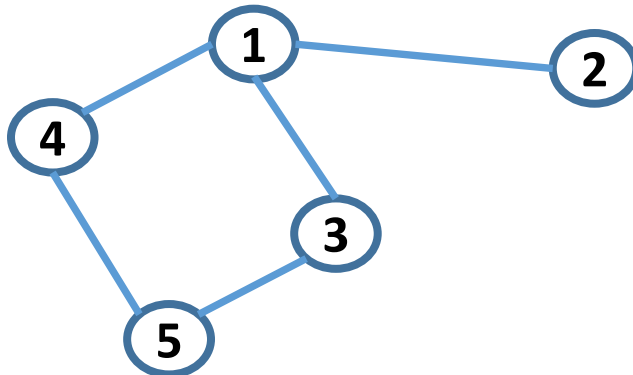
Breath First Search

- Starting from a root (say vertex 2)
- Insert the root into Queue

Loop until queue is empty

- poll() a vertex from Queue (it is 2)
- 2 has not been visited
- mark 2 as visited (insert 2 into visited)
- process the vertex
- push all its adjacent vertices in the queue (which is 1)

repeat till queue is empty



Before 1st loop

					visited
2					queue

At end of 1st loop

2					visited
1					queue

Breath First Search

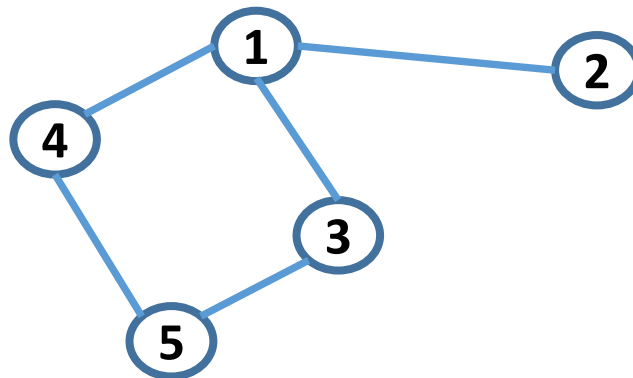
Loop until queue is empty

- poll() a vertex from Queue (it is 1)
- 1 has not been visited
- mark 1 as visited (insert 1 into visited)
- process the vertex
- push all its adjacent vertices in the queue (which are 2, 3 and 4)

Before 2nd loop

2					visited
1					queue

repeat till queue is empty



End of 2nd loop

2	1				visited
2	3	4			queue

Breath First Search

Loop until queue is empty

- poll() a vertex from Queue (It is 2. It has been visited before. Skip. Go to next loop)

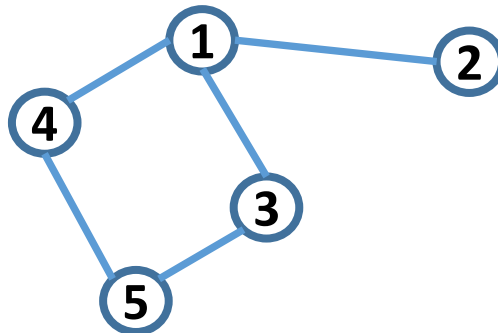
Before 3rd loop

2	1				visited
2	3	4			queue

Loop until queue is empty

- poll() a vertex from Queue (it is 3)
- 3 has not been visited
- mark 3 the as visited (insert 3 into visited)
- process the vertex
- push all its adjacent vertices in the queue (which are 1 and 5)

repeat till queue is empty



End of 4th loop

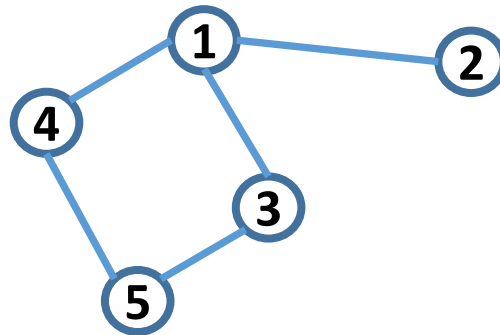
2	1	3			visited
4	1	5			queue

Breath First Search

Loop until queue is empty

- poll() a vertex from Queue (it is 4)
- 4 has not been visited
- mark 4 as visited (insert 4 into visited)
- process the vertex
- push all its adjacent vertices in the queue (which are 1 and 5)

repeat till queue is empty



Before 5th loop

2	1	3			visited
4	1	5			queue

End of 5th loop

2	1	3	4		visited
1	5	1	5		queue

Breath First Search

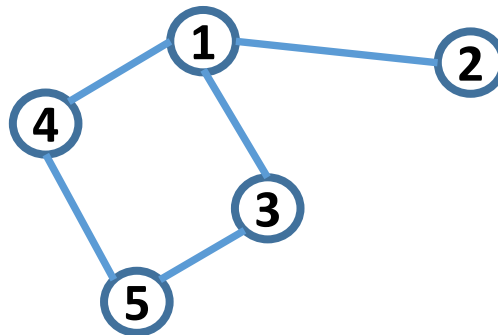
Loop until queue is empty

- poll() a vertex from Queue (It is 1. It has been visited before. Skip. Go to next loop)

Loop until queue is empty

- poll() a vertex from Queue (it is 5)
- 5 has not been visited
- Mark 5 the as visited (insert 5 into visited)
- process the vertex
- push all its adjacent vertices in the queue (which are 3 and 4)

repeat till queue is empty



Before 6th loop

2	1	3	4		visited
1	5	1	5		queue

End of 7th loop

2	1	3	4	5	visited
1	5	3	4		queue

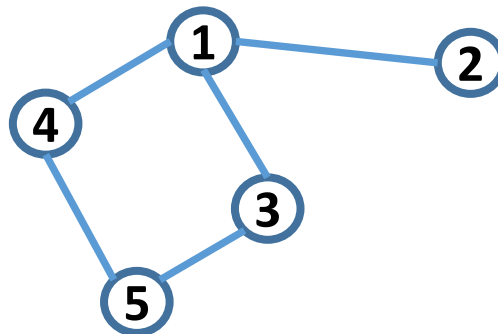
Breath First Search

Loop until queue is empty

- poll() a vertex from Queue (It is 1. It has been visited before. Skip. Go to next loop)
- poll() a vertex from Queue (It is 5. It has been visited before. Skip. Go to next loop)
- poll() a vertex from Queue (It is 3. It has been visited before. Skip. Go to next loop)
- poll() a vertex from Queue (It is 4. It has been visited before. Skip. Go to next loop)

Queue is empty. Stop.

All vertices have been visited in the order 2, 1, 3, 4 then 5.



Before 8th loop

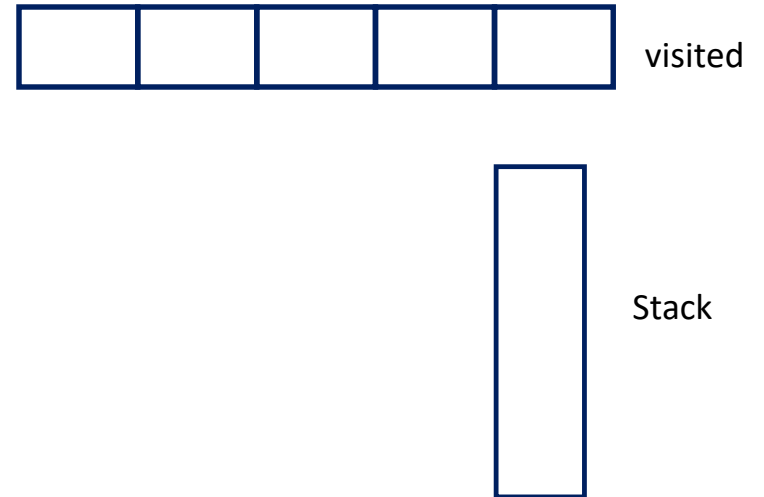
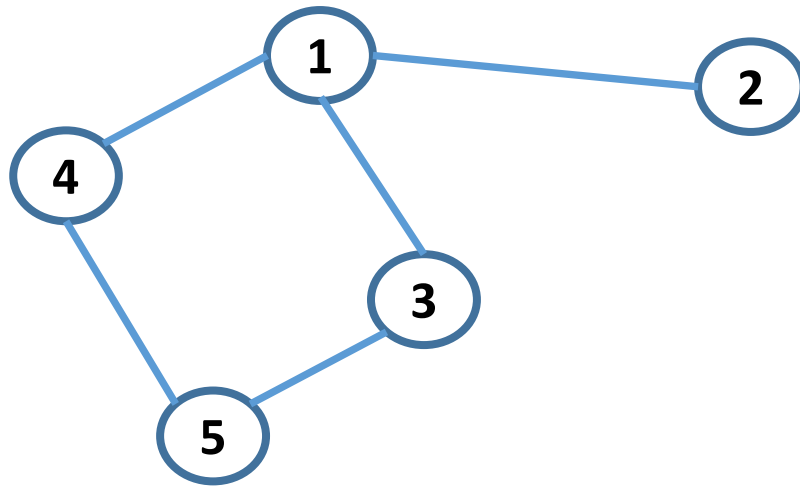
2	1	3	4	5	visited
1	5	3	4		queue

End of 11th loop

2	1	3	4	5	visited
					queue

Depth First Search

- The aim again is to visit all the vertices and every vertex can only be visited once
- Prepare a visited List (initially empty)
- Prepare a Stack (initially empty)



Breath First Search

Steps:

put the root (starting) vertex in stack

while (stack is not empty)

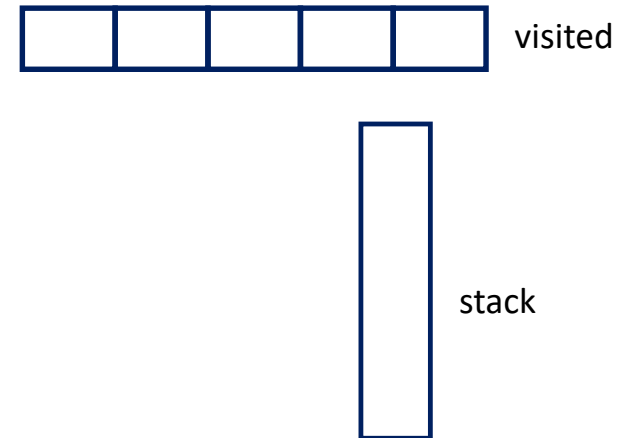
 pop() a vertex from stack

 if (vertex has not been visit before)

 process the vertex (whatever process is needed)

 push all the neighbours (from adjacent list) onto stack

repeat till stack is empty



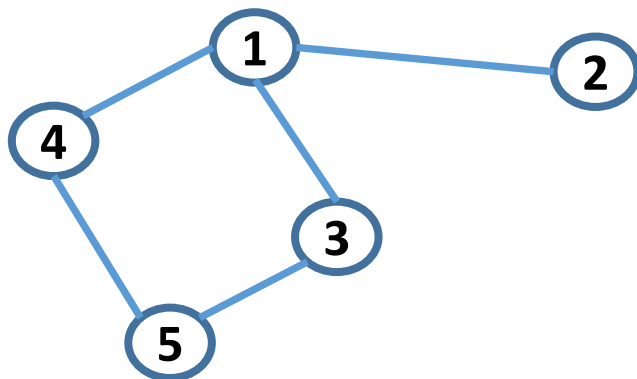
Depth First Search

- Starting from a root (say 2)
- Insert the root into Stack

Loop until stack is empty

- pop() a vertex from top of Stack (2 is only one in the stack now)
- 2 has not been visited
- mark the 2 as visited (insert 2 into visited)
- process 2 (can be as simple as print it out)
- push all its adjacent vertices onto stack (has only 1 adjacent, which is 1)

repeat till stack is empty



Before 1st loop



visited



stack

At end of 1st loop



visited



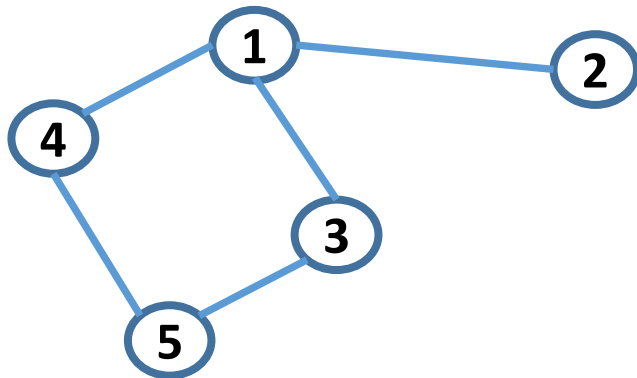
stack

Depth First Search

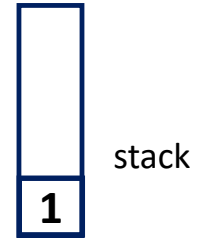
Loop until stack is empty

- pop() a vertex from top of Stack (1 is only one in the stack now)
- 1 has not been visited
- mark the 1 as visited (insert 1 into visited)
- process 1 (can be as simple as print it out)
- push all its adjacent vertices onto stack (which are 2, 3 and 4)

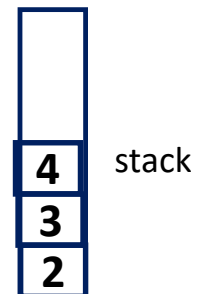
repeat till stack is empty



Before 2nd loop



At end of 2nd loop

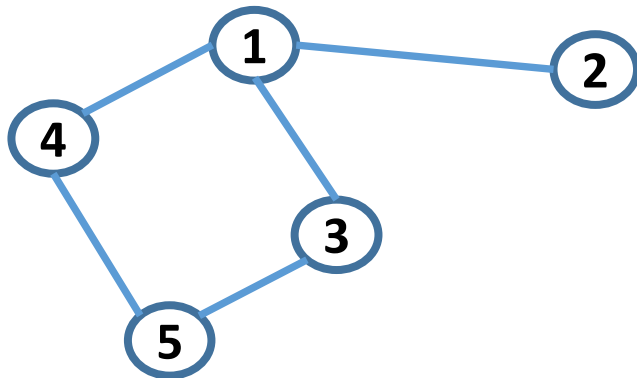


Depth First Search

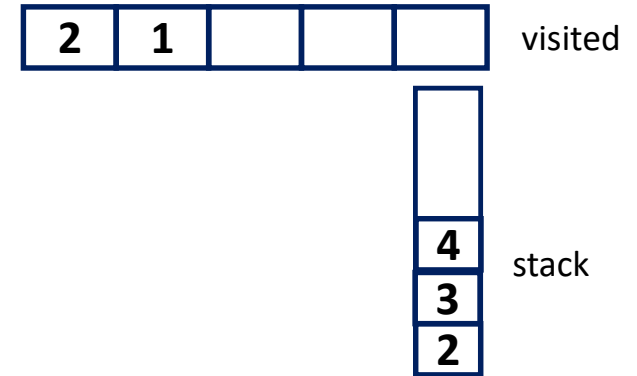
Loop until stack is empty

- pop() a vertex from top of Stack (it is 4)
- 4 has not been visited
- mark the 4 as visited (insert 4 into visited)
- process 4 (can be as simple as print it out)
- push all its adjacent vertices onto stack (which are 1 and 5)

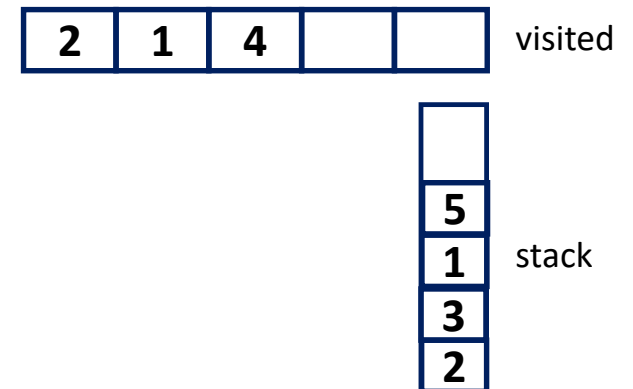
repeat till stack is empty



Before 3rd loop



At end of 3rd loop

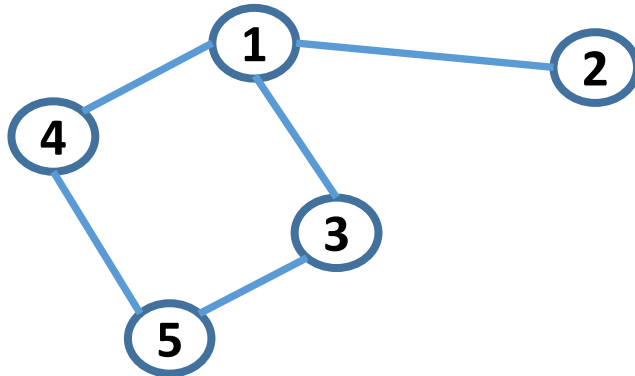


Depth First Search

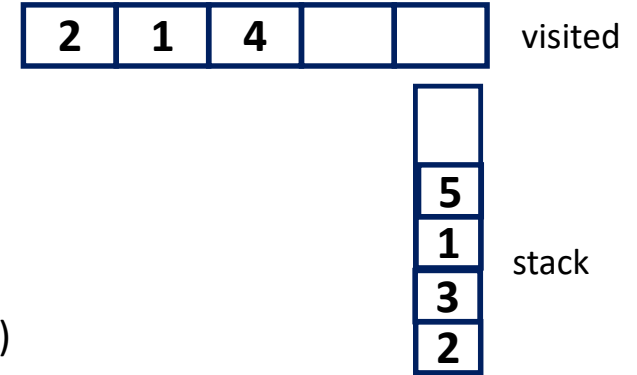
Loop until stack is empty

- pop() a vertex from top of Stack (it is 5)
- 5 has not been visited
- mark the 5 as visited (insert 5 into visited)
- process 5 (can be as simple as print it out)
- push all its adjacent vertices onto stack (which are 3 and 4)

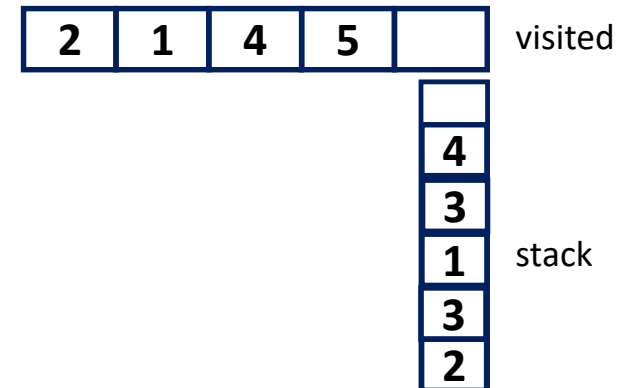
repeat till stack is empty



Before 4th loop



At end of 4th loop



Depth First Search

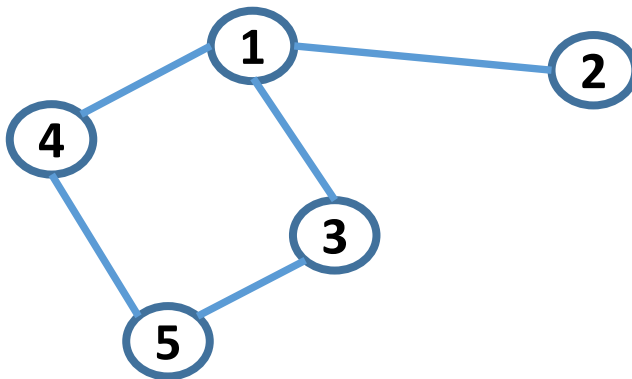
Loop until Stack is empty

- pop() a vertex from Stack (It is 4. It has been visited before. Skip. Go to next loop)

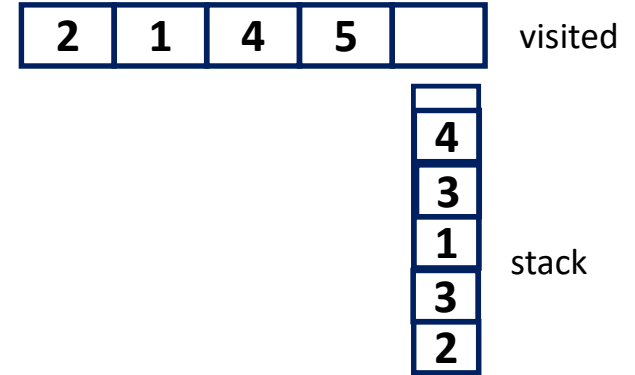
Loop until Stack is empty

- pop() a vertex from Stack (it is 3)
- 3 has not been visited
- mark 3 as visited (insert 3 into visited)
- process the vertex
- push all its adjacent vertices onto the Stack (which are 1 and 5)

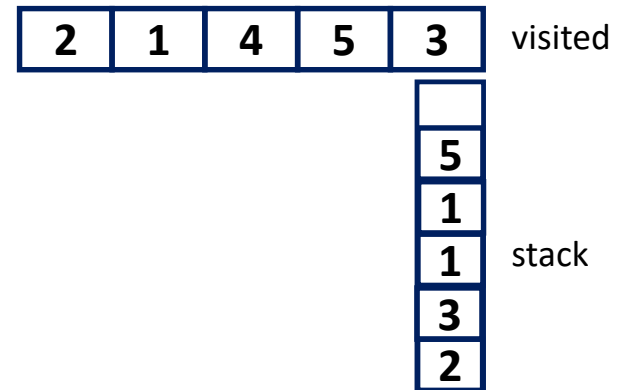
repeat till queue is empty



Before 5th loop



At end of 6th loop

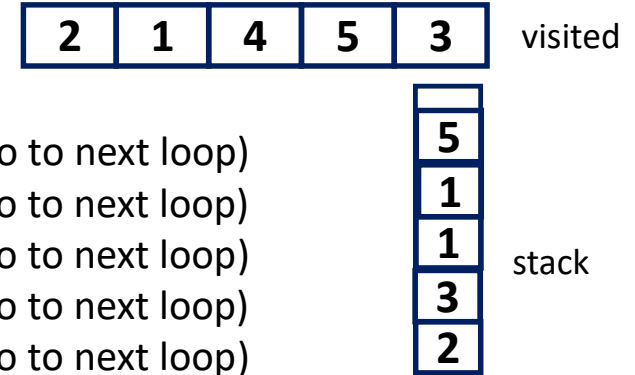


Depth First Search

Loop until queue is empty

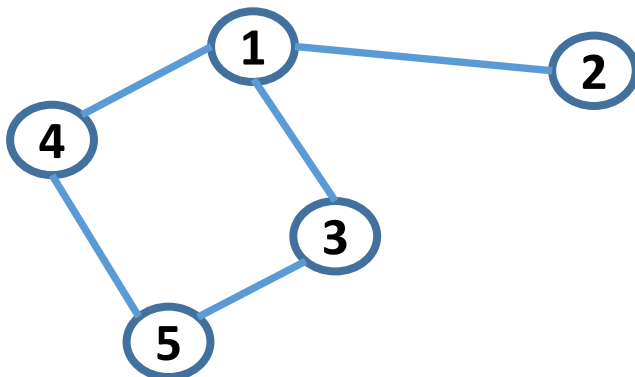
- pop() a vertex from Stack (It is 5. It has been visited before. Skip. Go to next loop)
- pop() a vertex from Stack (It is 1. It has been visited before. Skip. Go to next loop)
- pop() a vertex from Stack (It is 1. It has been visited before. Skip. Go to next loop)
- pop() a vertex from Stack (It is 3. It has been visited before. Skip. Go to next loop)
- pop() a vertex from Stack (It is 2. It has been visited before. Skip. Go to next loop)

Before 6th loop

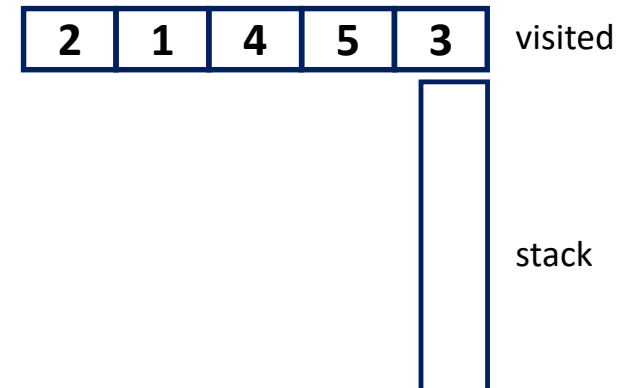


Stack is empty. Stop.

All vertices have been visited in the order 2, 1, 4, 5 then 3.

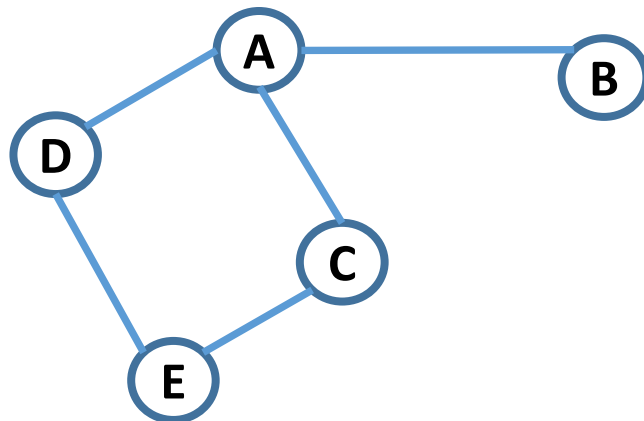


At end of 10th loop

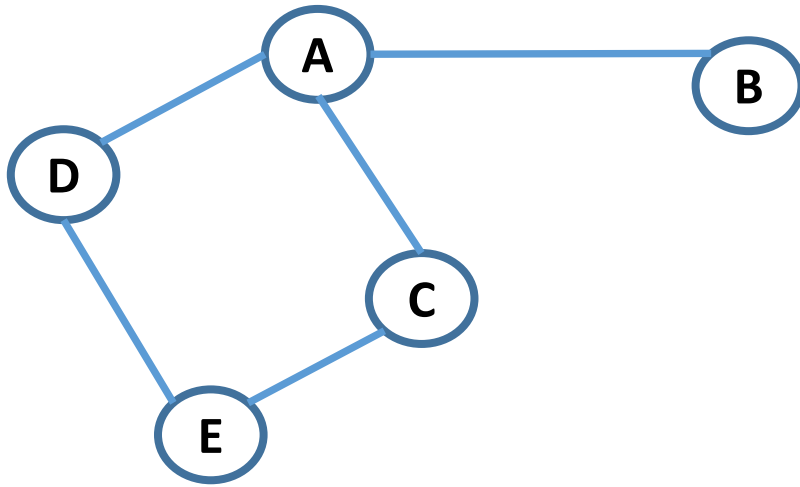


Implementing Traversal in BFS

```
class Vertex{  
    String label;  
    ArrayList<Vertex> adjList = new ArrayList<>();  
  
    Vertex (String label) {  
        this.label = label;  
    }  
}
```



Implementing Traversal in BFS

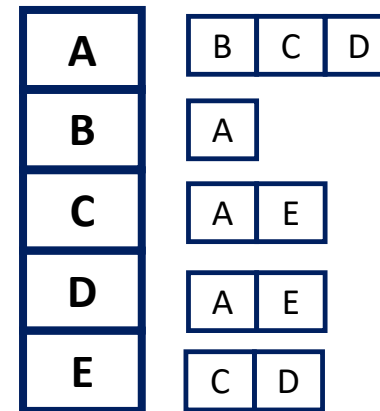


A	B	C	D
B	A		
C	A	E	
D	A	E	
E	C	D	

Implementing Traversal in BFS

Representing vertices and edges for the graph:

```
Vertex a = new Vertex("A");  
Vertex b = new Vertex("B");  
Vertex c = new Vertex("C");  
Vertex d = new Vertex("D");  
Vertex e = new Vertex("E");  
  
a.adjList.add(b);  
a.adjList.add(c); // what if d is added  
a.adjList.add(d); // before c?  
  
b.adjList.add(a);  
  
c.adjList.add(a);  
c.adjList.add(e);  
  
d.adjList.add(a);  
d.adjList.add(e);  
  
e.adjList.add(c);  
e.adjList.add(d);
```



Implementing Traversal in BFS

Prepare visited ArrayList and Queue.

Start by putting the root vertex (say B) into Queue.

```
ArrayList<Vertex> visited = new ArrayList<>();  
Queue<Vertex> q = new ArrayDeque<>();  
  
// set root  
q.add(b);
```

Implementing Traversal in BFS

Start the loop with the algorithm for BFT that we have covered.

```
while (!q.isEmpty()){
    Vertex theVertex = q.poll();
    if (!visited.contains(theVertex)){
        visited.add(theVertex);
        for (int i=0; i<theVertex.adjList.size(); i++){
            q.add(theVertex.adjList.get(i));
        }
    }
}

visited.forEach(z->System.out.print(z.label+" "));
```

Output:

B A C D E

Implementing Traversal in DFS

Instead of Queue, a Stack is used.

Start by putting the root vertex (say B) into Queue.

```
ArrayList<Vertex> visited = new ArrayList<>();  
Stack<Vertex> s = new Stack<>();  
  
// set root  
s.push(b);
```

Implementing Traversal in DFS

Similar to BFS:

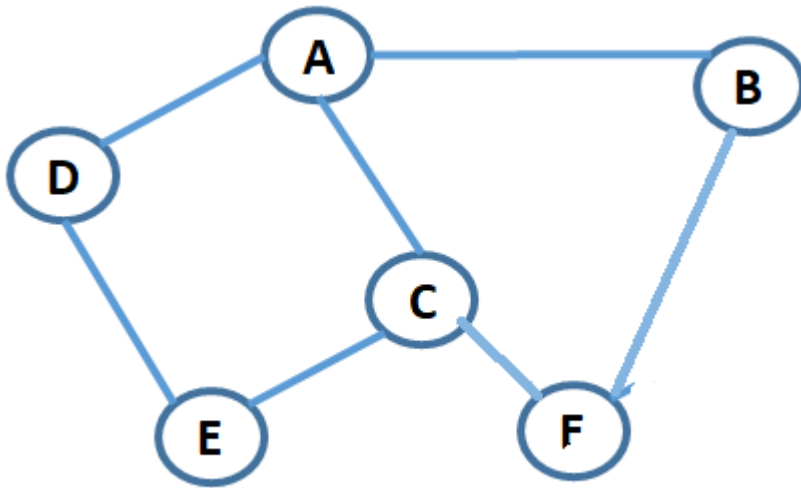
```
while (!s.isEmpty()){
    Vertex theVertex = s.pop();
    if (!visited.contains(theVertex)){
        visited.add(theVertex);
        for (int i=0; i<theVertex.adjList.size(); i++){
            s.add(theVertex.adjList.get(i));
        }
    }
}

visited.forEach(z->System.out.print(z.label+" "));
```

Output:

B A D E C

Try



```
Vertex a = new Vertex("A");  
Vertex b = new Vertex("B");  
Vertex c = new Vertex("C");  
Vertex d = new Vertex("D");  
Vertex e = new Vertex("E");  
Vertex f = new Vertex("F");
```

```
a.adjList.add(b);  
a.adjList.add(c);  
a.adjList.add(d);
```

```
b.adjList.add(a);
```

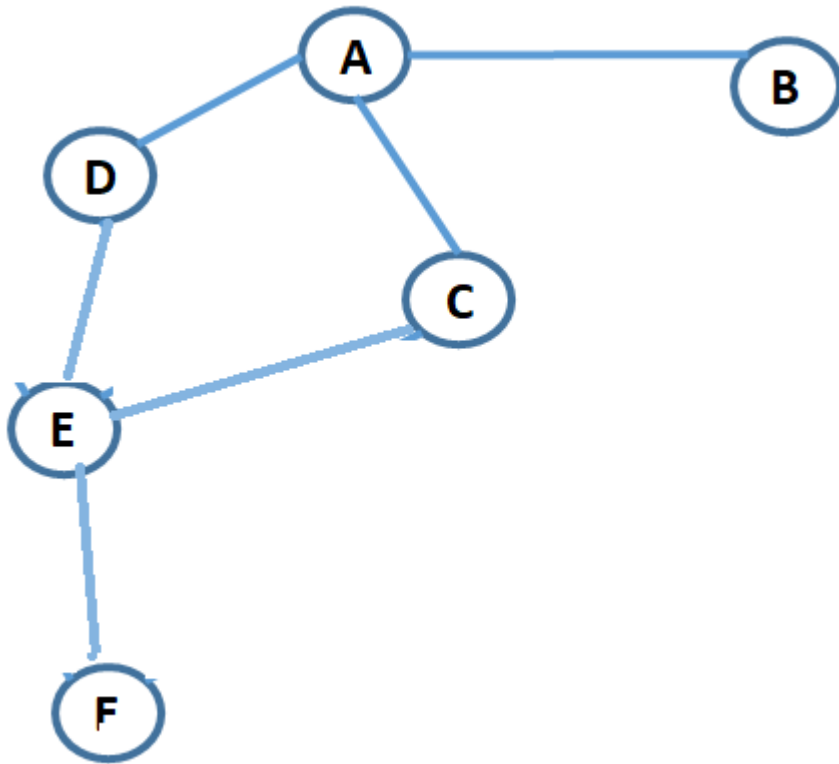
```
c.adjList.add(a);  
c.adjList.add(e);  
c.adjList.add(f);
```

```
d.adjList.add(a);  
d.adjList.add(e);
```

```
e.adjList.add(c);  
e.adjList.add(d);
```

```
f.adjList.add(c);  
f.adjList.add(b);
```

Try



```
Vertex a = new Vertex("A");  
Vertex b = new Vertex("B");  
Vertex c = new Vertex("C");  
Vertex d = new Vertex("D");  
Vertex e = new Vertex("E");  
Vertex f = new Vertex("F");
```

```
a.adjList.add(b);  
a.adjList.add(c);  
a.adjList.add(d);
```

```
b.adjList.add(a);
```

```
c.adjList.add(a);  
c.adjList.add(e);
```

```
d.adjList.add(a);  
d.adjList.add(e);
```

```
e.adjList.add(c);  
e.adjList.add(d);  
e.adjList.add(f);
```

```
f.adjList.add(e);
```