Assignment-5

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Download all python codes from

https://github.com/Sushma−AI1103/Assignment−1/blob/main/Assingment−5/simulation.py

1 Problem-gov/stats/2015/ques-1(e)

Using Central Limit theorem, show that

$$e^{-n}\sum_{k=0}^{n}\frac{n^k}{k!}=\frac{1}{2} \tag{1.0.1}$$

2 Solution

Let a discrete rv X having poission distribution , then **PMF** is given by

$$f(k;\lambda)=\Pr(X=k)=\frac{\lambda^k e^{-\lambda}}{k!} \tag{2.0.1}$$

Let X_1,X_2,X_3......X_n be the i.i.d rv with $X_i \sim \text{Pois}(1)$.

$$E(X_i)=\mu=\lambda=1 \tag{2.0.2}$$
$$var(X_i)=\sigma^2=\lambda=1 \tag{2.0.3}$$
$$\tag{2.0.4}$$

Let a random variable,

$$X=X_1+X_2+......+X_n \tag{2.0.5}$$
$$\implies X\sim Pois(n) \tag{2.0.6}$$

As CLT states that summation of i.i.d rvs from any distribution are normally distributed as sample size increases . For X

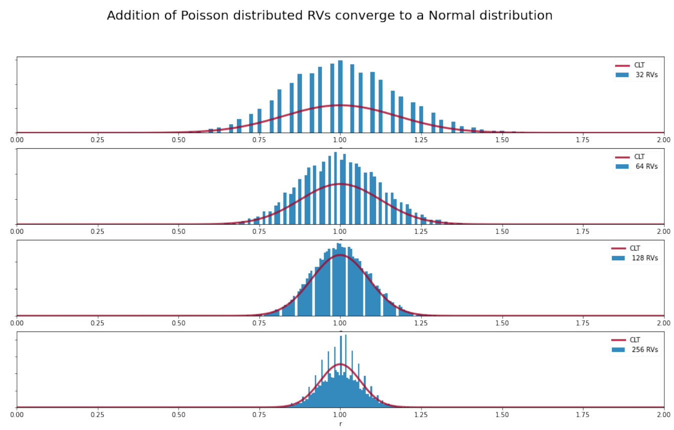
$$\lambda=n \tag{2.0.7}$$
$$\Pr(X=k)=\frac{n^k e^{-n}}{k!} \tag{2.0.8}$$

Now consider a another random variable, Z_n

$$Z_n=\frac{X-n\mu}{\sqrt{n\sigma^2}} \tag{2.0.9}$$

Then **CLT** states that Z_n converges to standard normal distribution as n goes to infinity i.e

$$\Pr(Z_n\le x)\xrightarrow{n\to\infty}=\Phi(x) \tag{2.0.10}$$



$$\implies e^{-n}\sum_{k=0}^{n}\frac{n^k}{k!}=\sum_{k=0}^{n}e^{-n}\frac{n^k}{k!} \tag{2.0.11}$$
$$=\sum_{k=0}^{n}\Pr(X=k) \tag{2.0.12}$$
$$=\Pr(X\le n) \tag{2.0.13}$$
$$=\Pr\left(\frac{X-n\mu}{\sqrt{n\sigma^2}}\le\frac{n-n\mu}{\sqrt{n\sigma^2}}\right) \tag{2.0.14}$$
$$=\Pr(Z_n\le 0) \tag{2.0.15}$$

Using (2.0.10),

$$\Pr(Z_n\le 0)\xrightarrow{n\to\infty}\Phi(0)=\frac{1}{2} \tag{2.0.16}$$