

Assignment-2

Sushma - CS20BTECH11051

Download all python codes from

https://github.com/Sushma-AI1103/AI1103-Assingment-2/blob/main/assingment_2.py

1 PROBLEM

79-Suppose the random variable U has uniform distribution on $[0,1]$ and $X = -2\ln(U)$. The density of X is

- 1) $f(x) = \begin{cases} \exp(-x) & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- 2) $f(x) = \begin{cases} 2\exp(-2x) & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- 3) $f(x) = \begin{cases} \frac{1}{2}\exp(\frac{-x}{2}) & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- 4) $f(x) = \begin{cases} \frac{1}{2} & x \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$

2 SOLUTIONS:

U - uniformly distributed random variable on $\in [0,1]$. Probability density function of U is:

$$f_U(u) = \begin{cases} 1 & u \in [0, 1] \\ 0 & \text{otherwise} \end{cases} \quad (2.0.1)$$

X is given by :

$$X = -2\ln(U) \quad (2.0.2)$$

$$\Rightarrow 0 \leq X \leq \infty \quad (2.0.3)$$

CDF of X is defined as

$$F_X(x) = \Pr(X \leq x) \quad (2.0.4)$$

$$= \Pr(-2\ln(U) \leq x) \quad (2.0.5)$$

$$= \Pr(\ln(U) \geq (-x)/2) \quad (2.0.6)$$

$$= \Pr(U \geq \exp(-x/2)) \quad (2.0.7)$$

$$= 1 - \Pr(U \leq \exp(-x/2)) \quad (2.0.8)$$

$$= 1 - \exp(-x/2) \quad (2.0.9)$$

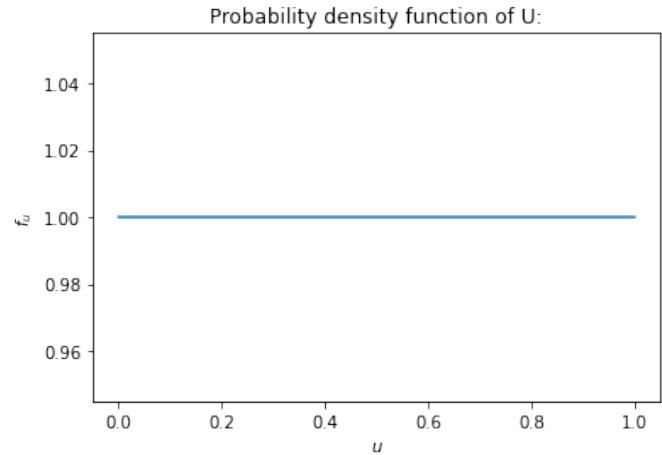


Fig. 4: PDF of U

where $x \in [0, \infty]$

PDF of X :

$$f_X(x) = \frac{d(F_X(x))}{dx} \quad (2.0.10)$$

$$= \frac{1}{2} \exp((-x)/2) \quad (2.0.11)$$

we have

$$0 \leq X \leq \infty \quad (2.0.12)$$

$$f_X(x) = \begin{cases} \frac{1}{2} \exp(\frac{-x}{2}) & x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.13)$$

\therefore answer will be option (3)

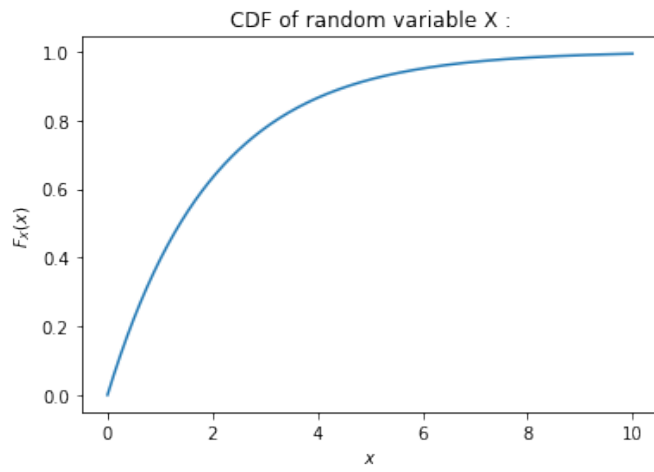


Fig. 4: CDF of X

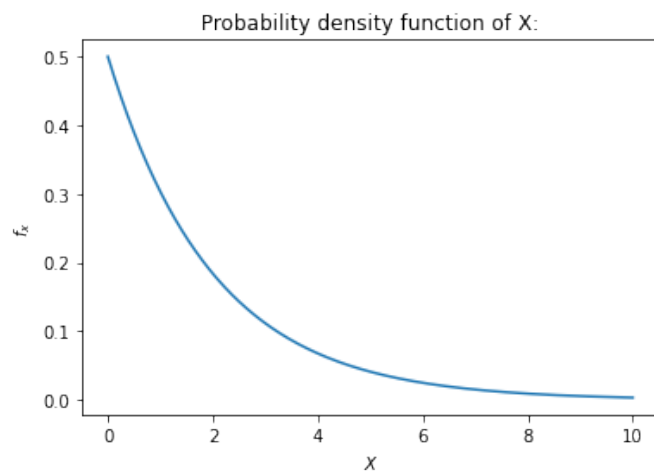


Fig. 4: PDF of X