Assignment 3

Sushma - CS20BTECH11051

1 Problem

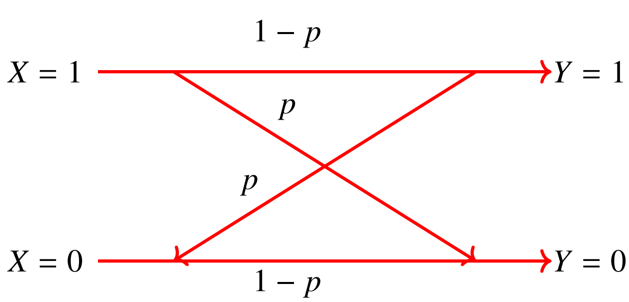
A binary symmetric channel (BSC) has a transition probability of $\frac{1}{8}$. If the binary symbol X is such that P $(X = 0) = \frac{9}{10}$, then the probability of error for an optimum receiver will be

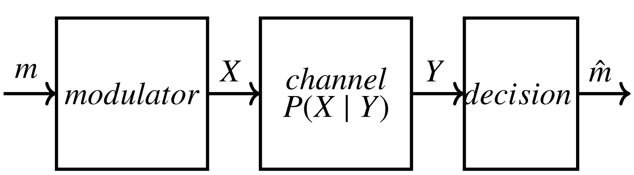
1) $\frac{7}{80}$
2) $\frac{63}{80}$
3) $\frac{63}{80}$
4) $\frac{1}{10}$

2 Solution

Probability of transition,p is given by

$$p = \frac{1}{8} \tag{2.0.1}$$
$$\Pr(X = 0) = \frac{9}{10} \tag{2.0.2}$$
$$\Pr(X = 1) = \frac{1}{10} \tag{2.0.3}$$





(here m and X can be considered similar)

$\therefore$ Probability of error is defined as

$$P_e = \Pr(\hat{m} \neq m) \tag{2.0.4}$$

Probability of being correct is defined as

$$P_c = 1 - P_e \tag{2.0.5}$$
$$= 1 - \Pr(\hat{m} \neq m) \tag{2.0.6}$$
$$= \Pr(\hat{m} = m) \tag{2.0.7}$$

Optimum detector maxmize P_c or equivalently minimize P_e
Probability of making correct decision, for a given received y

$$P_c = \Pr(\hat{m} = m) \tag{2.0.8}$$
$$= p(m_i \mid y)p(y) \tag{2.0.9}$$
$$= p(x_i \mid y)p(y) \tag{2.0.10}$$

Using Bayes theorem,

$$P_c = p(y \mid x_i)p(x_i) \tag{2.0.11}$$

To maximize P_c we use **Maximum a Posterior Detector (MAP)** rule , for a given Y

$$\hat{m} \implies m_i \;\; if \;\; \frac{p(y \mid x_i)p(x_i)}{p(y \mid x_j)p(x_j)} \geq 1 \tag{2.0.12}$$

Now , when Y = 1 then $\hat{m} = 0$ if

$$\frac{p(y = 1 \mid x = 0)p(x = 0)}{p(y = 1 \mid x = 1)p(x = 1)} \geq 1 \tag{2.0.13}$$
$$\implies \frac{p(y = 1 \mid x = 0)p(x = 0)}{p(y = 1 \mid x = 1)p(x = 1)} \tag{2.0.14}$$
$$= \frac{\frac{1}{8} \cdot \frac{9}{10}}{\frac{7}{8} \cdot \frac{1}{10}} \tag{2.0.15}$$
$$= \frac{9}{7} \geq 1 \tag{2.0.16}$$

when Y = 0 then $\hat{m} = 0$ if

$$\frac{p(y = 0 \mid x = 0)p(x = 0)}{p(y = 0 \mid x = 1)p(x = 1)} \geq 1 \tag{2.0.17}$$
$$\implies \frac{p(y = 0 \mid x = 0)p(x = 0)}{p(y = 0 \mid x = 1)p(x = 1)} \tag{2.0.18}$$
$$= \frac{\frac{7}{8} \cdot \frac{9}{10}}{\frac{1}{8} \cdot \frac{1}{10}} \tag{2.0.19}$$
$$= 63 \geq 1 \tag{2.0.20}$$

In both cases MAP detector suggest that message will be $\hat{m} = 0$

$\therefore$ probability of error

$$P_e = \Pr(\hat{m} \neq 0 \mid X = 0)\Pr(X = 0) + \Pr(\hat{m} \neq 1 \mid X = 1)\Pr(X = 1) \tag{2.0.21}$$

$$= 0 + 1 \cdot \frac{1}{10} \tag{2.0.22}$$

$$= \frac{1}{10} \tag{2.0.23}$$

So answer will be (D)