Name (Last, First):
Kunjangada Arun Sushma

NEU ID:
002473132

Assignment 9

NEU_COE_INFO6105_Fall2024

Instructions:

1. For answering **programming questions**, please use Adobe Acrobat to edit the pdf file in two steps **[See Appendix: Example Question and Answer]**:
    a. Copy and paste your R or Python code as text in the box provided (so that your teaching team can run your code);
    b. Screenshot your R or Python console outputs, save them as a .PNG image file, and paste/insert them in the box provided.
    c. Show all work—credit will not be given for code without showing it in action, including a screenshot of R or Python console outputs.
2. To answer **non-programming questions**, please type or handwrite your final answers clearly in the boxes. Show all work - credit will not be given for numerical solutions that appear without explanation in the space above the boxes. **You're encouraged to use R or Python to graph/plot the data and produce numerical summaries; please <u>append</u> your code and screenshot of the outputs at the end of your PDF submission.**
3. **[Total 78 pts = 75 pts + 3 Extra Credit pts]**

Grading Rubric

Each question is worth 3 points and will be graded as follows:

3 points: Correct answer with work shown

2 points: Incorrect answer but attempt shows some understanding (work shown)

1 point: Incorrect answer but an attempt was made (work shown), or **correct answer without explanation (work not shown)**

0 points: Left blank or made little to no effort/work not shown

Reflective Journal [3 pts]

(Copy and paste the link to your live Google doc in the box below)

https://docs.google.com/document/d/1ptEhnYHniNtT1yxDPcvXK7LpJaPGzi80BCSGZZhom7Y/edit?usp=sharing

I. Confidence Intervals for Proportions (7 x 3 = 21 pts)

1) A live action role-playing game (LARP) is a form of role-playing game where the participants physically portray their characters in a fictional setting, while interacting with each other in character. There are game rules to dictate the interactions between players and to determine the winner of the games.

A local gamemaster would like to determine how many people in the area would be interested in participating in his LARPing event. He sent out a survey to 1500 people in the area and asked if they would be interested in participating. 432 people said that they would be interested in participating in a LARPing event.

(a) State the parameter our confidence interval will estimate.

Answer: The parameter is the actual proportion of people in the area who would be intrested in participating in the LARPing event.

$p = \frac{432}{1500} = 0.288$

(b) Identify each of the conditions that must be met to use this procedure, and explain how you know that each one has been satisfied.

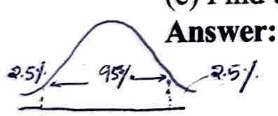
Answer: Random Sample: We assume that 1500 people we randomly selected from the area.

Independence: With a large population, we can assume individual respones are independent.

Normal: $np \geq 10$ and $n(1-p) \geq 10$. $p = 432/1500 = 0.288$

$n \times p = 1500 \times 0.288 = 432 \geq 10$. $n(1-p) = 1500(1-0.288) = 1068 \geq 10$. So the condition is Satisfied.

(c) Find the appropriate critical value and the standard error of the sample proportion.



$95 + 2.5 = 97.5\%$

0.975

Answer: For 95% confidence interval, the critical value (z^*) is 1.96 (Z score corresponding to 0.975)

Sample proportion $(\hat{p}) \Rightarrow \hat{p} = \frac{432}{1500} = 0.288$

Standard Error $(SE) \Rightarrow SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.288 \times (1-0.288)}{1500}} = 0.0116920486 \simeq 0.0117$

(d) Give the 95% confidence interval.

Answer: Confidence interval: $\hat{p} \pm z^* SE \Rightarrow \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.288(1-0.288)}{1500}} \simeq 0.0117$

$\hat{p} \pm z^* SE$

$= 0.288 \pm 1.96 \times 0.0117 = 0.288 \pm 0.022932 \simeq 0.288 \pm 0.0229$

95% confidence interval: $(0.288 - 0.0229, 0.288 + 0.0229) = (0.2651, 0.3109)$

(e) Interpret the confidence interval constructed in part (d) in the context of the problem.

Answer: We are 95% confident that the true proportion of people in the area who would be interested in participating in a LARPing event between (26.51% and 31.09%) 0.2651 to 0.3109

(f) This poll was conducted through email. Explain how undercoverage could lead to a biased estimate in this case, and speculate about the direction of the bias.

Answer:

Undercoverage bias due to email polling not covering certain group.
- Not everyone has email access, particularly older populations.
- Some people may not check their email regularly.
- Young people who are connected to the internet will be over represented in the pool.

2) The EPA wants to conduct a survey to determine the current approval rating on the government's handling of environmental action. From previous studies, they know that they need to poll 275 people to achieve their desired level of confidence. If they want to keep the same level of confidence but divide the margin of error in third, how many people will they have to poll?

Answer:

Margin of Error (ME):

$$ME = Z* SE = Z* \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}}$$

where $n_1 = 275$

$$\text{New } ME = \frac{(\text{Original } ME)}{3}$$

New Sample size $= n_2$

$$Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}} = 3*Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n_2}}$$

$$Z*\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n_1}} = 3*Z\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n_2}}$$

$$\frac{1}{\sqrt{n_1}} = 3\frac{1}{\sqrt{n_2}}$$

$$\sqrt{n_2} = 3\sqrt{n_1} \quad \text{(Square both sides)}$$

$$n_2 = 9n_1$$

$$n_2 = 9 \times 275$$

$$= \underline{2475} \text{ people}$$

∴ They will need to poll 2475 people to reduce the margin of error to one third of the original value while maintaining the same confidence level.

II. Significance Test for Proportions (7 x 3 = 21 pts)

1) Apple runs a quality control check on its products that come off the assembly line. Every 500th product is inspected for defects to make sure that the machines are calibrated correctly and working like they should. Due to machine deviations, they do expect 5% of products to be defective in a single day. The quality control specialist has to determine if a significant number of products is coming off the line defective.

(a) Define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

Answer:

The parameter of interest (p) is the true proportion of defective products that come off the assembly line in a single day.

Null hypothesis (H_0): The proportion of defective products is, $p = 0.05$

Alternative Hypothesis (H_a): The proportion of defective products is greater than, $p > 0.05$

The quality control specialist takes an SRS of products and finds that the sample proportion of defective products is 0.058, which produces a P-value of 0.027.

(b) Interpret the P-value in the context of the problem.

Answer:

Assuming that the true proportion of defective products is actually 5% ($p = 0.05$), the probability of observing a sample proportion of 0.058 or more extreme is 0.027 or 2.7%.

(c) What conclusion would you draw at the $\alpha = 0.05$ level? At the $\alpha = 0.01$ level?

Answer:

- At $\alpha = 0.05$: Since p value $(0.027) < \alpha\ (0.05)$, we reject null hypothesis (H_0)
- There is significant evidence that the proportion of defective products is greater than 5%.
- At $\alpha = 0.01$: Since Pvalue $(0.027) > \alpha\ (0.01)$ we do not reject the null hypothesis (H_0).
- We don't have sufficient evidence to conclude that the defect rate is significantly higher at 1% level.

2) According to the CD website, 88% of US adults aged 65 – 74 are fully vaccinated against COVID-19 (as of 11/2021). A researcher suspects that this proportion is lower in their community. She selects an SRS of 125 adults aged 65 – 74 in her community and asks if they are fully vaccinated. 100 of them say that they are fully vaccinated. Does the researcher have convincing evidence that the proportion is lower in her community? Perform a significance test using a 5% level of significance. **Explain using the 4 step process.**

Answer:

Step 1.

State the hypothesis:

Parameter of interest (p): The true proportion of fully vaccinated adults aged 65-74 in research community.

Null Hypothesis (H_0) $\Rightarrow p = 0.88$ (Vaccination rate in community same as national rate)

Alternative Hypothesis (H_a): $p < 0.88$ (vaccination rate in community is lower than the national rate)

Step 2.

Random Sample: The researcher selected a SRS of 125 adults.

Normal: $np \geq 10$ & $n(1-p) \geq 10$

$125 \times 0.88 = 110 \geq 10$

$125 \times (1-0.88) = 125(0.12) = 15 \geq 10$

Both are greater than 10, sample size is large enough

Independence: We assume population is large enough.

Step 3.

Sample proportion ($\hat{p}$): $\hat{p} = \frac{100}{125} = 0.80$

Standard Error (SE): $SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.88 \times 0.12}{125}} = 0.0290654434 \approx \underline{0.0291}$

Z Score:

$z = \frac{\hat{p} - p}{SE} = \frac{0.80 - 0.88}{0.0291} = -2.74914089 \approx \underline{-2.75}$

Step 4.

A Z Score of -2.75 corresponds to p-value of 0.0030

At $\alpha = 0.05$

$0.003 < 0.05$ we reject the null hypothesis

We have convincing evidence that the proportion of fully vaccinated adults aged 65-74 in the researcher's community is lower than reported national rate of 88%.

III. Errors and Power (3 x 3 = 9 pts)

Your local Starbucks is hoping to cut back on the wait time for their customers. The proportion of drive-through customers who wait longer than 3 minutes to get their coffee is p = 0.76. In an effort to reduce this, the manager assigns an extra employee to help make the orders in the morning. During the next month, the manager will select an SRS of customer wait times and determine if the proportion who are waiting longer than 3 minutes has decreased.

a) What is the parameter of interest in this problem?

Answer:

The parameter of interest is the true proportion of drive-Through customers who wait longer Than 3 minutes to get Their coffee after adding the extra employee.
$p = 0.76$

b) State the null and alternative hypotheses.

Answer:

Hypothesis:

Null Hypothesis (H_0): The proportion of customers who wait longer than 3 minutes has not decreased with extra employee. $H_0: p = 0.76$

Alternative Hypothesis: The proportion of customers who wait longer Than 3 minutes has decreased with the extra employee. $H_a: p < 0.76$

c) Describe a Type I and Type II error in this setting and explain the consequences of each.

Answer:

Type I Error: • Rejecting H_0 (null hypothesis) when it is actually true.

In this context: Concluding that the propotion of customers waiting more than 3 minutes has decreased when it actually hasn't changed.

Consequences: The manager might:

- Incorrectly belive the extra employee is helping.
- Keep paying for unnecessary extra labor costs
- Make incorrect staffing decisions based on false positive results.

Type II Error: • Failing to reject H_0 (null hypothesis) when it's actually false

In This context: Failing to detect that the proportion of customers waiting more than 3 minutes has actually decreased.

Consequences: The manager might:

- Miss evidence that the intervention is working.
- Unnecessary change or abandon an effective strategy.
- Lose oppurnity to improve customer satisified.
- Make incorrect staffing decisions based on false negative results.

IV. Confidence Intervals and Significance Tests (8 x 3 = 24 pts)

1) Is the ratio of males and females that attend a large university even? An SRS of 254 students that attend the college is taken and 145 of them were female. A 99% confidence interval for p = the proportion of females that attend the university is given by (0.491, 0.651)

(a) Use the confidence interval to draw a conclusion about the hypothesis H_0: p = 0.5 vs the alternative H_a: p ≠ 0.5.Be sure to indicate the appropriate significance level.

Answer:

Using 99% confidence interval (0.491, 0.651) to test hypothesis.
Null Hypothesis (H_0) ⇒ $H_0: p = 0.5$
Alternative Hypothesis (H_a) ⇒ $H_a: p \neq 0.5$
Since the 99% confidence interval (0.491, 0.651) includes 0.5, we fail to reject the null hypothesis at the 1% significance level.
This means we don't have strong evidence to say the ratio of males and females is even.

(b) What information is provided by the confidence interval that would not be provided by a test of significance alone?

Answer:

A confidence interval provides a range of plausible values for the true proportion of females attending the university, not just a single decision about whether it equals 0.5.
For example: the confidence interval (0.491, 0.651) tells us that we are 99% confidence the true proportion of female students falls between 49.1% and 65.1%.
A significance test alone would only tell us whether to reject or fail to reject the hypothesis $H_0: p = 0.05$, without providing this range.
Thus, the confidence interval gives more context by indicating the ~~proportion~~ potential values of the true proportion, helping us understand where the actual proportion likely falls relative to 0.5

2) In political campaigns, an attack ad ("mudslinging") is an advertisement whose message is designed to personally attack an opposing candidate in order to gain support for the candidate waging the attack. A recent candidate running for governor noticed his support in the polls had slipped, and only 45% of voters now supported him. His campaign ran the attack ad, hoping to boost his voter approval. Two weeks after the ad ran, the poll was run again and it found that out of the SRS of 450 voters, 233 supported the candidate. The 95% confidence interval for the proportion of voters who support him is (0.472, 0.564).

(a) Suppose his pollsters conducted a test of H_0: $p = 0.45$ vs the alternative H_a: $p \neq 0.45$, using $\alpha = 0.05$. Use the confidence interval to determine whether this test would reject or fail to reject the null hypothesis. Explain your reasoning.

Answer:

The candidate's support was 45% before an attack ad. After the ad, a poll with a sample of 450 voters showed that 233 support the candidate. The 95% confidence interval for support is (0.472, 0.564).

Null Hypothesis (H_0): The true support proportion $p = 0.45$

Alternative Hypothesis (H_a): The true support proportion $p \neq 0.45$.

The 95% confidence interval (0.472, 0.564) does not include 0.45, so we reject the null hypothesis at the 5% significance level.

(b) Find the P-value for the test described in part (a), and explain what it measures in the context of the problem.

Answer:

Sample proportion: $\hat{p} = \frac{233}{450} = 0.51777778 \simeq \underline{0.518}$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.518 - 0.45}{\sqrt{\frac{0.45 \times 0.55}{450}}} = \frac{0.068}{\sqrt{0.00055}} = 2.89952917424 \simeq \underline{2.89}$$

For $z = 2.89$, corresponds to a one-tail probability

For 2 tail the propability: P-value $= 2 \times 0.019 = 0.0038$

If the true proportion of voters supporting the candidate were 0.45, the probability of observing a sample proportion of 0.518 a more extreme is 0.0038 (or 0.38%.)

3) Netflix relies heavily on viewers when it comes to deciding if they are going to make another season of their shows. A recent article stated that Netflix has said they will only continue to make another season of a show if it has been watched by over 20% of its viewers in the first month of the show coming out. Recently, Marvel's Daredevil was canceled. Let p = the true proportion of viewers who watch the show.

(a) State the null and alternative hypotheses for this test.

Answer:

Let p represent true proportion of Netflix viewers who watched the show within the first month of its release.

Null Hypothesis (H_0): $p = 0.20$

Alternative Hypothesis (H_a): $p < 0.20$

(b) Describe a Type I and Type II error in this context, and explain the consequences of each type of error.

Answer:

Type I Error:

- Concluding that more than 20% watched the show when in fact only 20% or fewer did.
- Consequences: Netflix might invest in a new season that doesn't have enough viewers, leading to financial loss.

Type II Error:

- Failing to detect that more than 20% watched, so Netflix cancels a show with good viewership.
- Consequences: Netflix might miss out on continuing a popular show, potentially losing revenue and disappointing viewers.

(c) The network determines that for a sample size of 2000, the power of this test at a 5% significance level for H_0: $p = 0.20$ is only 0.39. Explain what the power of the test measures in the context of the problem.

Answer:

Power of 0.39 means that if the true viewership proportion is above 20%, there's only a 39% chance of correctly detecting this and continuing the show. This indicates a high risk of incorrectly canceling shows that actually meet the viewership thershhold.

(d) Based on your answers to (b) and (c), would $\alpha = 0.10$ or $\alpha = 0.01$ be a better significance level for this test? Explain your choice.

Answer:

* Based on the low power (0.39) and the consequence of errors, $\alpha = 0.10$ would be a better significance level for this test.
- Increasing α to 0.10 would increase the power of the test, making it more likely to detect when a show truly falls below the 20% thershold.
- While this increases the risk of Type I errors, it reduces the risk of Type II errors.

THE END