CHAPTER-1
RELATIONS AND FUNCTIONS

EXERCISE 1.4

1. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

(i) On $\mathbb{Z}^+$, define $a * b = a - b$

(ii) On $\mathbb{Z}^+$, define $a * b = ab$

(iii) On $\mathbb{R}$, define $a * b = ab^2$

(iv) On $\mathbb{Z}^+$, define $a * b = |a - b|$

(v) On $\mathbb{Z}^+$, define $a * b = a$

2. For each operation $*$ defined below, determine whether it is binary, commutative, or associative.

(i) On $\mathbb{Z}$, define $a * b = a - b$

(ii) On $\mathbb{Q}$, define $a * b = ab + 1$

(iii) On $\mathbb{Q}$, define $a * b = \frac{ab}{2}$

(iv) On $\mathbb{Z}^+$, define $a * b = 2^{ab}$

(v) On $\mathbb{Z}^+$, define $a * b = a^b$

(vi) On $\mathbb{R} - \{-1\}$, define $a * b = \frac{a}{b+1}$

3. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by

$$a \wedge b = \min(a, b).$$

Write the operation table of the operation $\wedge$.

4. Consider a binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table (Table 1.2).

(i) Compute $(2 * 3) * 4$ and $2 * (3 * 4)$.

(ii) Is $*$ commutative?

$*$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	3
4	1	2	1	4	1
5	1	1	3	1	5

(iii) Compute $(2 * 3) * (4 * 5)$.

5. Let $*'$ be the binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by

$$a *' b = \text{H.C.F. of } a \text{ and } b.$$

Is the operation $*'$ same as the operation $*$ defined in Exercise 4 above? Justify your answer.

6. Let $*$ be the binary operation on $\mathbb{N}$ given by $a * b = \text{L.C.M. of } a$ and b. Find

(i) $5 * 7$, $20 * 16$

(ii) Is $*$ commutative?

(iii) Is $*$ associative?

(iv) Find the identity of $*$ in $\mathbb{N}$

(v) Which elements of $\mathbb{N}$ are invertible for the operation $*$?

7. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a * b = \text{L.C.M. of } a$ and b a binary operation? Justify your answer.

8. Let $*$ be the binary operation on $\mathbb{N}$ defined by $a*b = \text{H.C.F. of } a$ and b. Is $*$ commutative? Is $*$ associative? Does there exist an identity for this binary operation on $\mathbb{N}$?

9. Let $*$ be a binary operation on the set $\mathbb{Q}$ of rational numbers as follows:

(i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

10. Find which of the operations given above has identity.

11. Let $A = \mathbb{N} \times \mathbb{N}$ and $*$ be the binary operation on A defined by

$$(a, b) * (c, d) = (a + c, b + d).$$

Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

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12. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation $*$ on a set $\mathbb{N}$, $a * a = a$ for all $a \in \mathbb{N}$.

(ii) If $*$ is a commutative binary operation on $\mathbb{N}$, then $a * (b * c) = (c * b) * a$.

13. Consider a binary operation $*$ on $\mathbb{N}$ defined as $a * b = a^3 + b^3$. Choose the correct answer.

(A) Is $*$ both associative and commutative?

(B) Is $*$ commutative but not associative?

(C) Is $*$ associative but not commutative?

(D) Is $*$ neither commutative nor associative?