

Assignment-5

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Download all python codes from

<https://github.com/Sushma-AI1103/Assignment-1/blob/main/Assingment-5/simulation.py>

1 PROBLEM-GOV/STATS/2015/QUES-1(E)

Using Central Limit theorem, show that

$$e^{-n} \sum_{k=0}^n \frac{n^k}{k!} = \frac{1}{2} \quad (1.0.1)$$

2 SOLUTION

Definition 1. Let a discrete rv X having poisson distribution , then **PMF** is given by

$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (2.0.1)$$

Let $X_1, X_2, X_3, \dots, X_n$ be the i.i.d rv with $X_i \sim \text{Pois}(1)$.

$$E(X_i) = \mu = \lambda = 1 \quad (2.0.2)$$

$$\text{var}(X_i) = \sigma^2 = \lambda = 1 \quad (2.0.3)$$

$$(2.0.4)$$

Let a random variable,

$$X = X_1 + X_2 + \dots + X_n \quad (2.0.5)$$

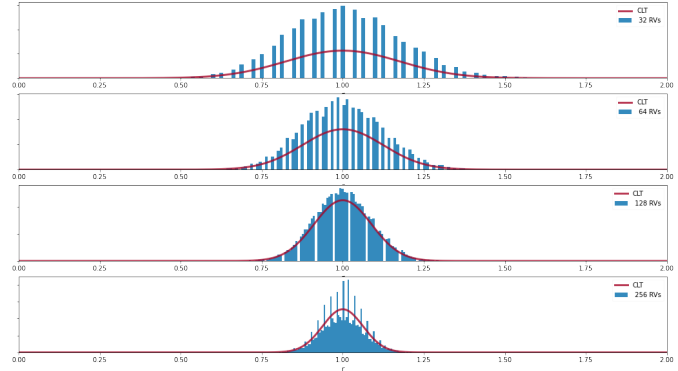
$$\implies X \sim \text{Pois}(n) \quad (2.0.6)$$

Theorem 2.1 (Classical central limit theorem). Let X_n be a sequence of independent, identically distributed (i.i.d.) random variables. Assume each X has finite mean, $E(X) = \mu$, and finite variance, $\text{Var}(X) = \sigma^2$. Let Z_n be the normalized average of the first n random variables.

$$Z_n = \frac{X - n\mu}{\sqrt{n\sigma^2}} \quad (2.0.7)$$

then Z_n converges in distribution to a standard normal distribution

Addition of Poisson distributed RVs converge to a Normal distribution



For X

$$\lambda = n \quad (2.0.8)$$

$$\Pr(X = k) = \frac{n^k e^{-n}}{k!} \quad (2.0.9)$$

By theorem , Z_n converges to standard normal distribution as n goes to infinity i.e

$$\Pr(Z_n \leq x) \xrightarrow{n \rightarrow \infty} \Phi(x) \quad (2.0.10)$$

Proof.

$$\implies e^{-n} \sum_{k=0}^n \frac{n^k}{k!} = \sum_{k=0}^n e^{-n} \frac{n^k}{k!} \quad (2.0.11)$$

$$= \sum_{k=0}^n \Pr(X = k) \quad (2.0.12)$$

$$= \Pr(X \leq n) \quad (2.0.13)$$

$$= \Pr\left(\frac{X - n\mu}{\sqrt{n\sigma^2}} \leq \frac{n - n\mu}{\sqrt{n\sigma^2}}\right) \quad (2.0.14)$$

$$= \Pr(Z_n \leq 0) \quad (2.0.15)$$

Using (2.0.10),

$$\Pr(Z_n \leq 0) \xrightarrow{n \rightarrow \infty} \Phi(0) = \frac{1}{2} \quad (2.0.16)$$

□