# TS

March 25, 2018

In [1]:
```
import numpy as np
from math import ceil,log,log10,sqrt,exp
import matplotlib.pyplot as plt
from scipy.stats import beta
import pandas as pd
import dill
```

In [2]:
```
# Saving & Loading Variables
filename = 'globalsave.pkl'
# dill.load_session(filename)
```

In [3]:
```
# for reproducibility
np.random.seed(1234)
```

In [4]:
```
# UCB Implementation given horizon (time steps), #replications, True arm means & Type 
# For plotting average % Optimal arm pulls
def UCB(horizon,replications,arms_prob,ucbtype,optimalpulls):

    optimal_arm = 0

    optimal_arm_pulls_per_round = np.zeros([horizon,replications]) # Stores % optimal 
    regret_per_round = np.zeros([horizon,replications]) # Stores regret for every time

    for r in range(replications):
        arm_means = [0]*len(arms_prob) # Initializing arm means & pulls to 0
        arm_pulls = [0]*len(arms_prob)
        t = 0
        #initially playing each arm once
        for i in range(len(arms_prob)):
            arm_pulls[i]+=1
            temp = np.random.binomial(1,arms_prob[i])
            arm_means[i] += (temp - arm_means[i])/arm_pulls[i] # Updating arm means es
            if optimalpulls == 'Avg':
                optimal_arm_pulls_per_round[t][r] = arm_pulls[optimal_arm]*100.0/(t+1)
            else:
                if i == optimal_arm: # Incrementing % optimal arm pulls if current arm
                    optimal_arm_pulls_per_round[t][r] += 1
            regret_per_round[t][r] = (arms_prob[optimal_arm] - arms_prob[i]) # Storing
```

```python
            t+=1

        while t < horizon:
            #Picking arm according to UCB algorithm or UCB' algorithm
            if ucbtype == 1:
                UCBEstimate = arm_means + np.sqrt(2*np.log(t)/arm_pulls)
                arm_selected = np.argmax(UCBEstimate)
            else:
                UCBEstimate = arm_means + np.sqrt(2*np.log(horizon)/arm_pulls)
                arm_selected = np.argmax(UCBEstimate)

            arm_pulls[arm_selected] += 1
            temp = np.random.binomial(1, arms_prob[arm_selected]) # Updating arm means
            arm_means[arm_selected] += (temp - arm_means[arm_selected]) /arm_pulls[arm_
            regret_per_round[t][r] = (arms_prob[optimal_arm] - arms_prob[arm_selected]
            if optimalpulls == 'Avg':
                optimal_arm_pulls_per_round[t][r] = arm_pulls[optimal_arm]*100.0/(t+1)
            else:
                if arm_selected == optimal_arm: # Incrementing % optimal arm pulls if
                    optimal_arm_pulls_per_round[t][r] += 1
            t+=1


    # Calculating Mean and Standard Error for % optimal arm pulls
    optimal_arm_means_stderr = np.zeros([horizon,2]) # Store % optimal arm means & std
    optimal_arm_means_stderr[:,0] = np.mean(optimal_arm_pulls_per_round,axis=1)
    optimal_arm_means_stderr[:,1] = (np.std(optimal_arm_pulls_per_round, axis=1)/sqrt(
    if optimalpulls == 'Avg':
        optimal_arm_percentage = sum(optimal_arm_means_stderr[:,0])/horizon
        optimal_arm_pulls_sum = np.mean(optimal_arm_pulls_per_round,axis=1)
    else:
        optimal_arm_percentage = sum(optimal_arm_means_stderr[:,0])/horizon*100
        optimal_arm_pulls_sum = np.cumsum(optimal_arm_means_stderr[:,0])/horizon*100
    print("\nTotal Optimal arm pulls :",sum(optimal_arm_means_stderr[:,0]),'and percen



    # Calculating Mean and Standard Error for commulative regret
    regret_means_stderr = np.zeros([horizon,2]) # Store regret means & stderr in (hori
    regret_means_stderr[:,0] = np.mean(regret_per_round,axis=1)
    regret_means_stderr[:,1] = (np.std(regret_per_round, axis=1)/sqrt(replications))
    total_regret = sum(regret_means_stderr[:,0])
    regret_per_round_sum = np.cumsum(regret_means_stderr[:,0])
    print("Total Regret :",total_regret)


    return regret_per_round_sum,regret_means_stderr, optimal_arm_pulls_sum,optimal_arm
```

In [5]:
```python
# Implemention Thompson Sampling
def TS(horizon,replications,arms_prob,alpha,beta,optimalpulls):
    optimal_arm = 0
    optimal_arm_pulls_per_round = np.zeros([horizon,replications])
    regret_per_round = np.zeros([horizon,replications])
    savepoints = (0,1000,5000,9999)
    success_ret = np.zeros([len(savepoints),len(arms_prob)])
    faliure_ret = np.zeros([len(savepoints),len(arms_prob)])

    for r in range(replications):
        arm_pulls = [0]*len(arms_prob)
        success = np.array(alpha)
        failure = np.array(beta)
        t = 0
        s = 0
        while t < horizon:
            if t in savepoints and r == replications-1:
                success_ret[s] = success
                faliure_ret[s] = failure
                s+=1

            #Picking arm according to Posterior distribution
            sample_means = [0]*len(arms_prob)
            for i in range(len(arms_prob)):
                sample_means[i] = np.random.beta(success[i],failure[i])
            arm_selected = np.argmax(sample_means)


            arm_pulls[arm_selected] += 1
            temp = np.random.binomial(1, arms_prob[arm_selected])
            success[arm_selected] += temp
            failure[arm_selected] += 1 - temp

            if optimalpulls == 'Avg':
                optimal_arm_pulls_per_round[t][r] = arm_pulls[optimal_arm]*100.0/(t+1)
            else:
                if arm_selected == optimal_arm: # Incrementing % optimal arm pulls if
                    optimal_arm_pulls_per_round[t][r] += 1

            regret_per_round[t][r] = (arms_prob[optimal_arm] - arms_prob[arm_selected]

            t+=1


    # Calculating Mean and Standard Error for % optimal arm pulls
    optimal_arm_means_stderr = np.zeros([horizon,2])
    optimal_arm_means_stderr[:,0] = np.mean(optimal_arm_pulls_per_round,axis=1)
    optimal_arm_means_stderr[:,1] = (np.std(optimal_arm_pulls_per_round, axis=1)/sqrt(
```

```
    if optimalpulls == 'Avg':
        optimal_arm_percentage = sum(optimal_arm_means_stderr[:,0])/horizon
        optimal_arm_pulls_sum = np.mean(optimal_arm_pulls_per_round,axis=1)
    else:
        optimal_arm_percentage = sum(optimal_arm_means_stderr[:,0])/horizon*100
        optimal_arm_pulls_sum = np.cumsum(optimal_arm_means_stderr[:,0])/horizon*100

    print("\nTotal Optimal arm pulls :",sum(optimal_arm_means_stderr[:,0]),'and percen


    # Calculating Mean and Standard Error for commulative regret
    regret_means_stderr = np.zeros([horizon,2])
    regret_means_stderr[:,0] = np.mean(regret_per_round,axis=1)
    regret_means_stderr[:,1] = (np.std(regret_per_round, axis=1)/sqrt(replications))
    total_regret = sum(regret_means_stderr[:,0])
    regret_per_round_sum = np.cumsum(regret_means_stderr[:,0])
    print("Total Regret :",total_regret)


    return success_ret,faliure_ret,regret_per_round_sum,regret_means_stderr, optimal_a
```

In [6]:

```
# Calculating Lower bound Regret
def calculate_lower_bound():
    arms_prob = [[0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4], [0.5, 0.48, 0.48,

    n = 10000
    gap_dependent_regret = np.zeros([3,n])
    gap_independent_regret = np.zeros([3,n])
    for i in range(3):
        k = len(arms_prob[i])
        for j in range(1,n):
            dep_reg = 0
            for g in range(len(arms_prob[i])):
                gap = arms_prob[i][0] - arms_prob[i][g]
                if gap > 0 and dep_reg >= 0:
                    dep_reg += log(j*gap**2)/(8*gap) #n*gap * exp(-4*j*gap**2/(k-1))/4
                    if dep_reg < 0:
                        dep_reg = 0
            gap_dependent_regret[i][j] = dep_reg

            indep_reg = sqrt((k-1)*j/8)*exp(-1/2)/4
            gap_independent_regret[i][j] = indep_reg
    return gap_dependent_regret,gap_independent_regret


gap_dependent_regret,gap_independent_regret = calculate_lower_bound()
```

```
        print('gap_dependent_regret')
        print(gap_dependent_regret)

        print('\ngap_independent_regret')
        print(gap_independent_regret)
```

```
gap_dependent_regret
[[  0.           0.           0.          ...,  51.80478909  51.80591437
   51.80703954]
 [  0.           0.           0.          ...,  77.96218028  77.96780669
   77.97343253]
 [  0.           0.           0.          ...,   5.13966203   5.13973496
    5.13980789]]

gap_independent_regret
[[  0.           0.16083073   0.227449    ...,  16.0806602   16.08146446
   16.08226867]
 [  0.           0.16083073   0.227449    ...,  16.0806602   16.08146446
   16.08226867]
 [  0.           0.07581633   0.10722049 ...,   7.58049592   7.58087505
    7.58125416]]
```

```
In [7]: # Plotting % Cummulative Optimal Arm Pulls Vs Time steps with error bars
        def plotCumOptimalArmPulls(horizon,optimal_arm_means_stderr,optimal_arm_pulls_sum,probl
            x = np.arange(horizon)
            ind = [i for i in range(0,horizon,step)]

            for i in range(m_len):
                plt.errorbar(x[ind],optimal_arm_pulls_sum[i,ind], optimal_arm_means_stderr[i,in
                             linestyle='-', marker='x',capsize=4,capthick=1.5,elinewidth=1.5)
            plt.xlabel('Steps')
            plt.ylabel('% Cummulative Optimal Arm Pulls')
            plt.legend(['UCB',"TS with prior mean=0.5","TS with prior mean=0.2"],loc=0,frameon=
            plt.title(arms_prob[problem])
            plt.xlim((0,10000))
            plt.ylim((0,100))
            plt.savefig('CummOptimalArmPulls_'+str(problem)+optimalpulls+'.png',dpi=300)
            plt.show()

            print("optimal_arm_stderr")
            print(optimal_arm_means_stderr[:,[500,2000,5000,8000,9500],1])
```

```
In [8]: # Plotting % Average Optimal Arm Pulls Vs Time steps with error bars
        def plotAvgOptimalArmPulls(horizon,optimal_arm_means_stderr,optimal_arm_pulls_sum,probl
            x = np.arange(horizon)
            ind = [i for i in range(0,horizon,step)]
```

```
        for i in range(m_len):
            plt.errorbar(x[ind],optimal_arm_pulls_sum[i,ind], optimal_arm_means_stderr[i,in
                         linestyle='-', marker='x',capsize=4,capthick=1.5,elinewidth=1.5)
        plt.xlabel('Steps')
        plt.ylabel('% Average Optimal Arm Pulls')
        plt.legend(['UCB',"TS with prior mean=0.5","TS with prior mean=0.2"],loc=0,frameon=
        plt.title(arms_prob[problem])
        plt.xlim((0,10000))
        plt.ylim((0,100))
        plt.savefig('AvgOptimalArmPulls_'+str(problem)+optimalpulls+'.png',dpi=300)
        plt.show()

        print("optimal_arm_means_stderr")
        print(optimal_arm_means_stderr[:,[500,2000,5000,8000,9500],1])
```

In [9]:
```
    # Plotting Cumulative Regret Vs Time steps with error bars
    def plotCummRegret(horizon,regret_means_stderr,regret_per_round_sum,problem,step):
        labels = ["Gap-dependent lower bound","Gap-independent lower bound",'UCB',"TS with
        x = np.arange(horizon)
        ind = [i for i in range(0,horizon,step)]

        plt.plot(x[ind],gap_dependent_regret[problem][ind],label = labels[0])
        plt.plot(x[ind],gap_independent_regret[problem][ind],label = labels[1])

        for i in range(m_len):
            plt.errorbar(x[ind],regret_per_round_sum[i,ind], regret_means_stderr[i,ind,1],l
                     linestyle='-', marker='x',capsize=4,capthick=1.5,elinewidth=1.5)

        plt.xlabel('Steps')
        plt.ylabel('Cumulative Regret')
        plt.legend(loc=0,frameon=False)
        plt.title(arms_prob[problem])
        plt.xlim((0,10000))
    #     plt.ylim((0,80))
        plt.savefig('CumulativeRegret_'+str(problem)+optimalpulls+'.png',dpi=300)
        plt.show()

        print("regret_means_stderr")
        print(regret_means_stderr[:,[500,2000,5000,8000,9500],1])
        print("regret_per_round_sum")
        print(regret_per_round_sum[:,[500,2000,5000,8000,9500]])
        print("gap_dependent_regret")
        print(gap_dependent_regret[problem,[500,2000,5000,8000,9500]])
        print("gap_independent_regret")
        print(gap_independent_regret[problem,[500,2000,5000,8000,9500]])
```

In [10]:
```
def plot_arm_distribution(alpha_values,beta_values):
```

```
    savepoints = [1,1000,5000,10000]
    s = 0
    for p in range(len(alpha_values)):

        fig, ax = plt.subplots(figsize=(10, 6))
        m = alpha_values[p] / (alpha_values[p] + beta_values[p])
        print("Mean at time t = "+str(savepoints[s])+" is: ",m)

        x = np.linspace(0, 1, 1000)[1:-1]
        i = 1
        for a, b in zip(alpha_values[p], beta_values[p]):
            plt.plot(x, beta.pdf(x,a,b),
                     label=r'Arm %d : $\alpha=%d,\ \beta=%d$' % (i, a, b))
            i+=1

        plt.ylim(0, 20)
        plt.xticks(np.arange(0, 1.1,0.1))
        plt.xlabel(r'$ \theta $')
        plt.ylabel(r'$p(\theta|\alpha,\beta)$')
        plt.title('Problem '+str(problem+1)+' at t = '+str(savepoints[s]))
        s+=1
        plt.legend(loc=0)
        plt.savefig('Arm_dist_t_'+str(savepoints[s-1])+'_'+str(problem+1)+'_'+optimalp
        plt.show()
```

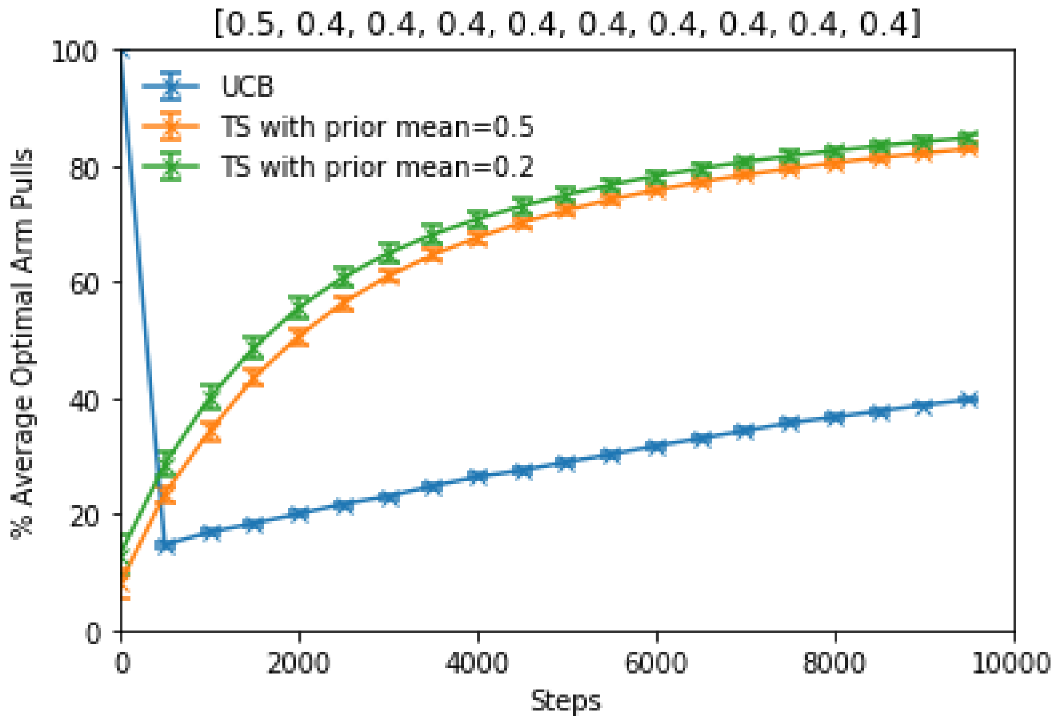
```
In [11]: # For Printing table
         from IPython.display import HTML, display

         def tableIt(data):
             print(pd.DataFrame(data))
```

```
In [12]: horizon = 10000
         replications = 100

         arms_prob = [[0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4], [0.5, 0.48, 0.48, 0.4
         types = ['UCB','TS M0.5','TS M0.2']
         m_len = len(types)
         success = [1,1]
         failure = [1,4]
         optimalpulls = 'Avg'

         for problem in range(3): # Repeating for 3 problems
             optimal_arm_pulls_sum = np.zeros([m_len,horizon]) # Storing variables returned by
             regret_per_round_sum = np.zeros([m_len,horizon])
             optimal_arm_means_stderr = np.zeros([m_len,horizon,2])
             regret_means_stderr = np.zeros([m_len,horizon,2])
             optimal_arm_percentage = np.zeros([m_len])
```

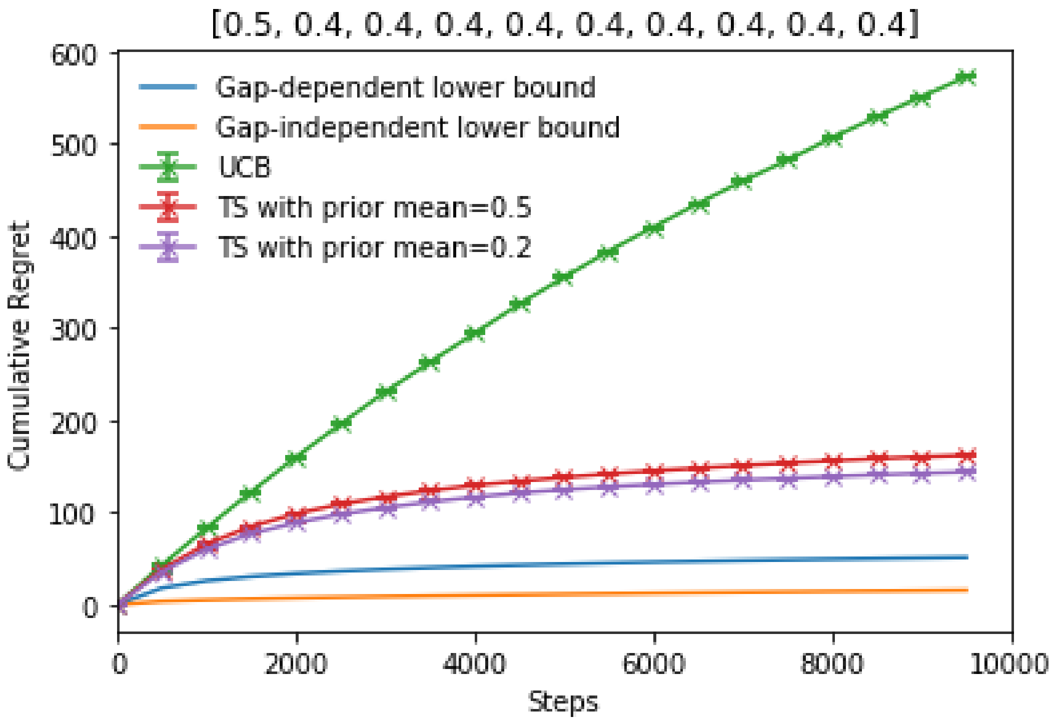
```
            total_regret = np.zeros([m_len])
            success_ret = np.zeros([2,4,len(arms_prob[problem])])
            failure_ret = np.zeros([2,4,len(arms_prob[problem])])


            regret_per_round_sum[0,:],regret_means_stderr[0,:,:], optimal_arm_pulls_sum[0,:],
            success_ret[0,:,:],failure_ret[0,:,:],regret_per_round_sum[1,:],regret_means_stde
            success_ret[1,:,:],failure_ret[1,:,:],regret_per_round_sum[2,:],regret_means_stde



            step = 500
            print("\n")
            print("optimal_arm_percentage")
            tableIt(optimal_arm_percentage)
            print("\n")
            print("total_regret")
            tableIt(total_regret)

            # Calling function to plot % Average Optimal Arm Pulls & Commulative regret with
            plotAvgOptimalArmPulls(horizon,optimal_arm_means_stderr,optimal_arm_pulls_sum,prob
            plotCummRegret(horizon,regret_means_stderr,regret_per_round_sum,problem,step)
            plot_arm_distribution(success_ret[0],failure_ret[0])
            plot_arm_distribution(success_ret[1],failure_ret[1])
```

```
Total Optimal arm pulls : 283612.675268 and percentage is : 28.3612675268
Total Regret : 593.993

Total Optimal arm pulls : 652499.168408 and percentage is : 65.2499168408
Total Regret : 163.781

Total Optimal arm pulls : 683731.231325 and percentage is : 68.3731231325
Total Regret : 145.473


optimal_arm_percentage
           0
0  28.361268
1  65.249917
2  68.373123


total_regret
         0
0  593.993
1  163.781
```

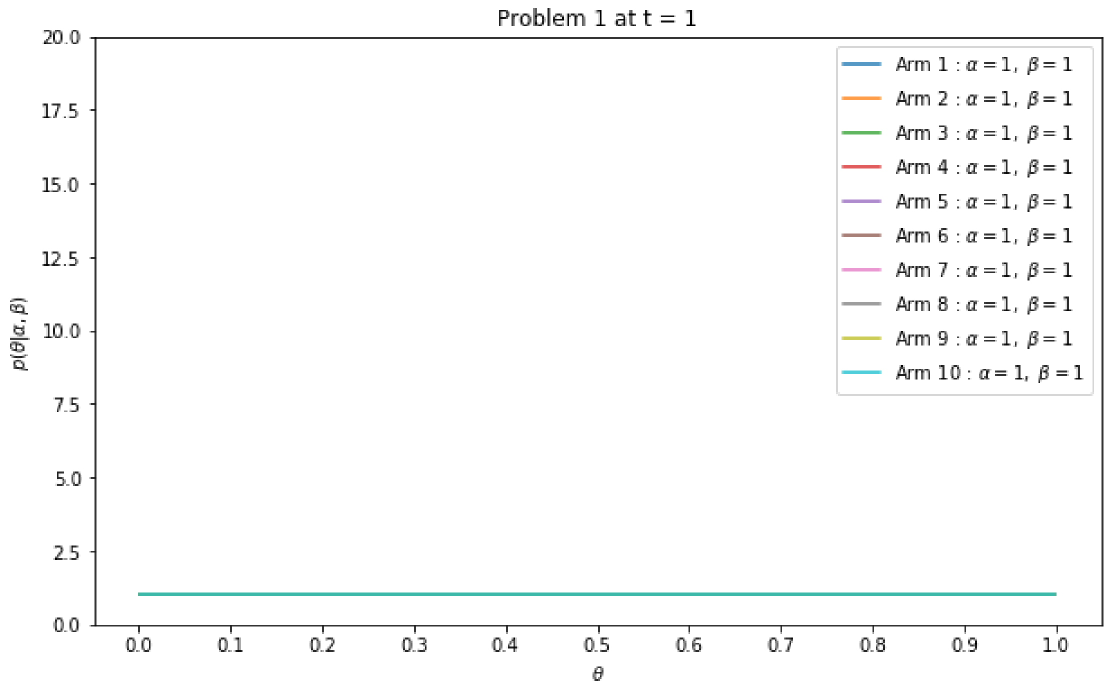
```
2  145.473
```



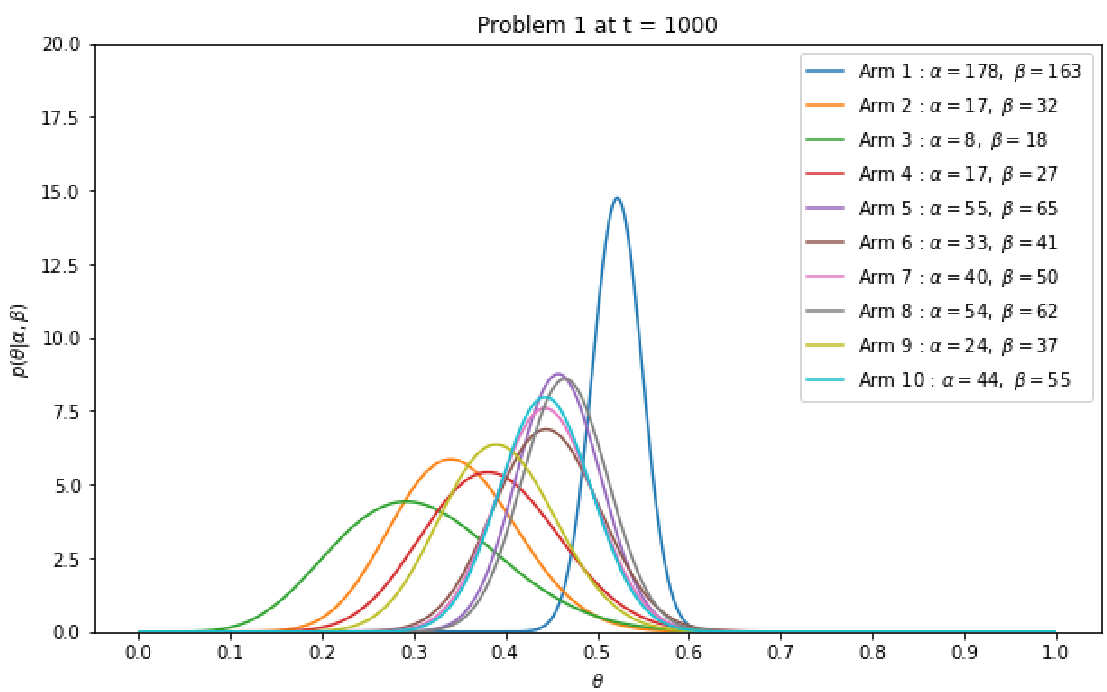

```
optimal_arm_means_stderr
[[ 0.37932051  0.43692854  0.44865863  0.45474807  0.435826  ]
 [ 1.37482323  1.32860494  0.6824189   0.44764855  0.38093131]
 [ 2.01556859  1.74196148  1.1765315   0.72305736  0.6041692 ]]
```



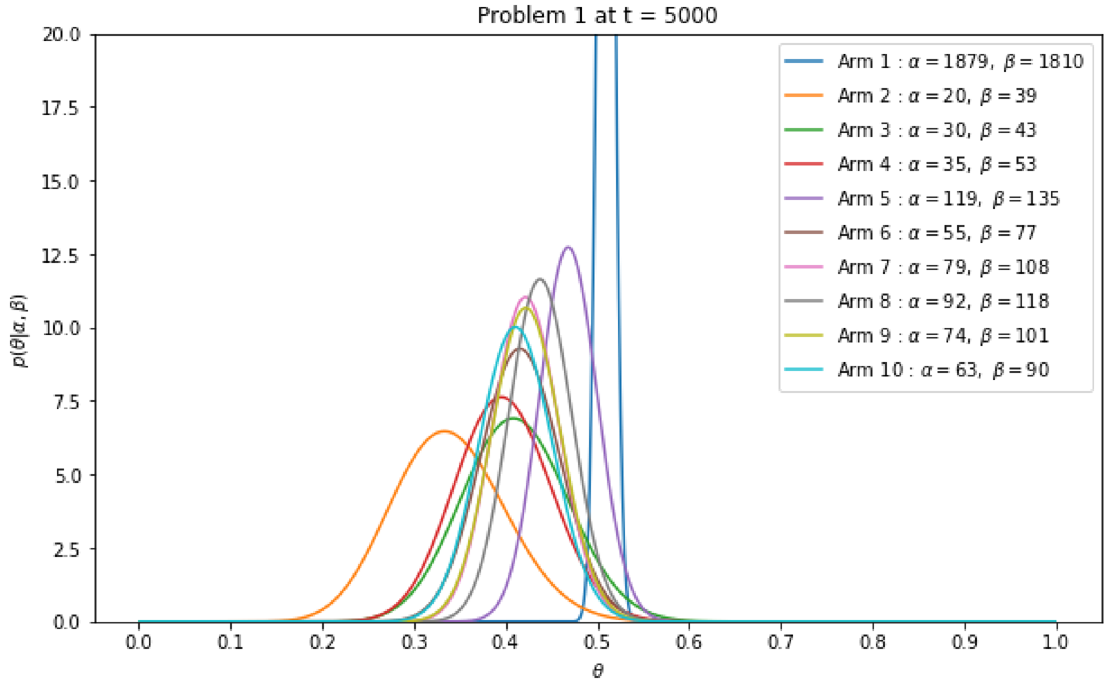

```
regret_means_stderr
[[ 0.00324962  0.00392301  0.00497494  0.00491833  0.00462493]
 [ 0.00448999  0.00384187  0.00255147  0.00099499  0.00099499]
 [ 0.00495076  0.00420833  0.00237487  0.00217945  0.00255147]]
regret_per_round_sum
[[  42.736  160.02   355.174  506.605  572.967]
 [  38.318   98.747  138.166  156.021  161.991]
 [  35.768   88.902  124.882  139.08   144.003]]
gap_dependent_regret
[ 18.10617651  33.70198808  44.01025881  49.29779964  51.23111503]
gap_independent_regret
[  3.59628442   7.19256883  11.37244987  14.38513767  15.67584034]
Mean at time t = 1 is:  [ 0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5]
```



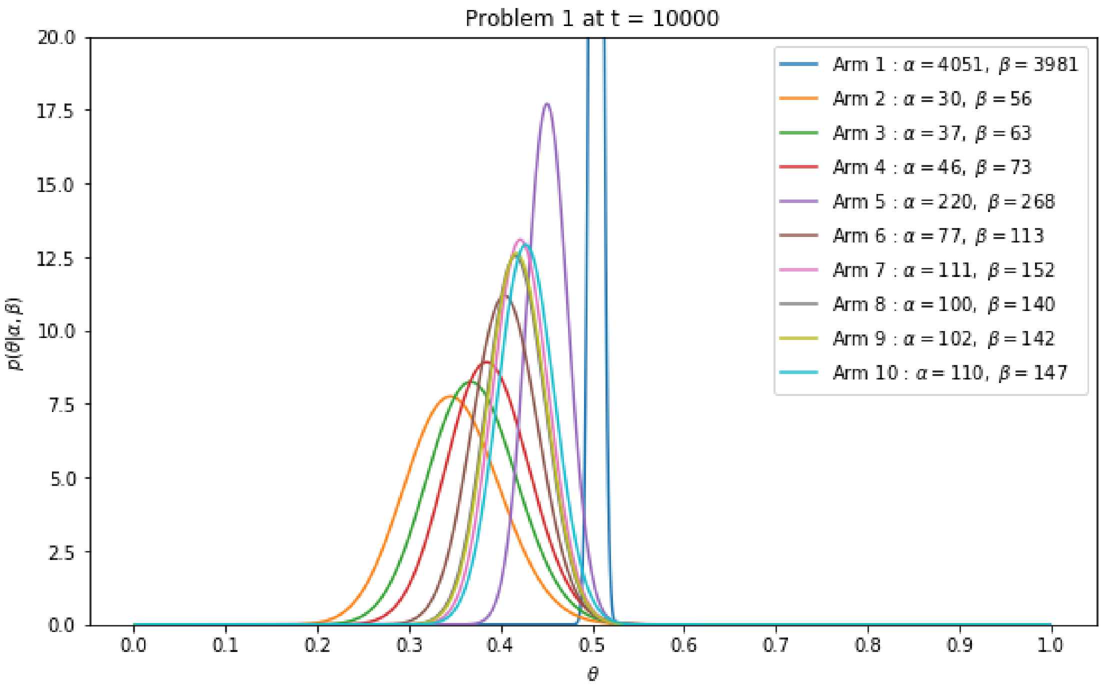

```
Mean at time t = 1000 is:  [ 0.52199413  0.34693878  0.30769231  0.38636364  0.45833333  0.4459
  0.44444444  0.46551724  0.39344262  0.44444444]
```



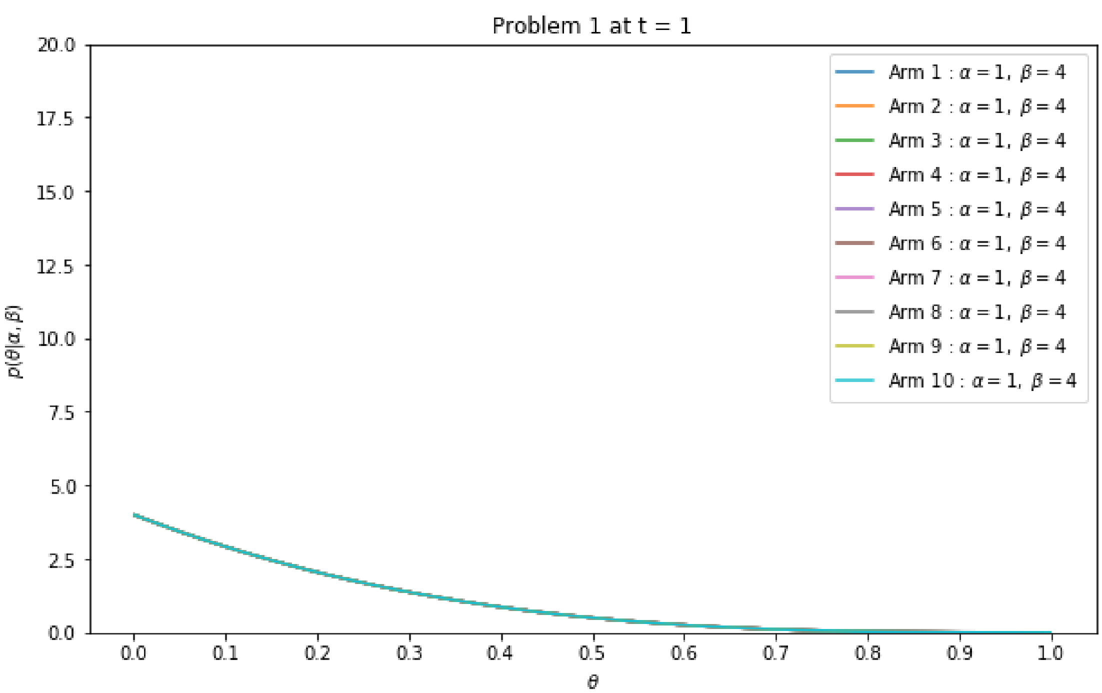
```
Mean at time t = 5000 is:  [ 0.50935213  0.33898305  0.4109589   0.39772727  0.46850394  0.416
  0.42245989  0.43809524  0.42285714  0.41176471]
```



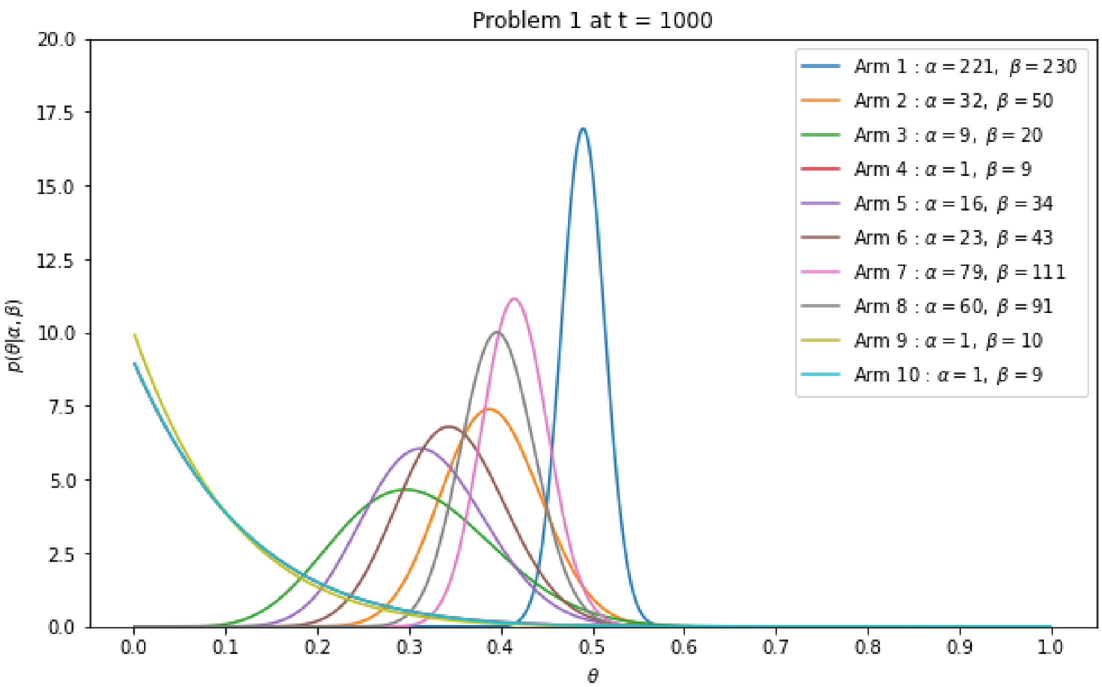

```
Mean at time t = 10000 is:  [ 0.50435757  0.34883721  0.37        0.38655462  0.45081967  0.40
  0.42205323  0.41666667  0.41803279  0.42801556]
```



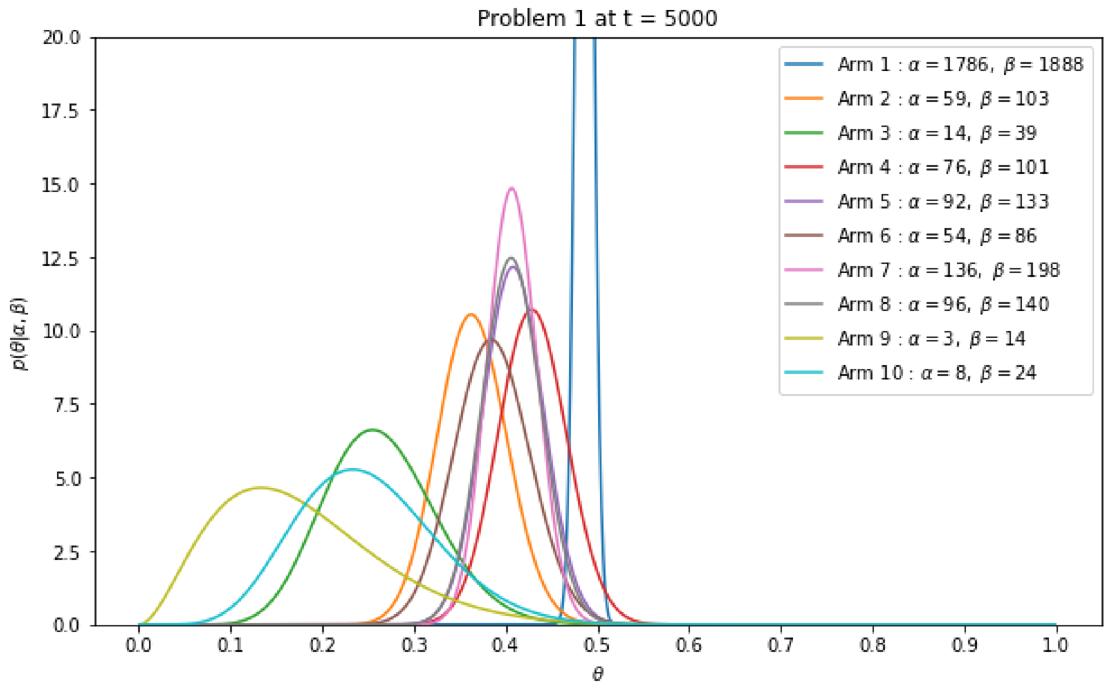

```
Mean at time t = 1 is:  [ 0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2]
```



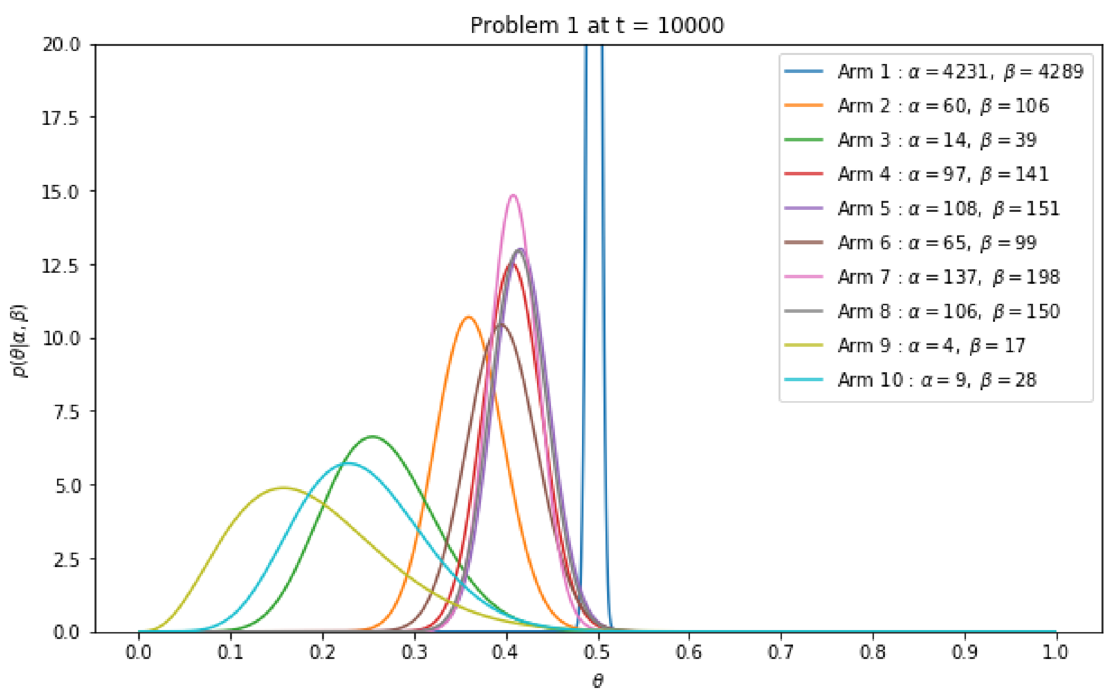
```
Mean at time t = 1000 is:  [ 0.49002217  0.3902439   0.31034483  0.1         0.32        0.3484
  0.41578947  0.39735099  0.09090909  0.1       ]
```



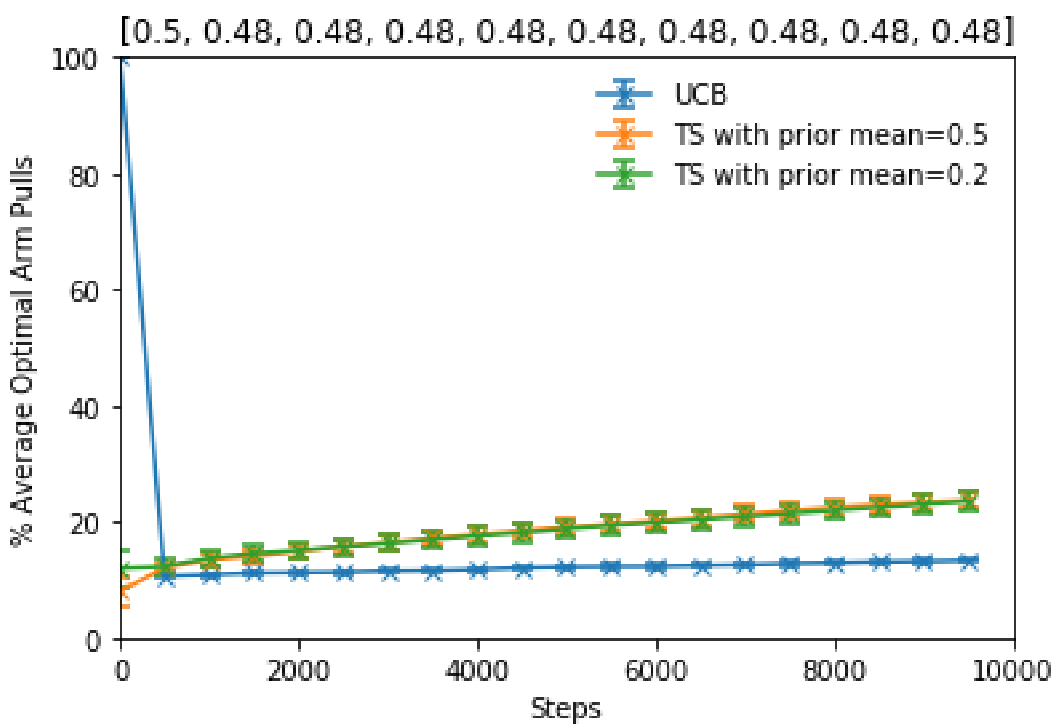

```
Mean at time t = 5000 is:  [ 0.48611867  0.36419753  0.26415094  0.42937853  0.40888889  0.385
  0.40718563  0.40677966  0.17647059  0.25      ]
```



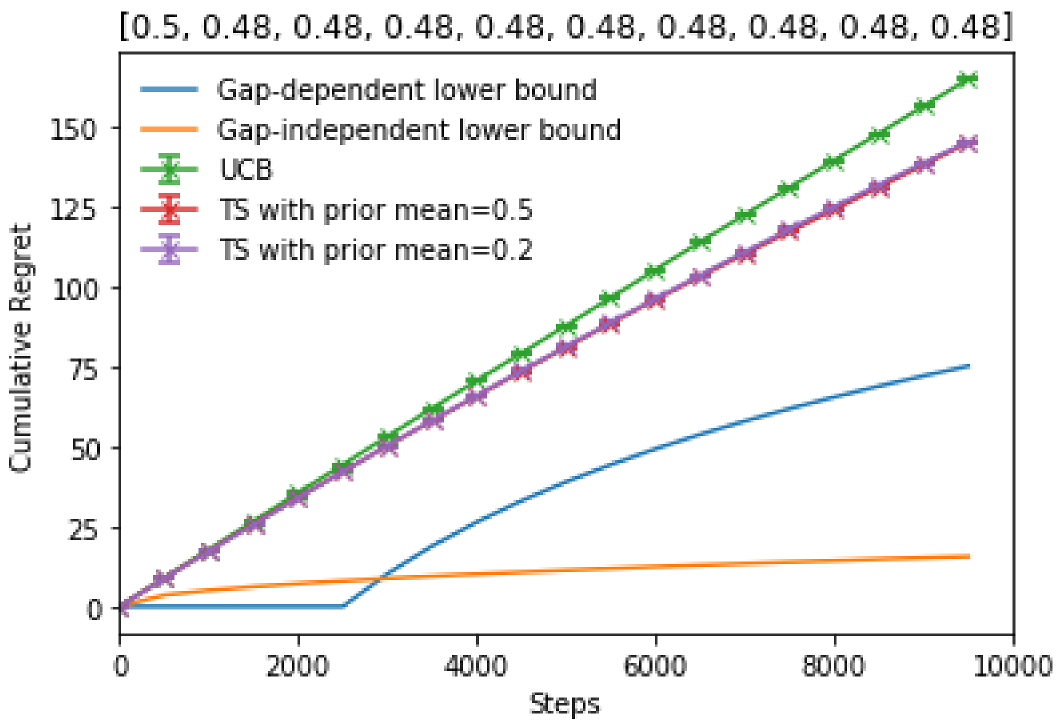

```
Mean at time t = 10000 is:  [ 0.49659624  0.36144578  0.26415094  0.40756303  0.41698842  0.396
  0.40895522  0.4140625   0.19047619  0.24324324]
```



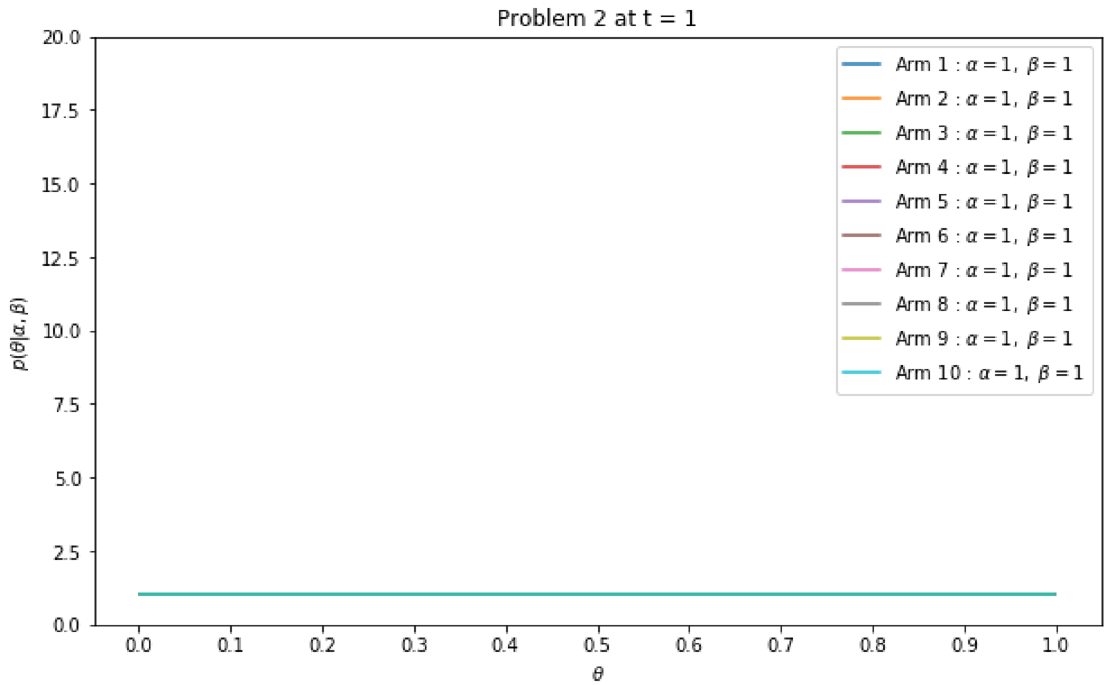
```
Total Optimal arm pulls : 120911.777501 and percentage is : 12.0911777501
Total Regret : 173.3864

Total Optimal arm pulls : 186662.859336 and percentage is : 18.6662859336
Total Regret : 151.8254

Total Optimal arm pulls : 184985.5294 and percentage is : 18.49855294
Total Regret : 151.7754


optimal_arm_percentage
           0
0  12.091178
1  18.666286
2  18.498553


total_regret
          0
0  173.3864
1  151.8254
2  151.7754
```



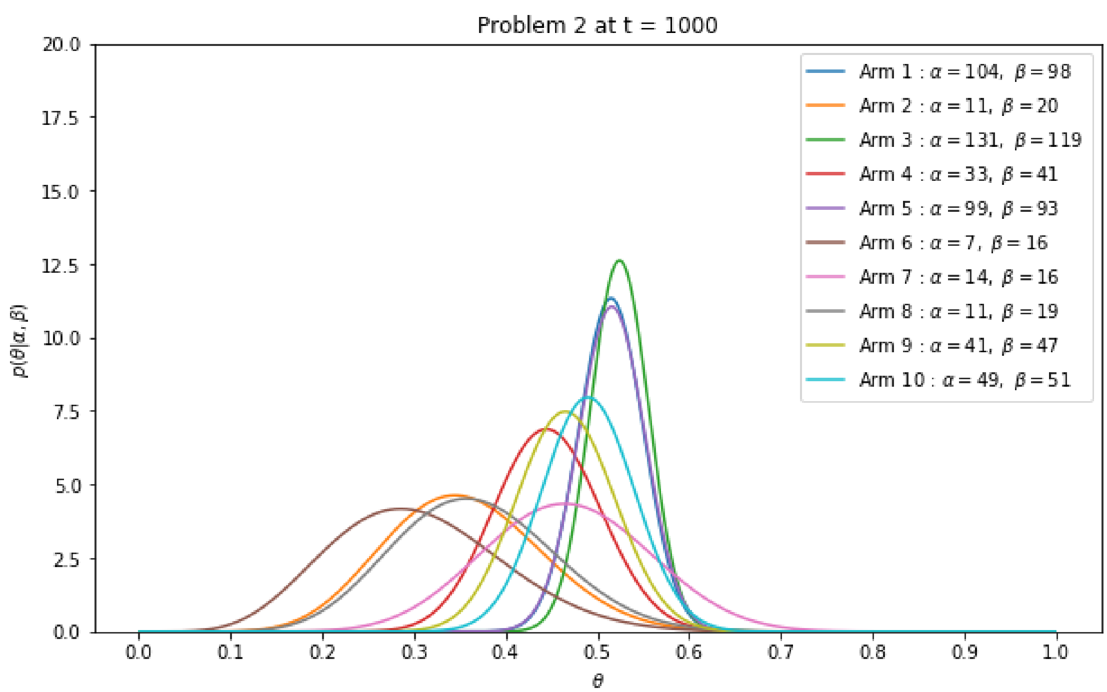
```
optimal_arm_means_stderr
[[ 0.28318001  0.28274657  0.29367604  0.30803299  0.30607703]
 [ 0.85373802  1.1549887   1.30079714  1.3876149   1.41370241]
 [ 1.26060158  1.35384415  1.46942818  1.54023541  1.56892188]]
```



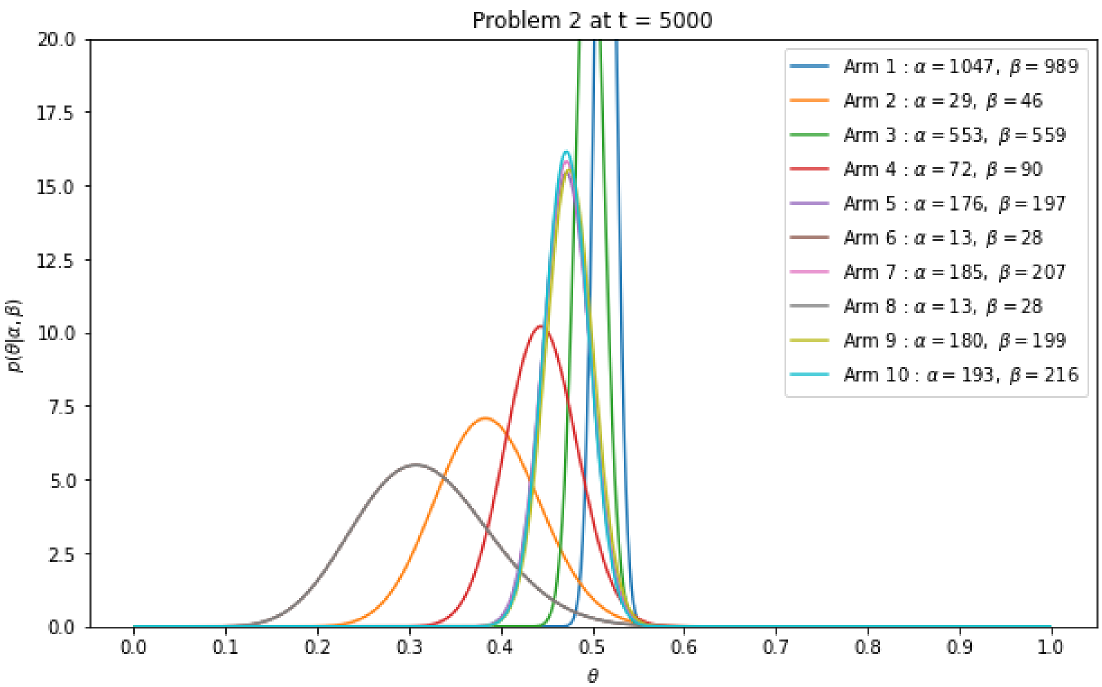

```
regret_means_stderr
[[ 0.00069397  0.00043589  0.00064992  0.00071414  0.00069397]
 [ 0.00071414  0.00084167  0.00081462  0.00092499  0.00094742]
 [ 0.00073321  0.0008      0.00084167  0.00091652  0.00096   ]]
regret_per_round_sum
[[   8.952     35.5116    87.7602   139.3692   164.7034]
 [   8.807     34.0506    80.9434   123.9802   144.9186]
 [   8.7878    33.9774    81.2388   124.8678   145.241 ]]
gap_dependent_regret
[  0.           0.          38.98952891  65.42723305  75.09381    ]
gap_independent_regret
[  3.59628442   7.19256883  11.37244987  14.38513767  15.67584034]
Mean at time t = 1 is:  [ 0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5]
```



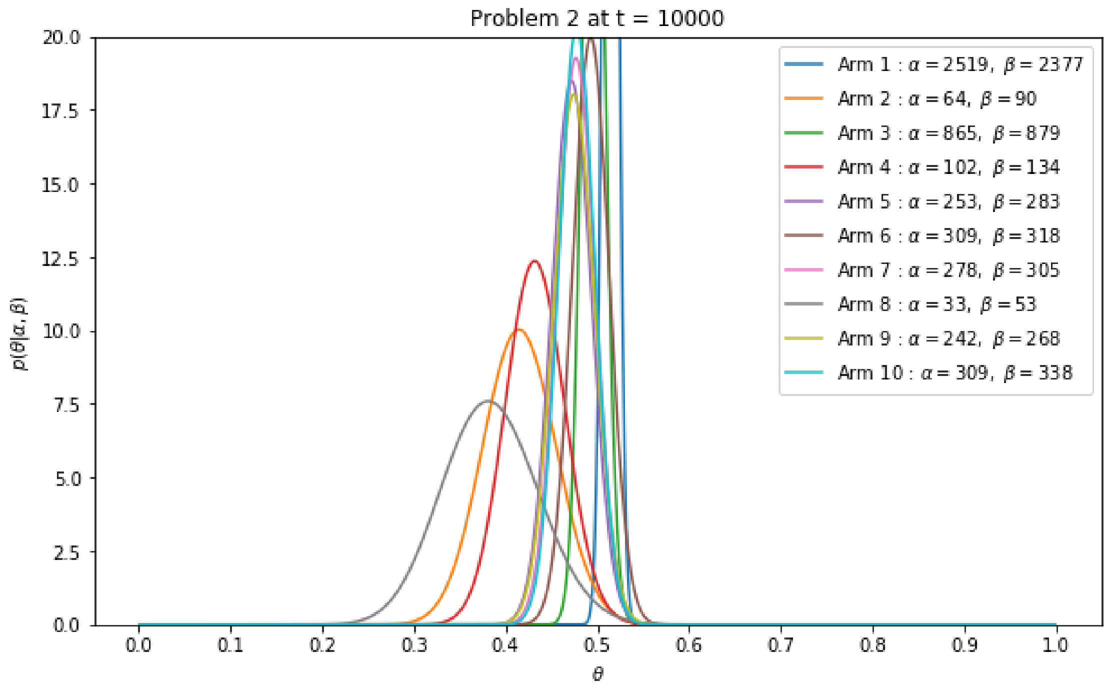

```
Mean at time t = 1000 is:  [ 0.51485149  0.35483871  0.524       0.44594595  0.515625    0.3043
  0.46666667  0.36666667  0.46590909  0.49      ]
```



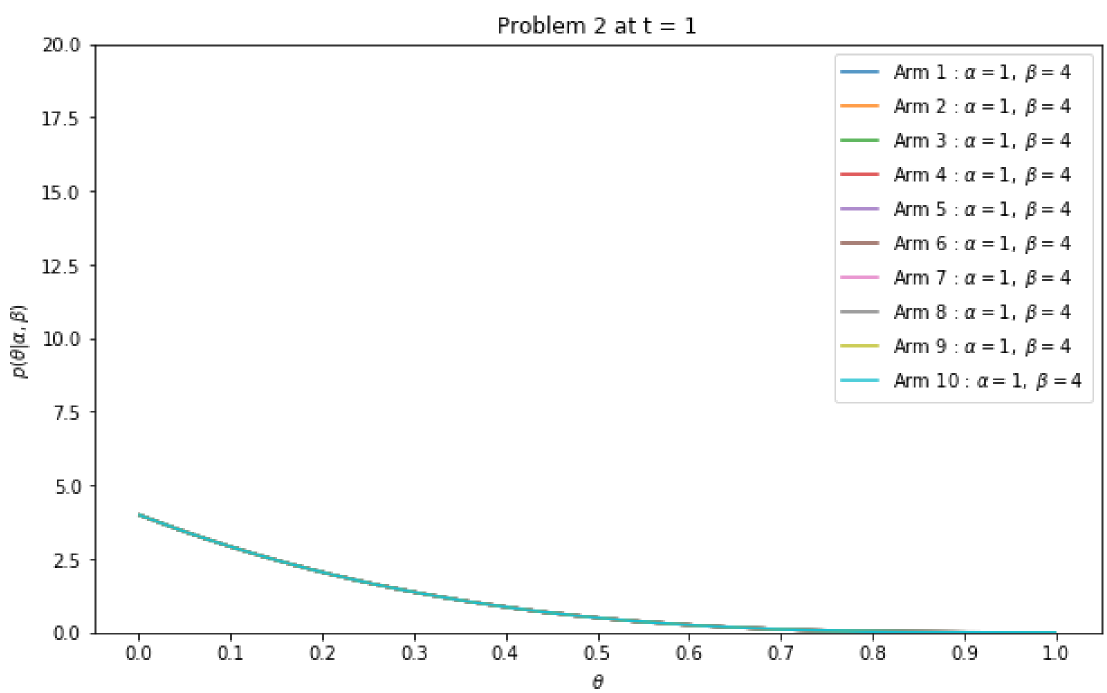
```
Mean at time t = 5000 is:  [ 0.51424361  0.38666667  0.49730216  0.44444444  0.47184987  0.317
  0.47193878  0.31707317  0.47493404  0.47188264]
```



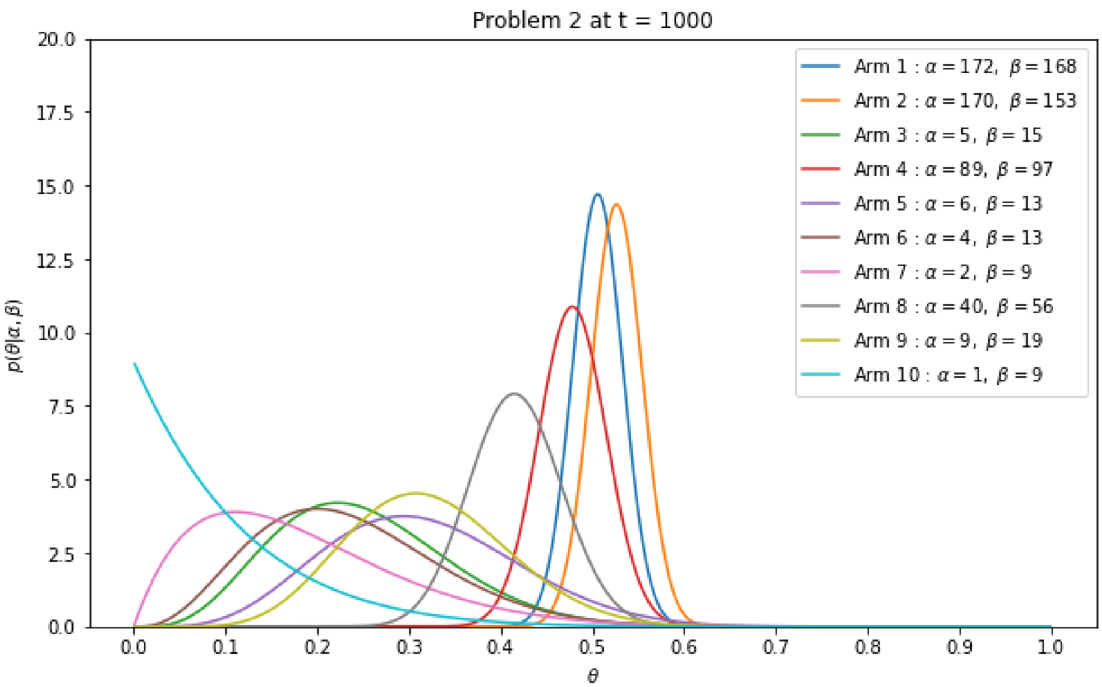

```
Mean at time t = 10000 is:  [ 0.51450163  0.41558442  0.49598624  0.43220339  0.47201493  0.49
  0.47684391  0.38372093  0.4745098   0.47758887]
```



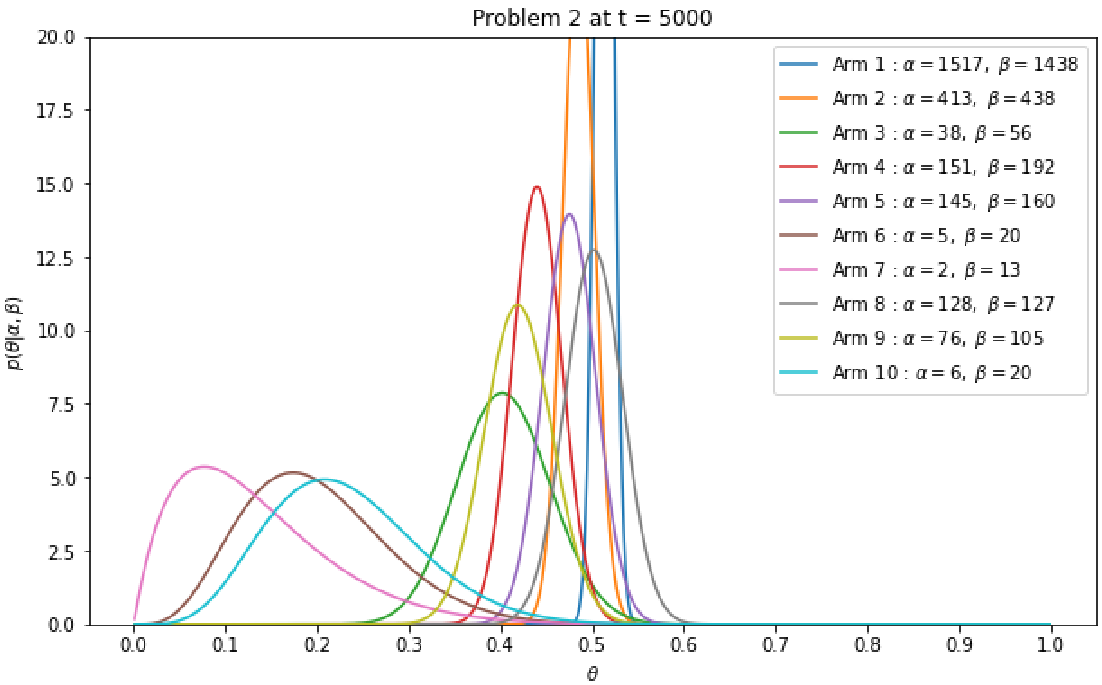

```
Mean at time t = 1 is:  [ 0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2]
```



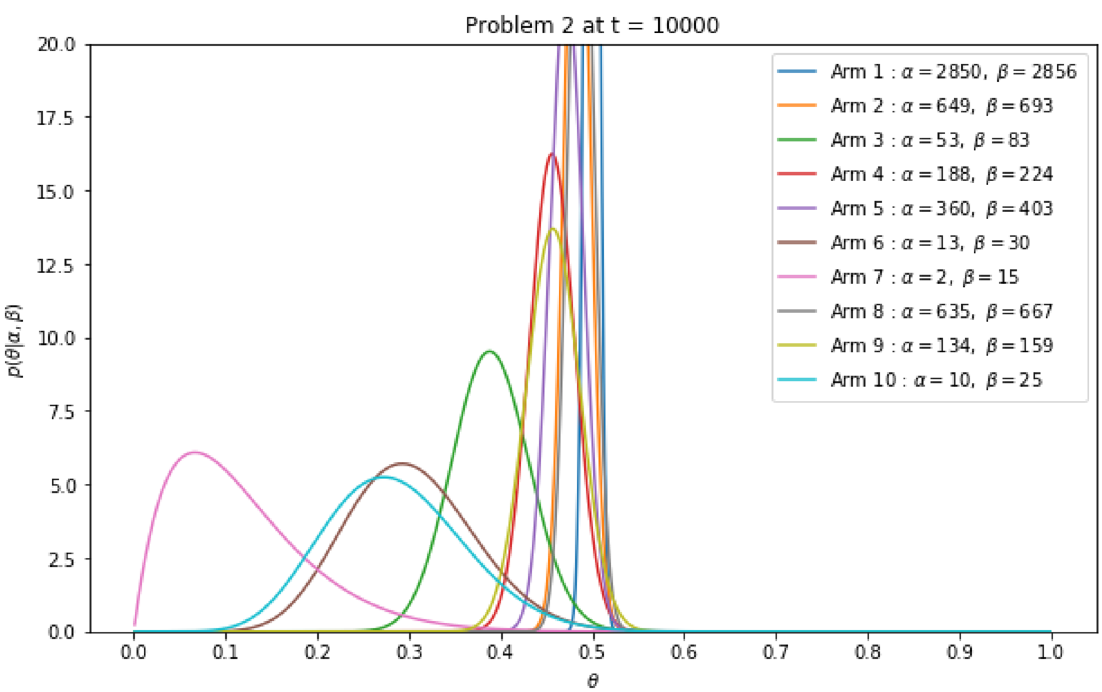
```
Mean at time t = 1000 is:  [ 0.50588235  0.52631579  0.25        0.47849462  0.31578947  0.2352
  0.18181818  0.41666667  0.32142857  0.1       ]
```



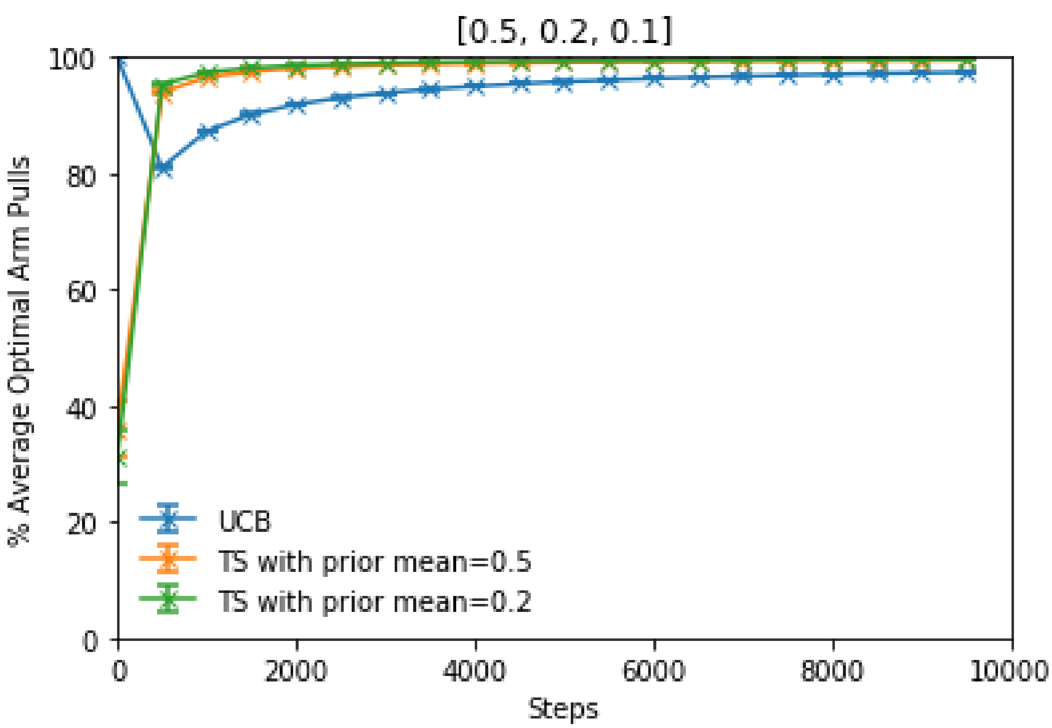

```
Mean at time t = 5000 is:  [ 0.51336717  0.4853114   0.40425532  0.44023324  0.47540984  0.2
  0.13333333  0.50196078  0.4198895   0.23076923]
```



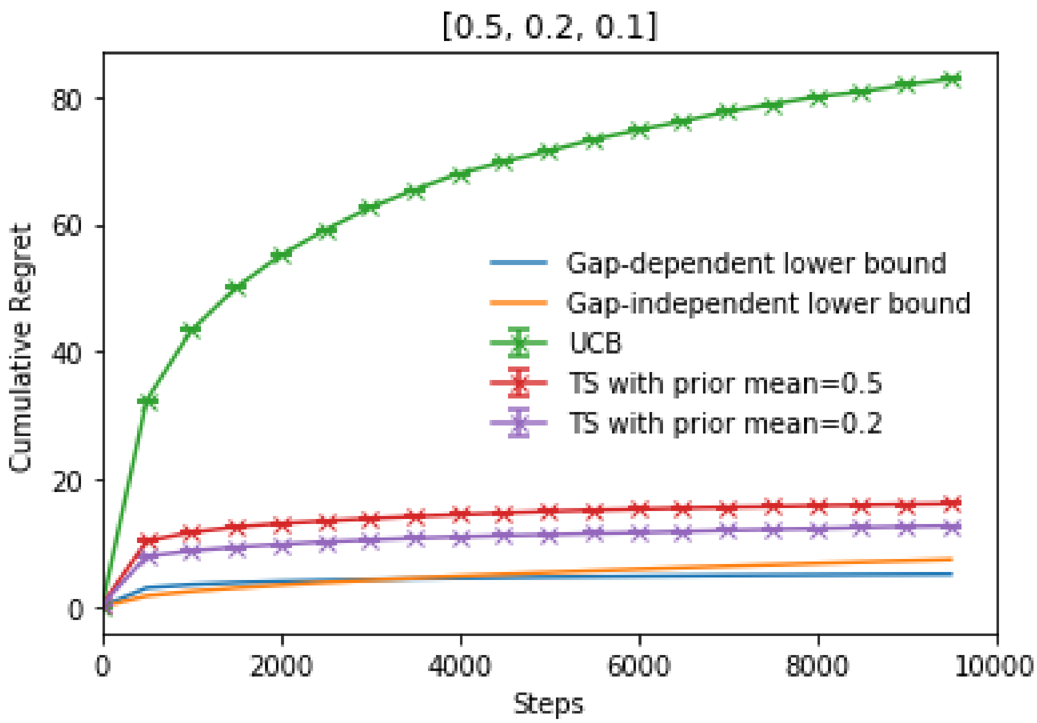

```
Mean at time t = 10000 is:  [ 0.49947424  0.48360656  0.38970588  0.45631068  0.47182176  0.302
  0.11764706  0.48771121  0.45733788  0.28571429]
```



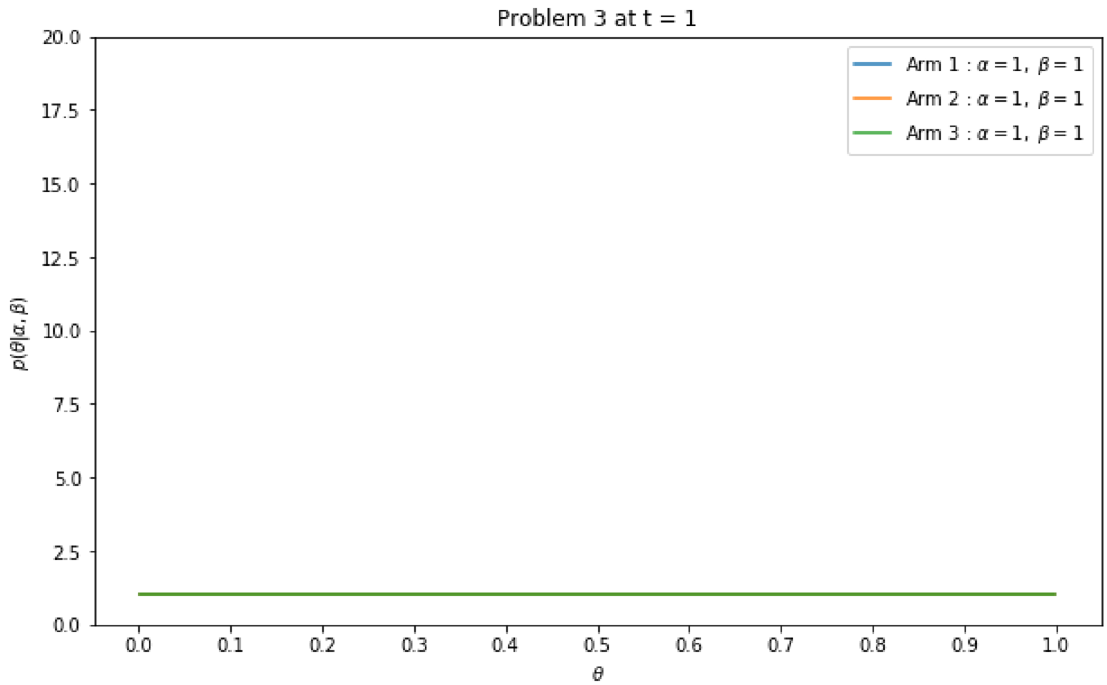
```
Total Optimal arm pulls : 934693.661168 and percentage is : 93.4693661168
Total Regret : 83.607

Total Optimal arm pulls : 980563.54202 and percentage is : 98.056354202
Total Regret : 16.274

Total Optimal arm pulls : 984437.531967 and percentage is : 98.4437531967
Total Regret : 12.754


optimal_arm_percentage
           0
0  93.469366
1  98.056354
2  98.443753


total_regret
        0
0  83.607
1  16.274
2  12.754
```



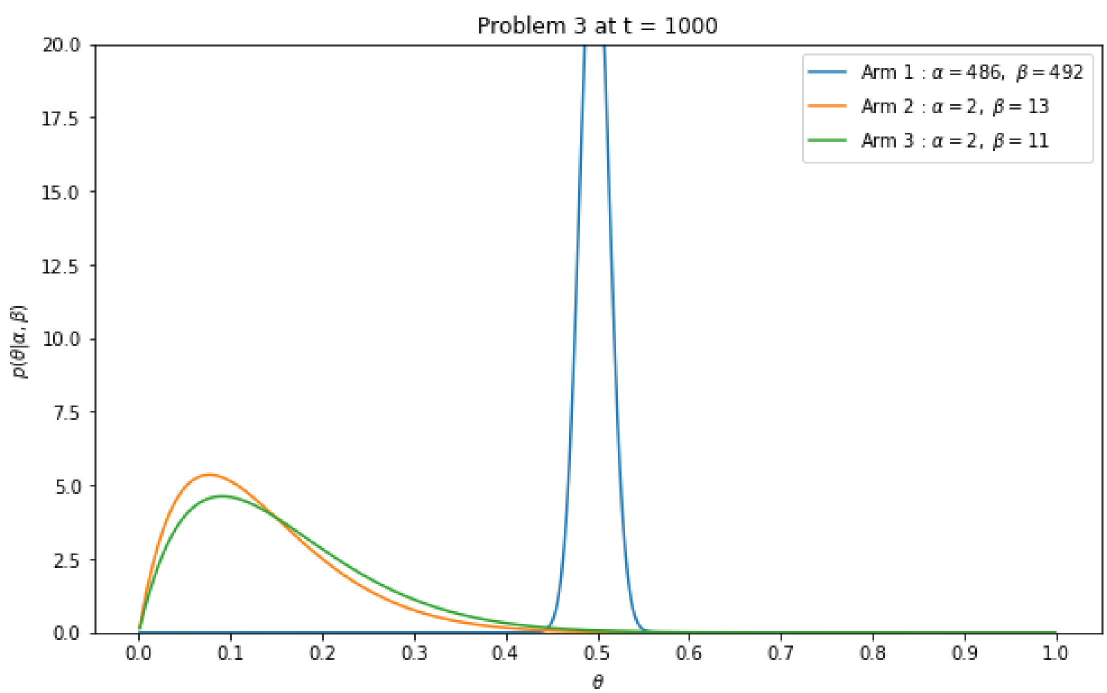
```
optimal_arm_means_stderr
[[ 0.30812797  0.11263889  0.05128719  0.03391501  0.02926298]
 [ 0.32196661  0.0802825   0.03251039  0.02044813  0.01726094]
 [ 0.29373779  0.07279044  0.02945255  0.01821061  0.01522703]]
```



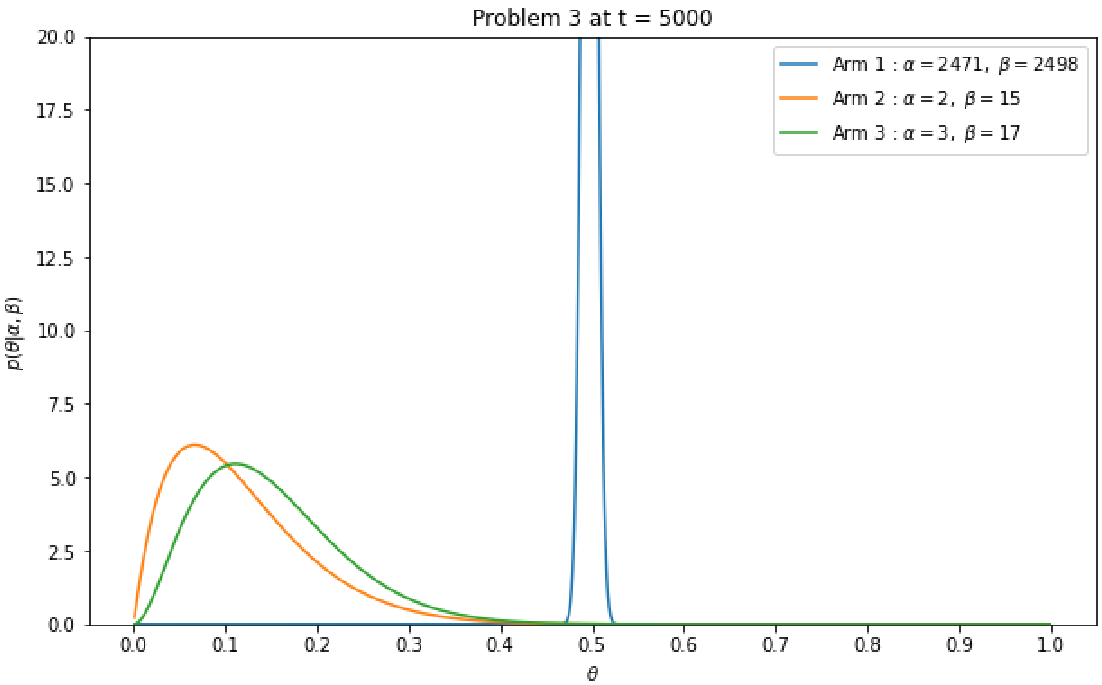

```
regret_means_stderr
[[ 0.00964365  0.00397995  0.00397995  0.00298496  0.00298496]
 [ 0.00298496  0.00298496  0.          0.          0.        ]
 [ 0.00298496  0.          0.          0.          0.        ]]
regret_per_round_sum
[[ 32.377  55.085  71.444  79.962  82.712]
 [ 10.293  13.056  14.944  15.859  16.159]
 [  7.919   9.769  11.291  12.24   12.671]]
gap_dependent_regret
[ 2.95549269  3.96633233  4.63446099  4.97717197  5.10247945]
gap_independent_regret
[ 1.69530473  3.39060946  5.36102428  6.78121893  7.38966201]
Mean at time t = 1 is:  [ 0.5  0.5  0.5]
```



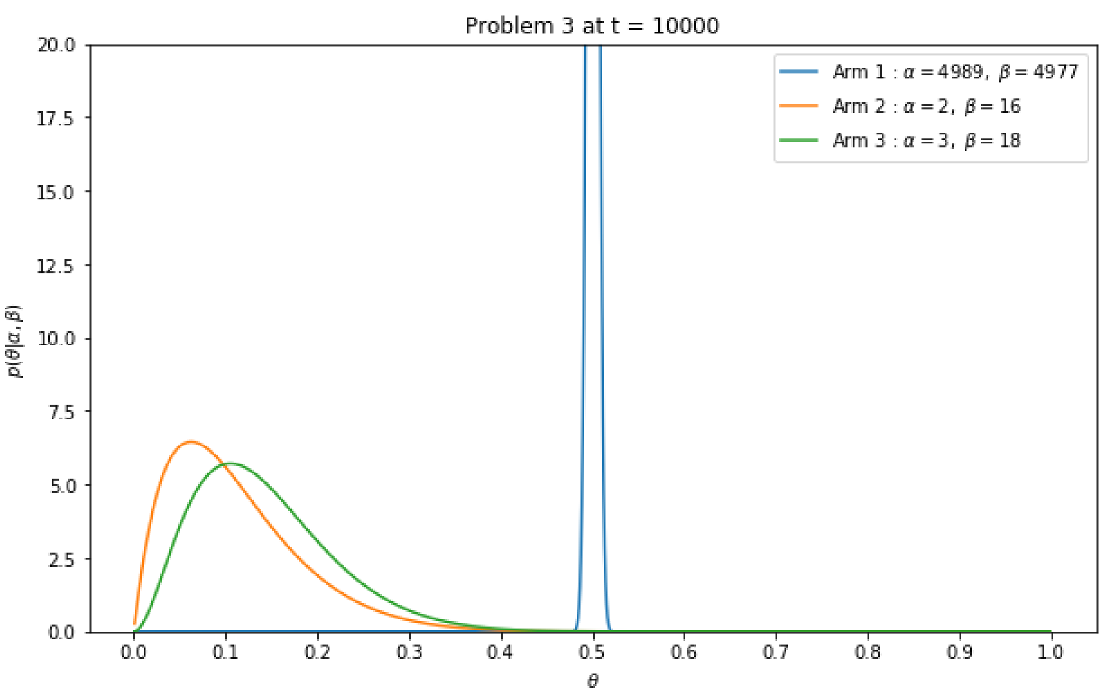

```
Mean at time t = 1000 is:  [ 0.49693252  0.13333333  0.15384615]
```



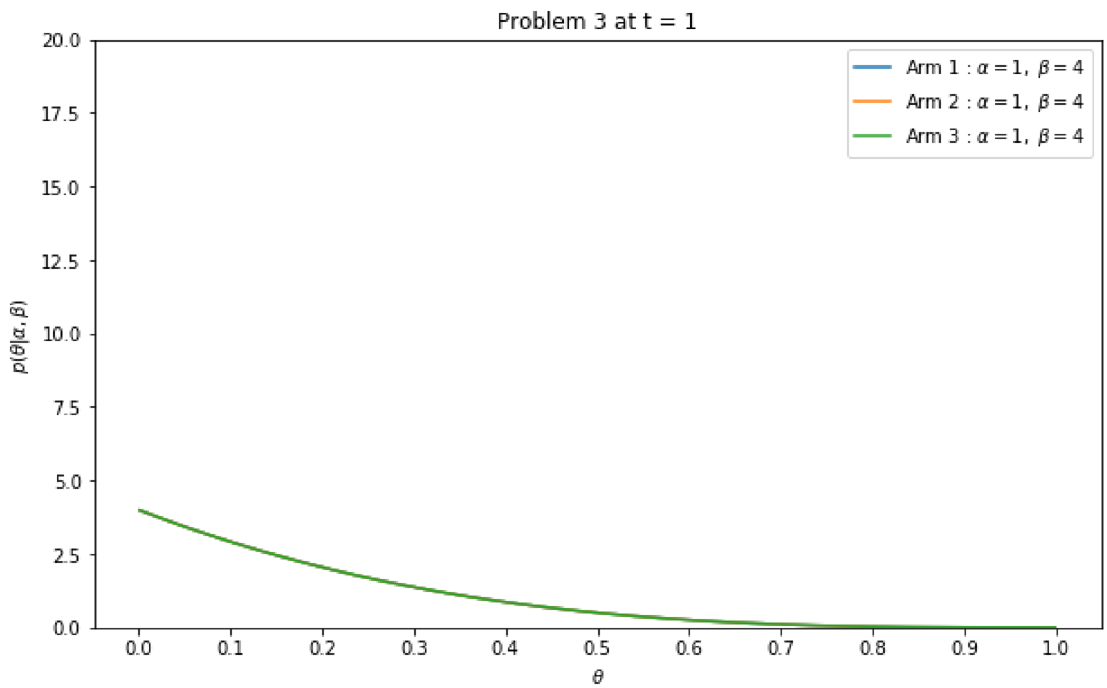
```
Mean at time t = 5000 is:  [ 0.49728316  0.11764706  0.15       ]
```



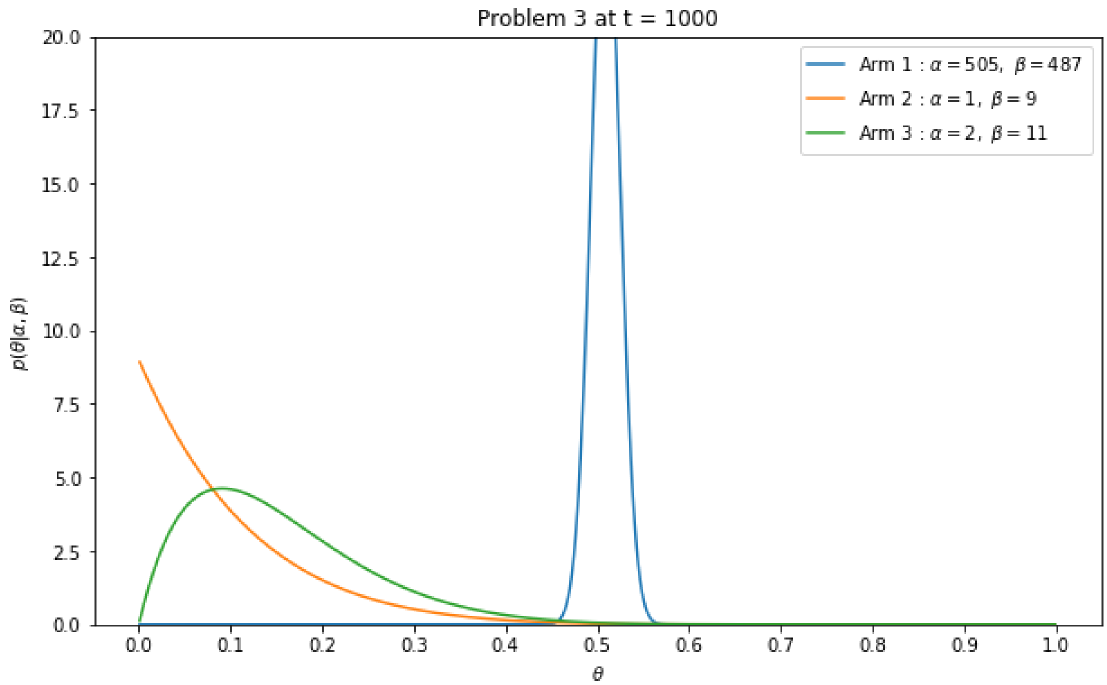

```
Mean at time t = 10000 is:  [ 0.50060205  0.11111111  0.14285714]
```



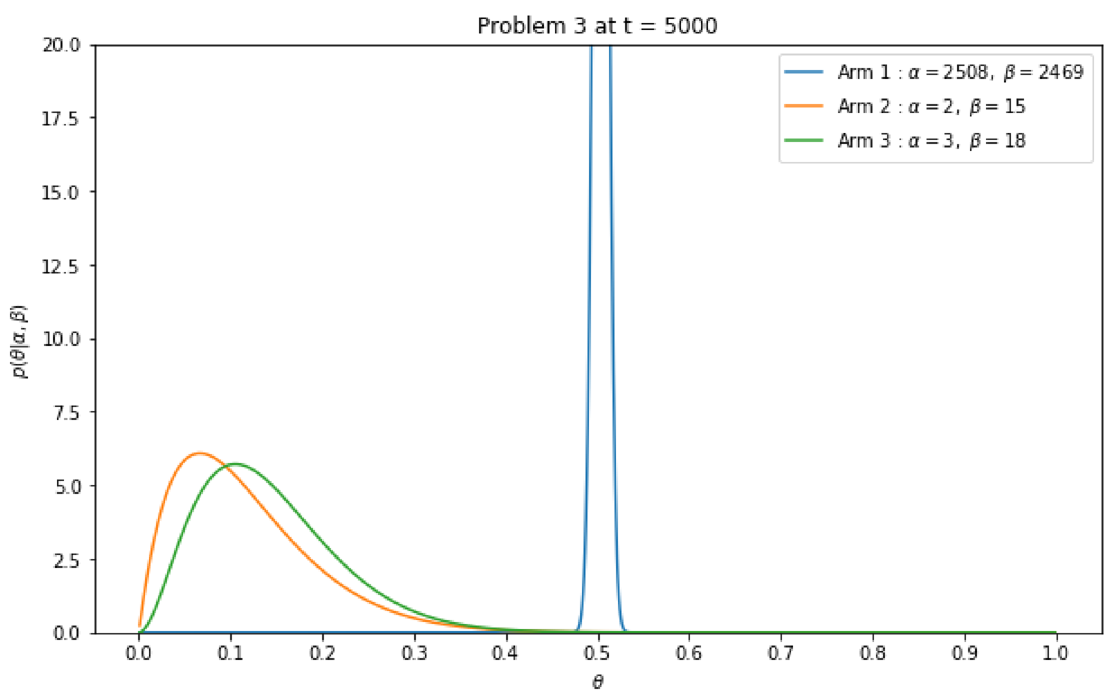
```
Mean at time t = 1 is:  [ 0.2  0.2  0.2]
```



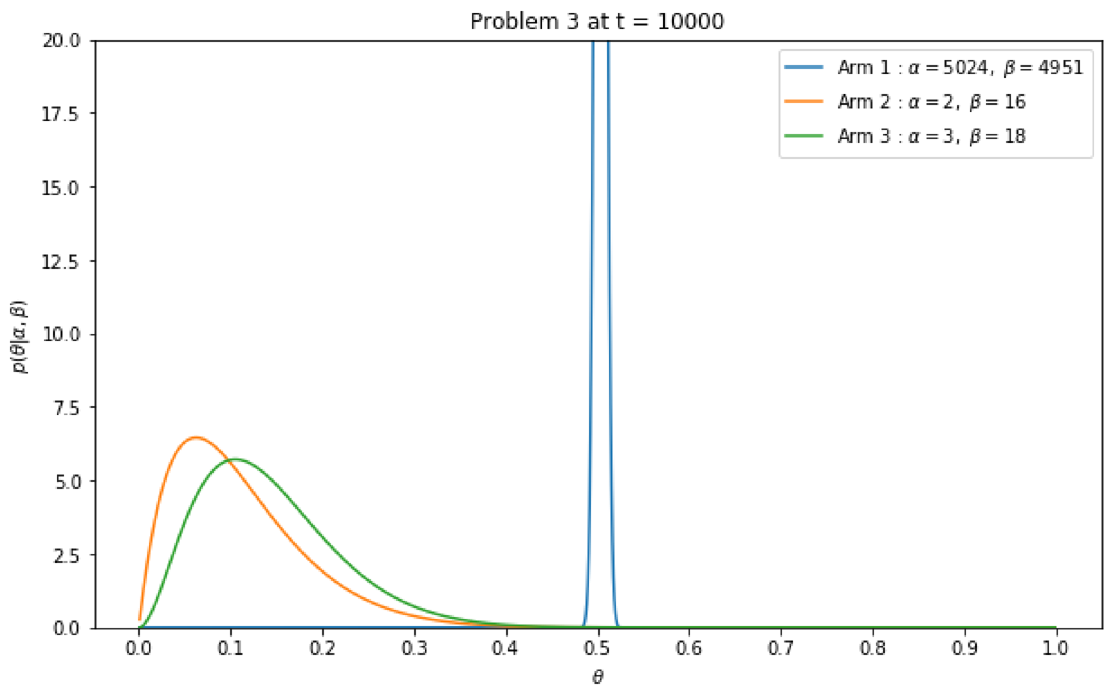

```
Mean at time t = 1000 is:  [ 0.50907258  0.1         0.15384615]
```




```
Mean at time t = 5000 is:  [ 0.50391802  0.11764706  0.14285714]
```



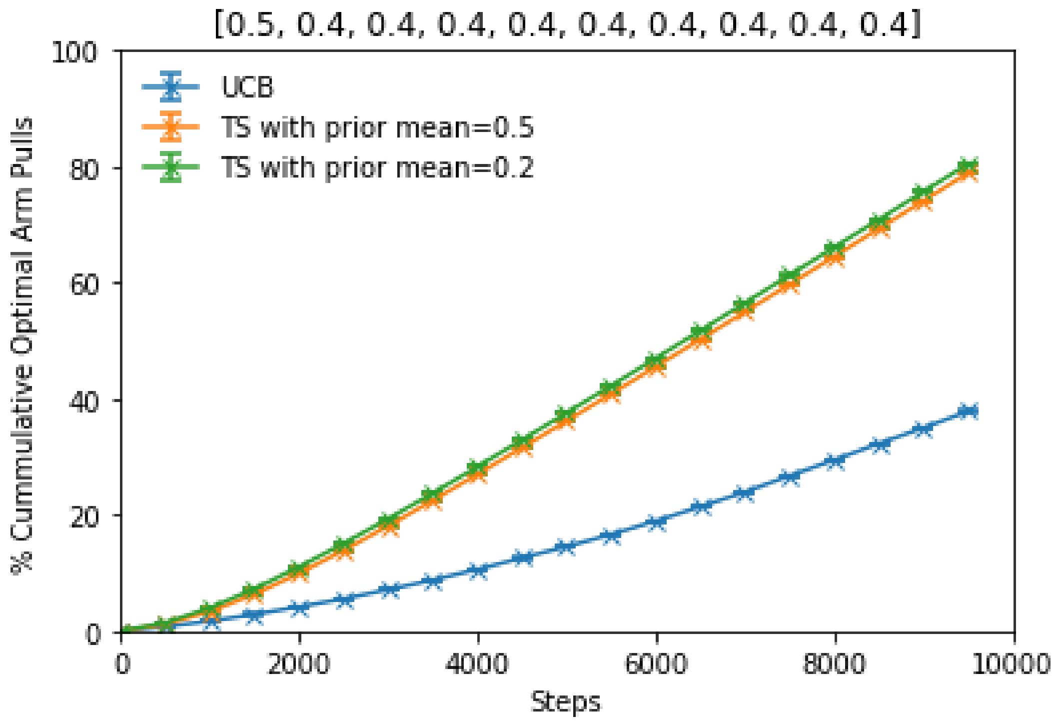
```
Mean at time t = 10000 is:  [ 0.50365915  0.11111111  0.14285714]
```



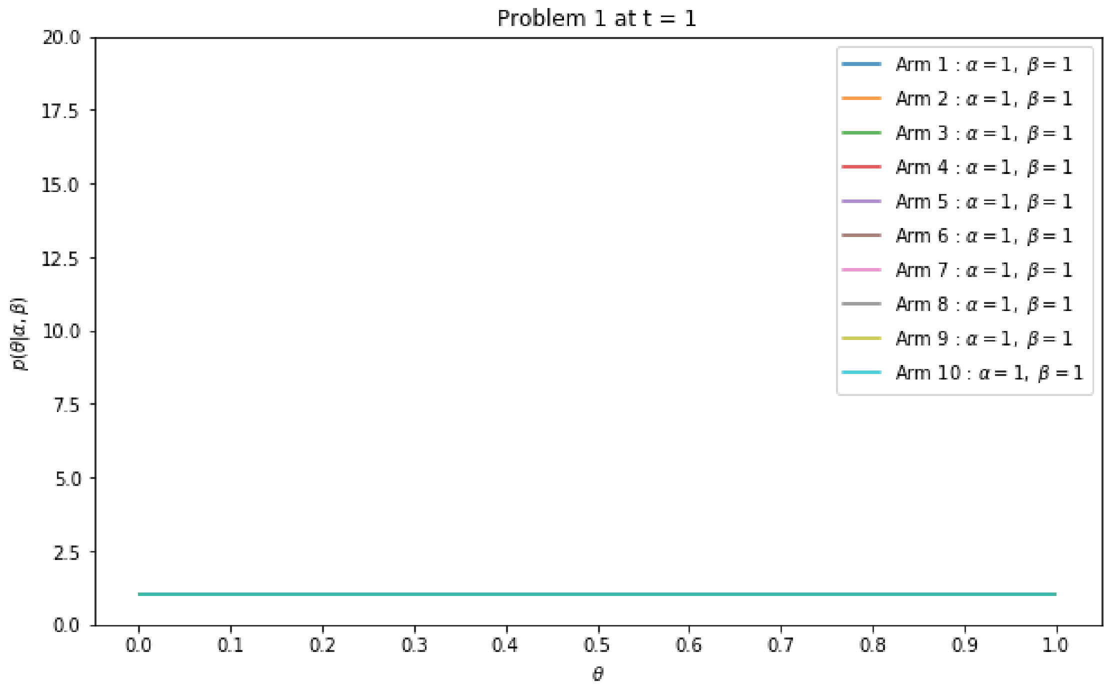

```
In [13]: horizon = 10000
         replications = 100

         arms_prob = [[0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4], [0.5, 0.48, 0.48, 0.4
         types = ['UCB','TS M0.5','TS M0.2']
         m_len = len(types)
         success = [1,1]
         failure = [1,4]
         optimalpulls = 'Cum'

         for problem in range(3): # Repeating for 3 problems
             optimal_arm_pulls_sum = np.zeros([m_len,horizon]) # Storing variables returned by
             regret_per_round_sum = np.zeros([m_len,horizon])
             optimal_arm_means_stderr = np.zeros([m_len,horizon,2])
             regret_means_stderr = np.zeros([m_len,horizon,2])
             optimal_arm_percentage = np.zeros([m_len])
             total_regret = np.zeros([m_len])
             success_ret = np.zeros([2,4,len(arms_prob[problem])])
             failure_ret = np.zeros([2,4,len(arms_prob[problem])])


             regret_per_round_sum[0,:],regret_means_stderr[0,:,:], optimal_arm_pulls_sum[0,:],
```

```
success_ret[0,:,:],failure_ret[0,:,:],regret_per_round_sum[1,:],regret_means_stder
success_ret[1,:,:],failure_ret[1,:,:],regret_per_round_sum[2,:],regret_means_stder


step = 500
print("\n")
print("optimal_arm_percentage")
tableIt(optimal_arm_percentage)
print("\n")
print("total_regret")
tableIt(total_regret)


# Calling function to plot % Optimal Arm Pulls Vs Time steps with error bars
plotCumOptimalArmPulls(horizon,optimal_arm_means_stderr,optimal_arm_pulls_sum,prob
plot_arm_distribution(success_ret[0],failure_ret[0])
plot_arm_distribution(success_ret[1],failure_ret[1])
```

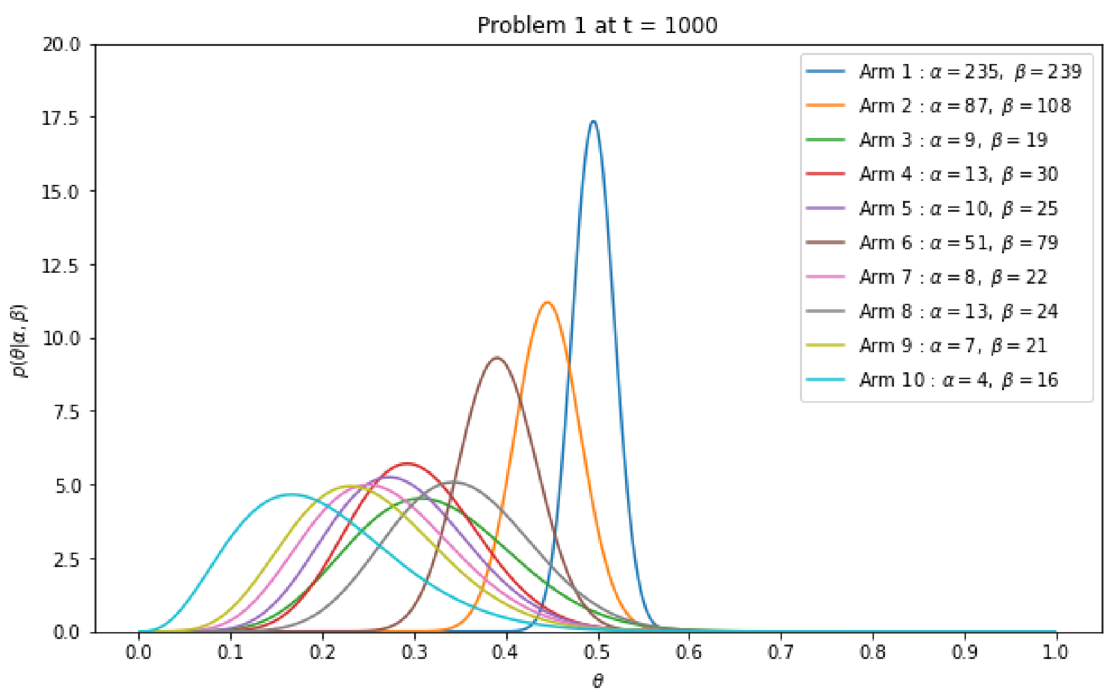
```
Total Optimal arm pulls : 4085.01 and percentage is : 40.8501
Total Regret : 591.499

Total Optimal arm pulls : 8373.46 and percentage is : 83.7346
Total Regret : 162.654

Total Optimal arm pulls : 8530.6 and percentage is : 85.306
Total Regret : 146.94


optimal_arm_percentage
         0
0  40.8501
1  83.7346
2  85.3060


total_regret
         0
0  591.499
1  162.654
2  146.940
```



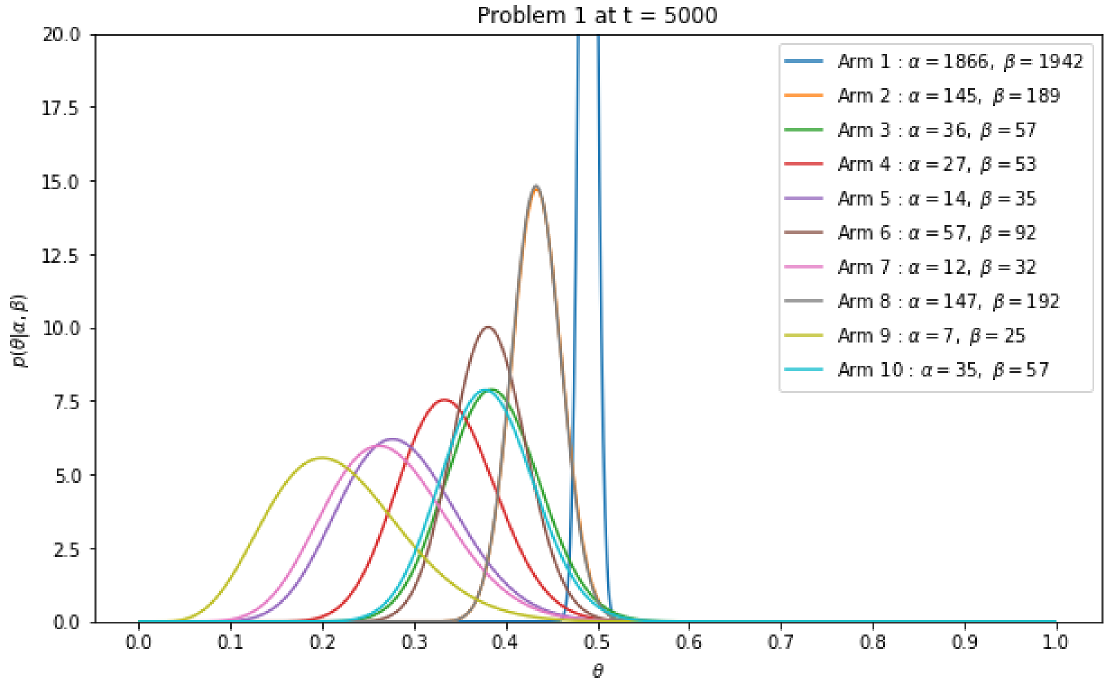

```
optimal_arm_stderr
[[ 0.03249615  0.04439595  0.04918333  0.04828043  0.04950758]
 [ 0.04737088  0.04439595  0.02179449  0.02179449  0.01705872]
 [ 0.04877499  0.03756328  0.02712932  0.00994987  0.01959592]]
Mean at time t = 1 is:  [ 0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5]
```



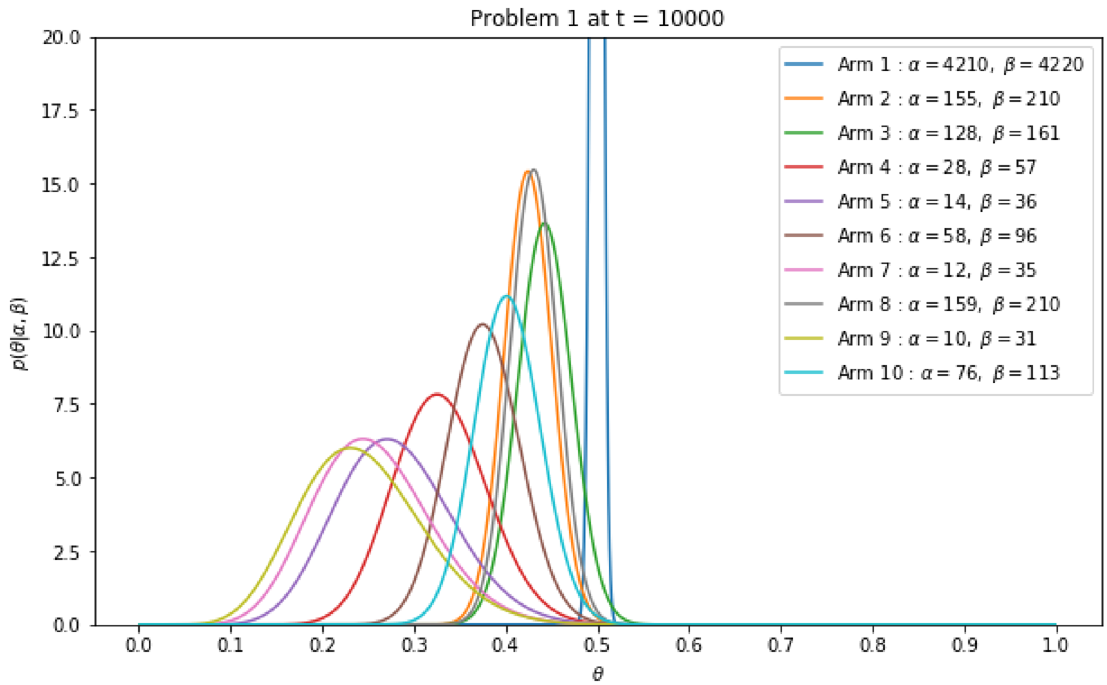

```
Mean at time t = 1000 is:  [ 0.49578059  0.44615385  0.32142857  0.30232558  0.28571429  0.392
  0.26666667  0.35135135  0.25        0.2       ]
```



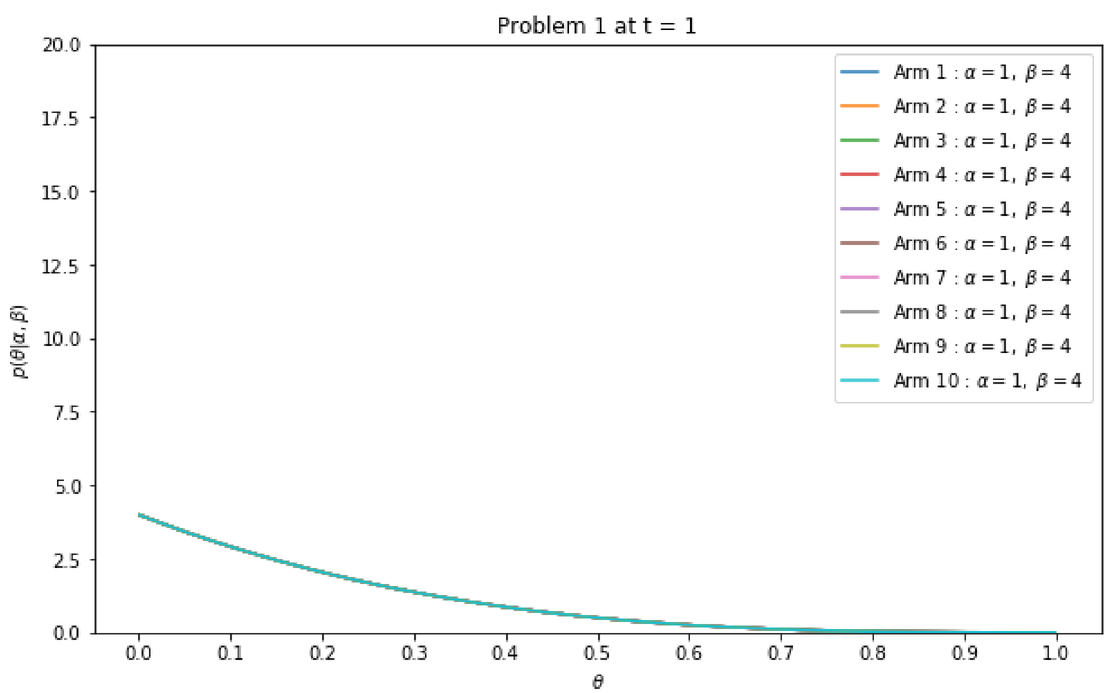
```
Mean at time t = 5000 is:  [ 0.49002101  0.43413174  0.38709677  0.3375      0.28571429  0.3825
  0.27272727  0.43362832  0.21875     0.38043478]
```



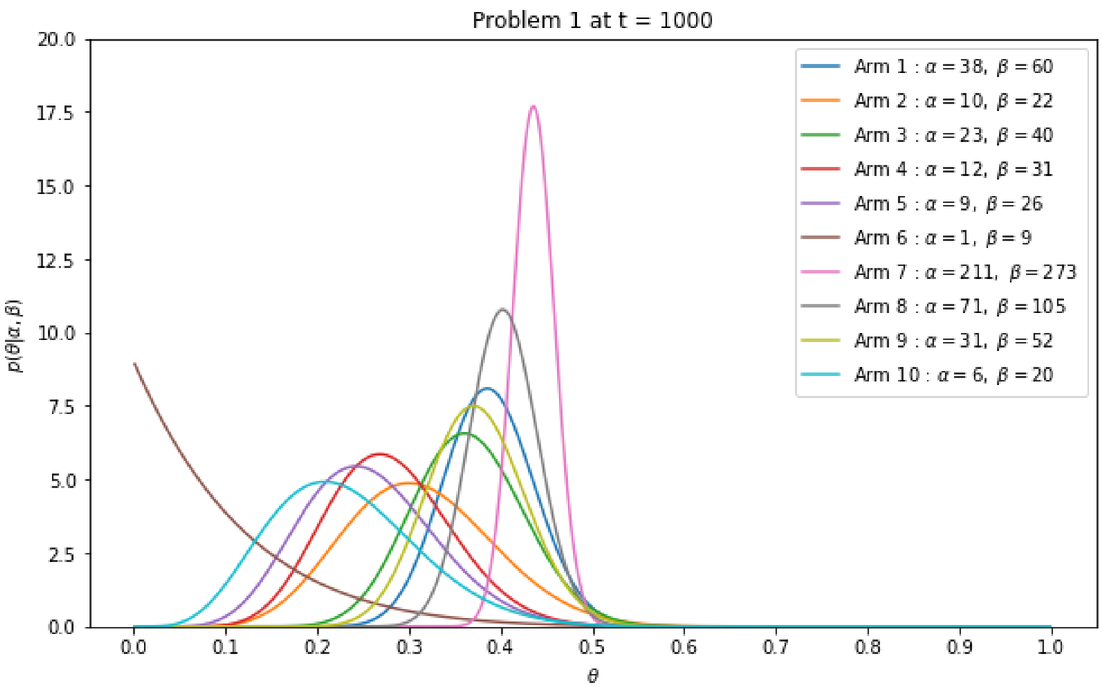

```
Mean at time t = 10000 is:  [ 0.49940688  0.42465753  0.44290657  0.32941176  0.28        0.376
  0.25531915  0.43089431  0.24390244  0.4021164 ]
```



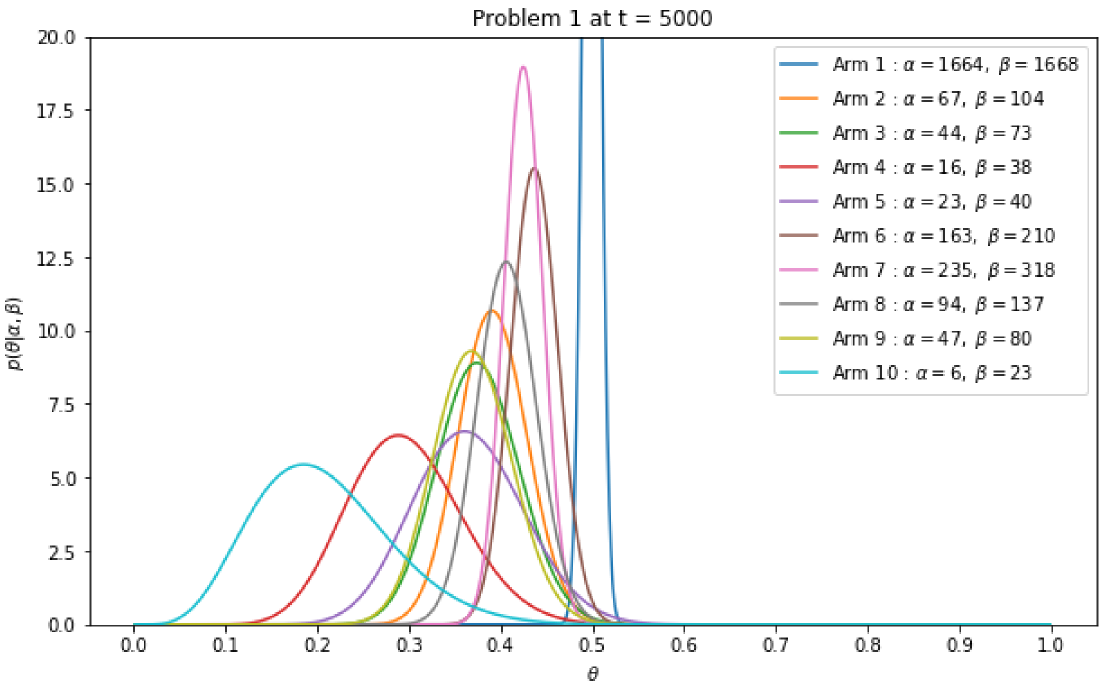

```
Mean at time t = 1 is:  [ 0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2]
```



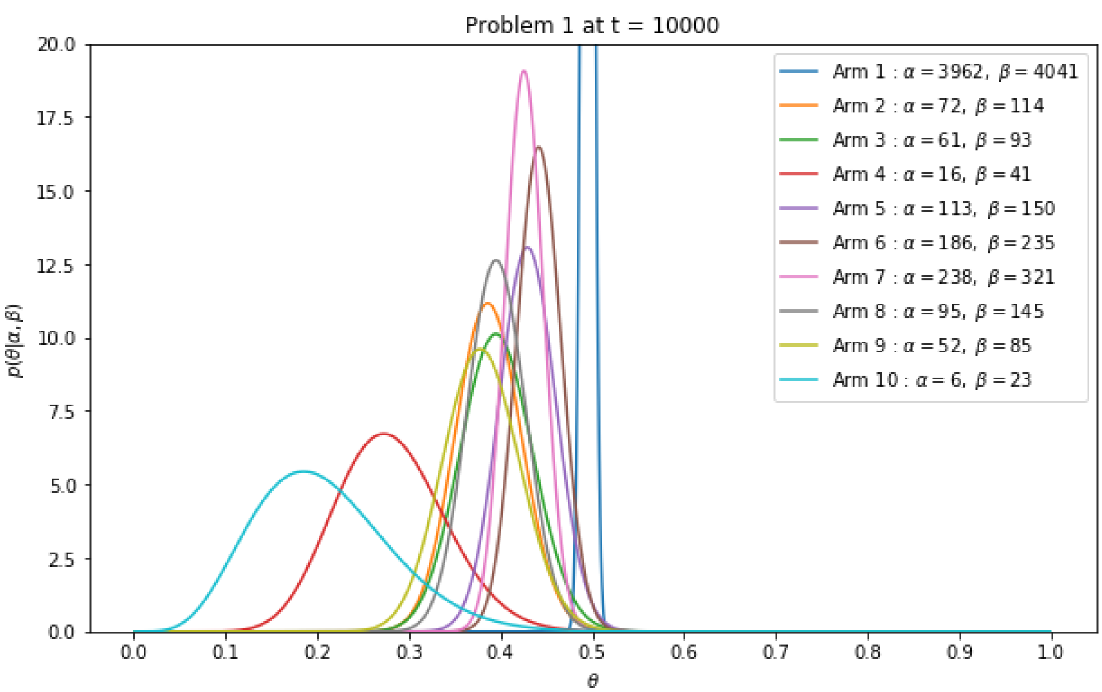
```
Mean at time t = 1000 is:  [ 0.3877551   0.3125      0.36507937  0.27906977  0.25714286  0.1
  0.43595041  0.40340909  0.37349398  0.23076923]
```



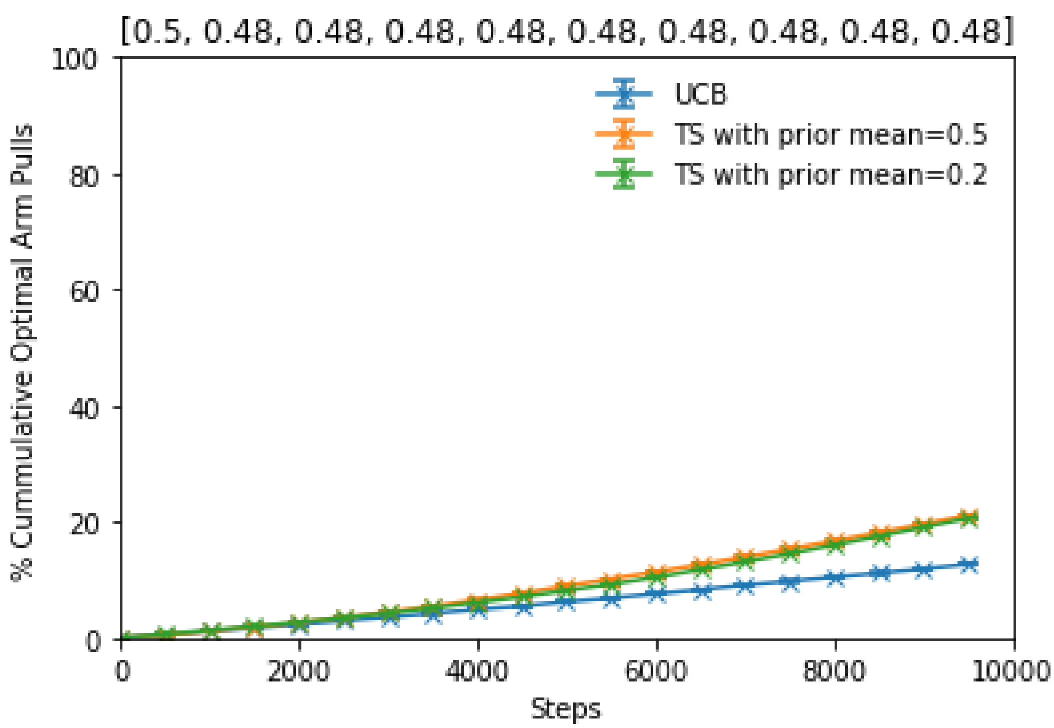

```
Mean at time t = 5000 is:  [ 0.49939976  0.39181287  0.37606838  0.2962963   0.36507937  0.4369
  0.42495479  0.40692641  0.37007874  0.20689655]
```



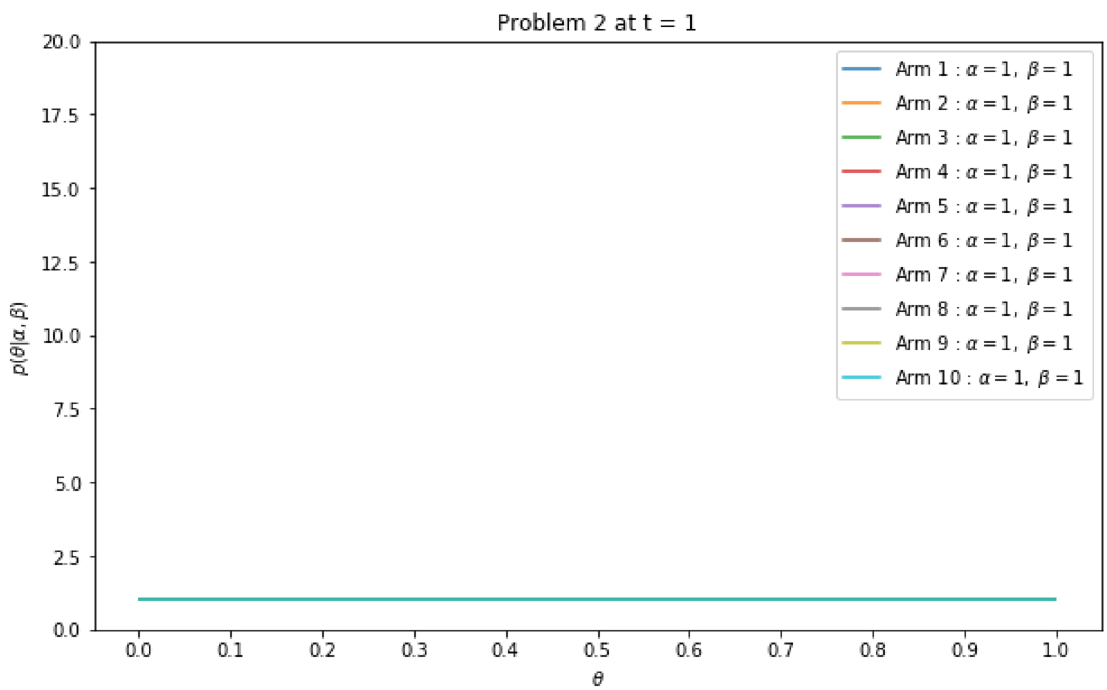

```
Mean at time t = 10000 is:  [ 0.49506435  0.38709677  0.3961039   0.28070175  0.42965779  0.44
  0.42576029  0.39583333  0.37956204  0.20689655]
```



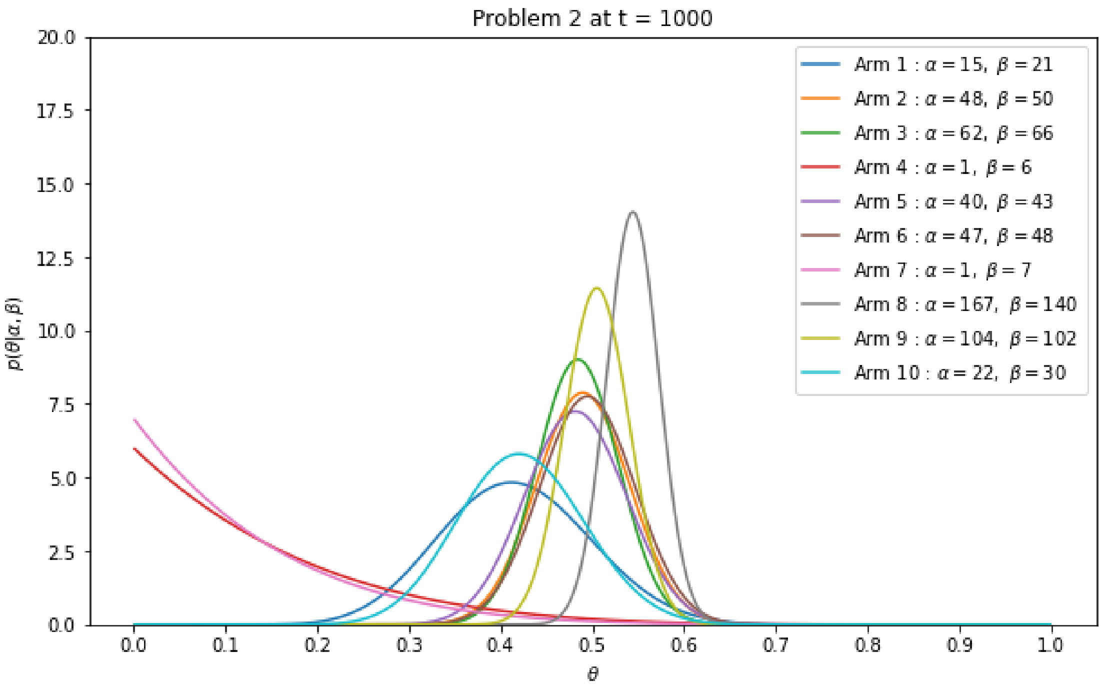
```
Total Optimal arm pulls : 1351.15 and percentage is : 13.5115
Total Regret : 172.977

Total Optimal arm pulls : 2266.69 and percentage is : 22.6669
Total Regret : 154.6662

Total Optimal arm pulls : 2225.47 and percentage is : 22.2547
Total Regret : 155.4906


optimal_arm_percentage
         0
0  13.5115
1  22.6669
2  22.2547


total_regret
          0
0  172.9770
1  154.6662
2  155.4906
```



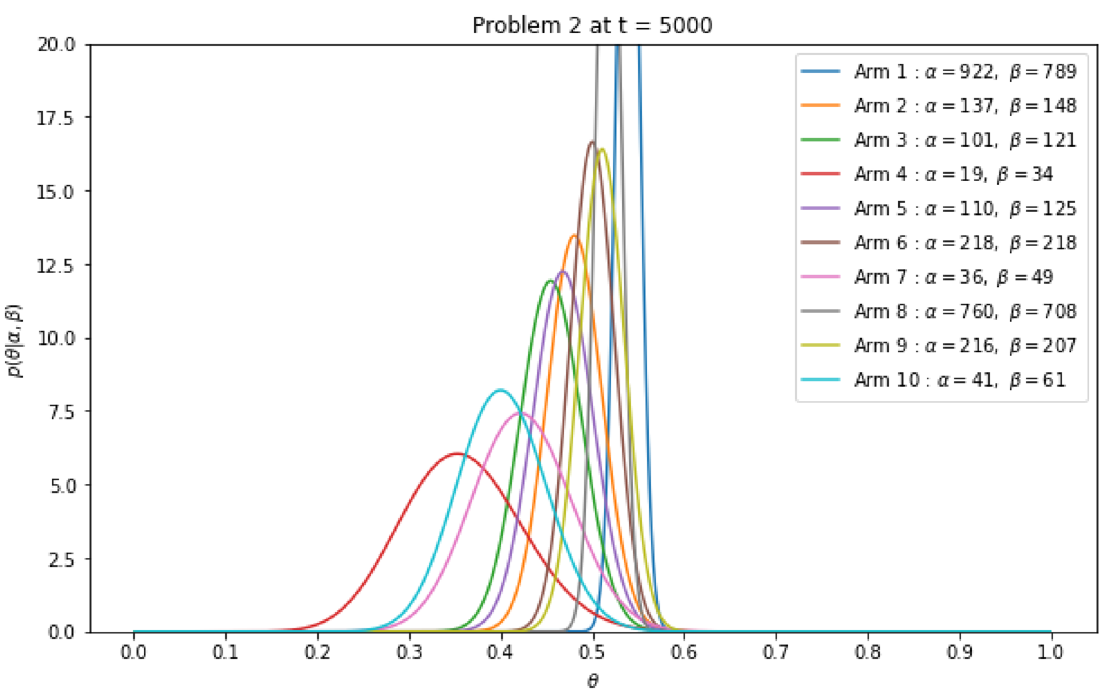
```
optimal_arm_stderr
[[ 0.03        0.03249615  0.03666061  0.0346987   0.03570714]
 [ 0.03249615  0.04        0.04702127  0.04439595  0.04624932]
 [ 0.03        0.03128898  0.04208325  0.04828043  0.04582576]]
Mean at time t = 1 is:  [ 0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5]
```



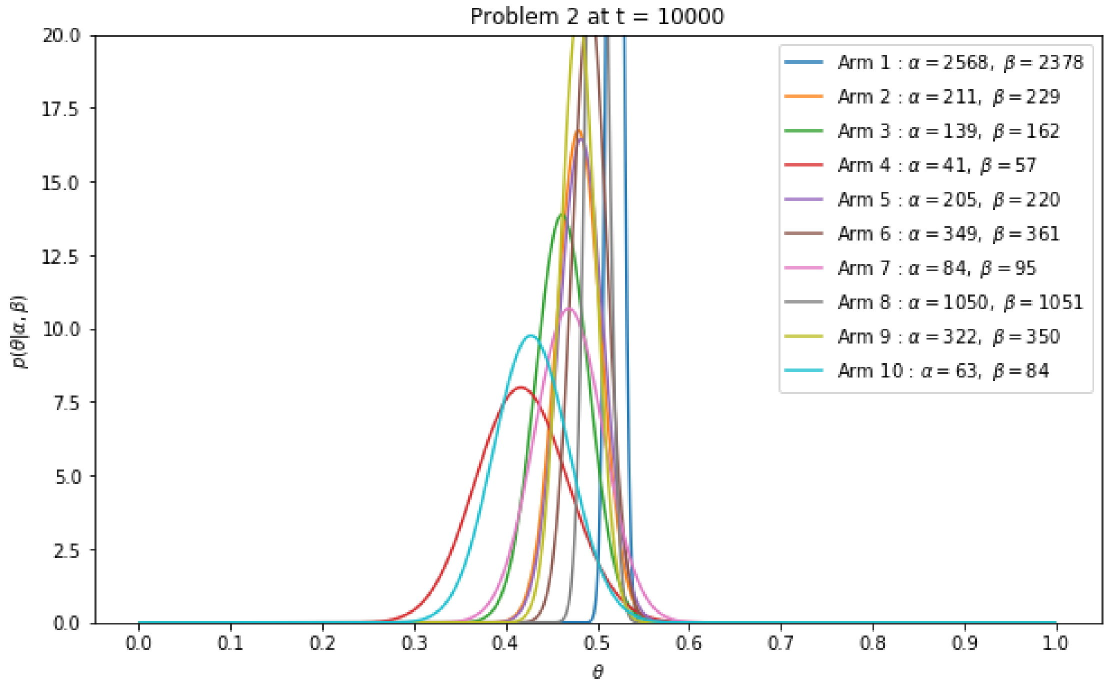

```
Mean at time t = 1000 is:  [ 0.41666667  0.48979592  0.484375    0.14285714  0.48192771  0.494
  0.125       0.54397394  0.50485437  0.42307692]
```



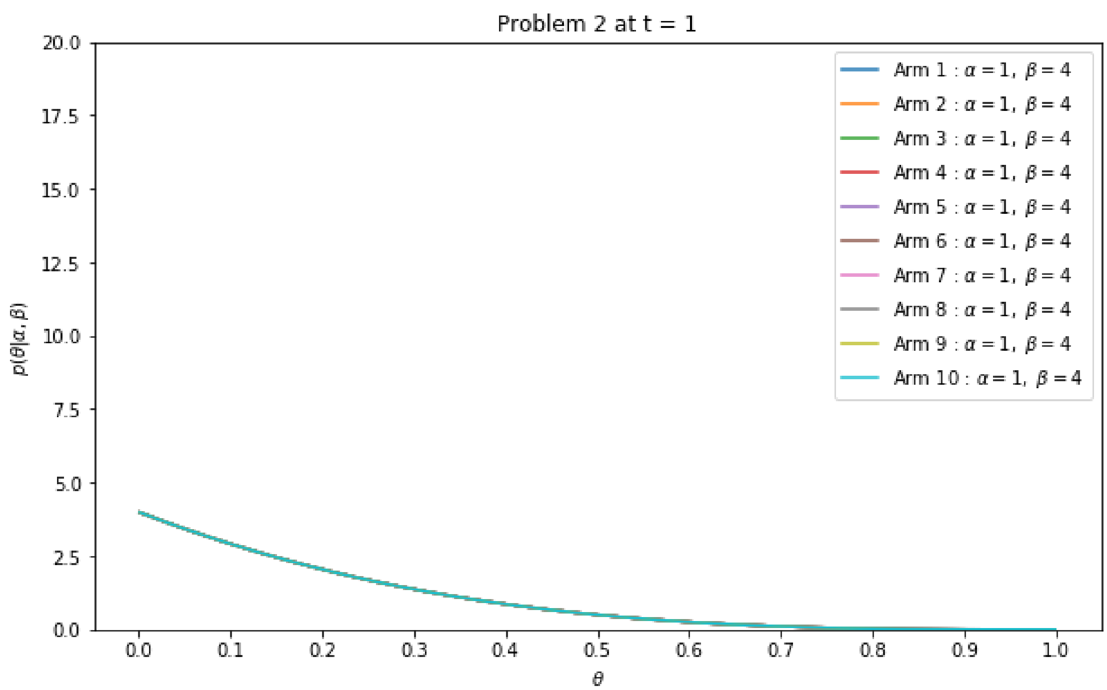

```
Mean at time t = 5000 is:  [ 0.53886616  0.48070175  0.45495495  0.35849057  0.46808511  0.5
  0.42352941  0.51771117  0.5106383   0.40196078]
```



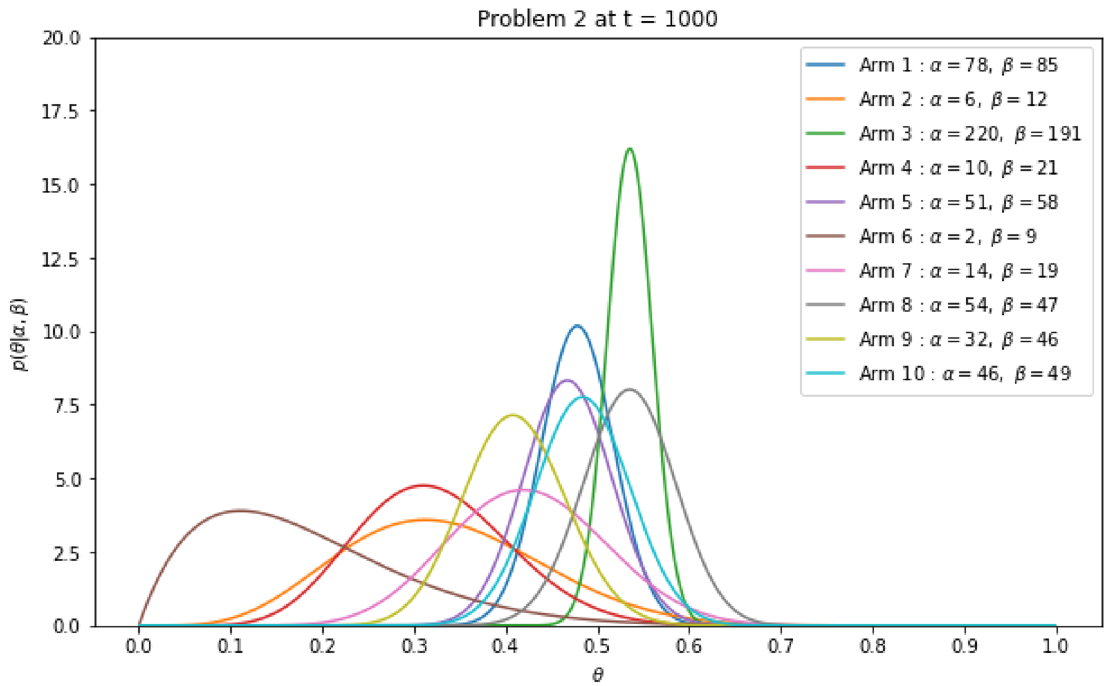
```
Mean at time t = 10000 is:  [ 0.51920744  0.47954545  0.46179402  0.41836735  0.48235294  0.49
  0.46927374  0.49976202  0.47916667  0.42857143]
```



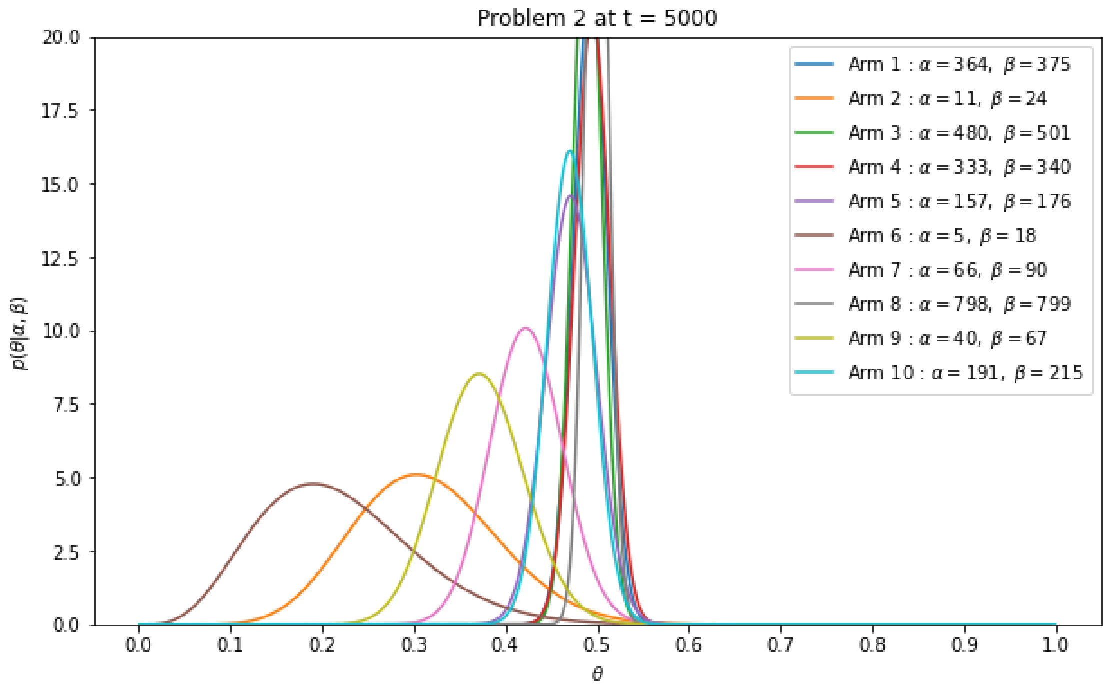

```
Mean at time t = 1 is:  [ 0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2  0.2]
```



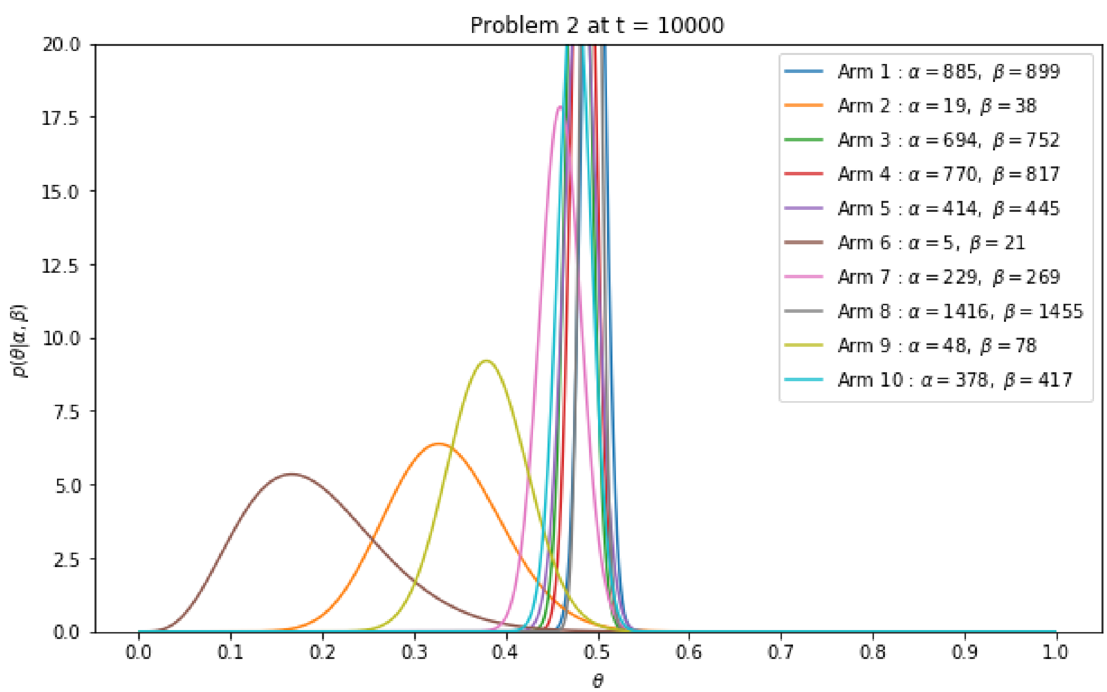
```
Mean at time t = 1000 is:  [ 0.47852761  0.33333333  0.53527981  0.32258065  0.46788991  0.1818
  0.42424242  0.53465347  0.41025641  0.48421053]
```



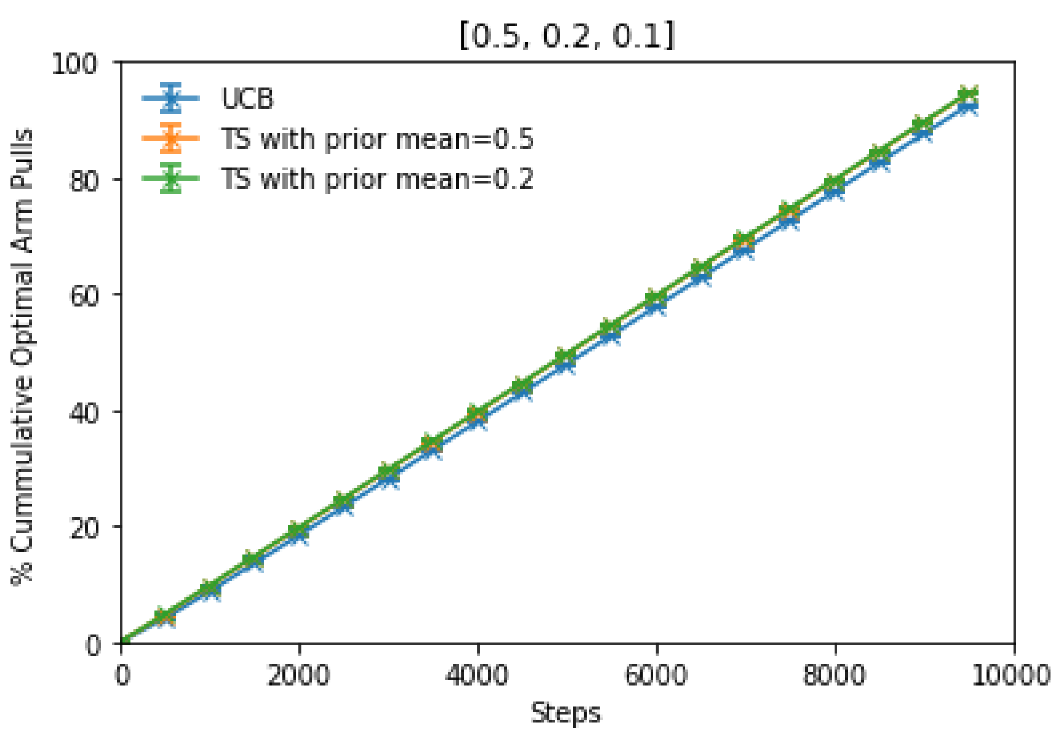

```
Mean at time t = 5000 is:  [ 0.49255751  0.31428571  0.48929664  0.49479941  0.47147147  0.2173
  0.42307692  0.49968691  0.37383178  0.47044335]
```



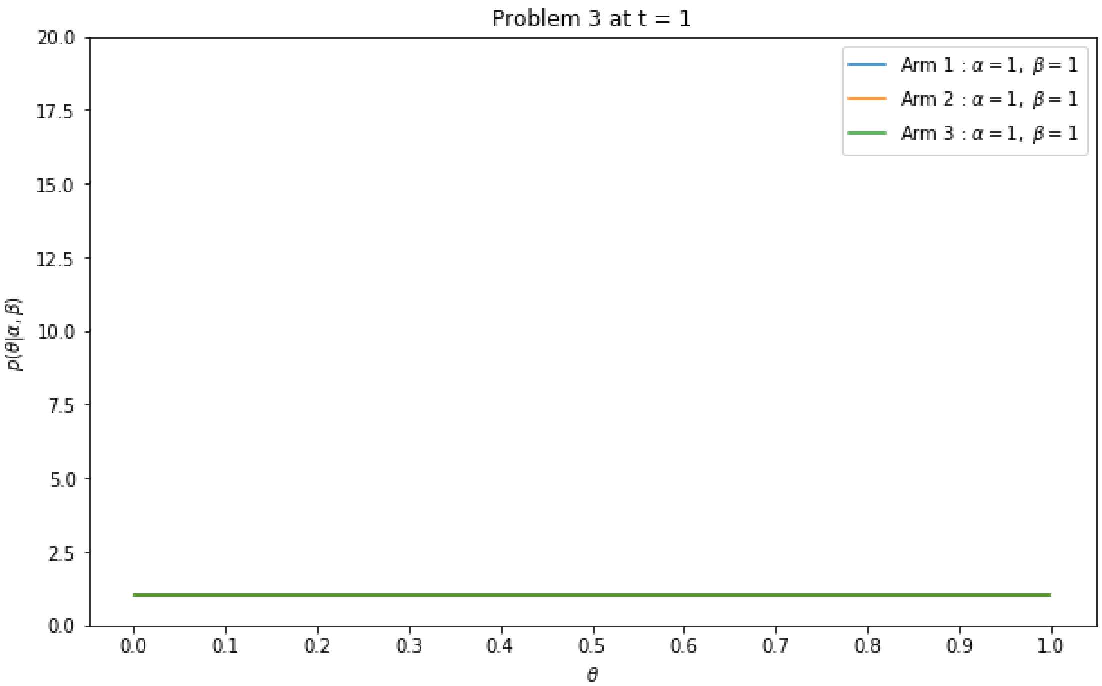

```
Mean at time t = 10000 is:  [ 0.49607623  0.33333333  0.47994467  0.48519219  0.48195576  0.192
  0.45983936  0.49320794  0.38095238  0.4754717 ]
```



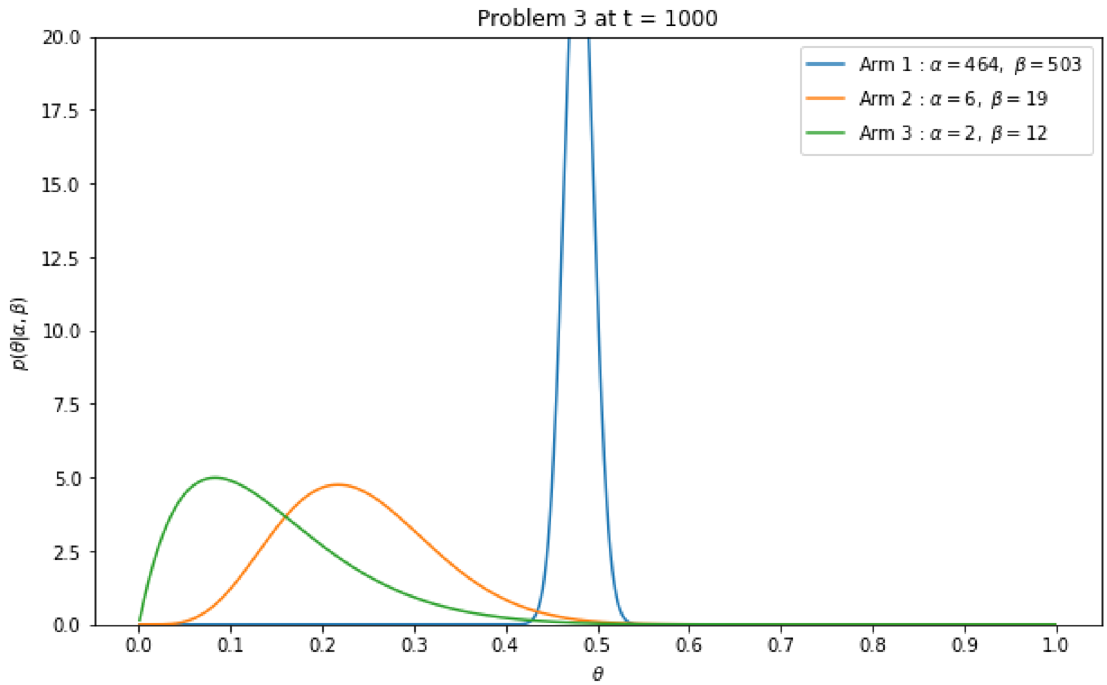
```
Total Optimal arm pulls : 9749.23 and percentage is : 97.4923
Total Regret : 84.45

Total Optimal arm pulls : 9952.5 and percentage is : 99.525
Total Regret : 16.055

Total Optimal arm pulls : 9963.59 and percentage is : 99.6359
Total Regret : 12.261


optimal_arm_percentage
         0
0  97.4923
1  99.5250
2  99.6359


total_regret
        0
0  84.450
1  16.055
2  12.261
```



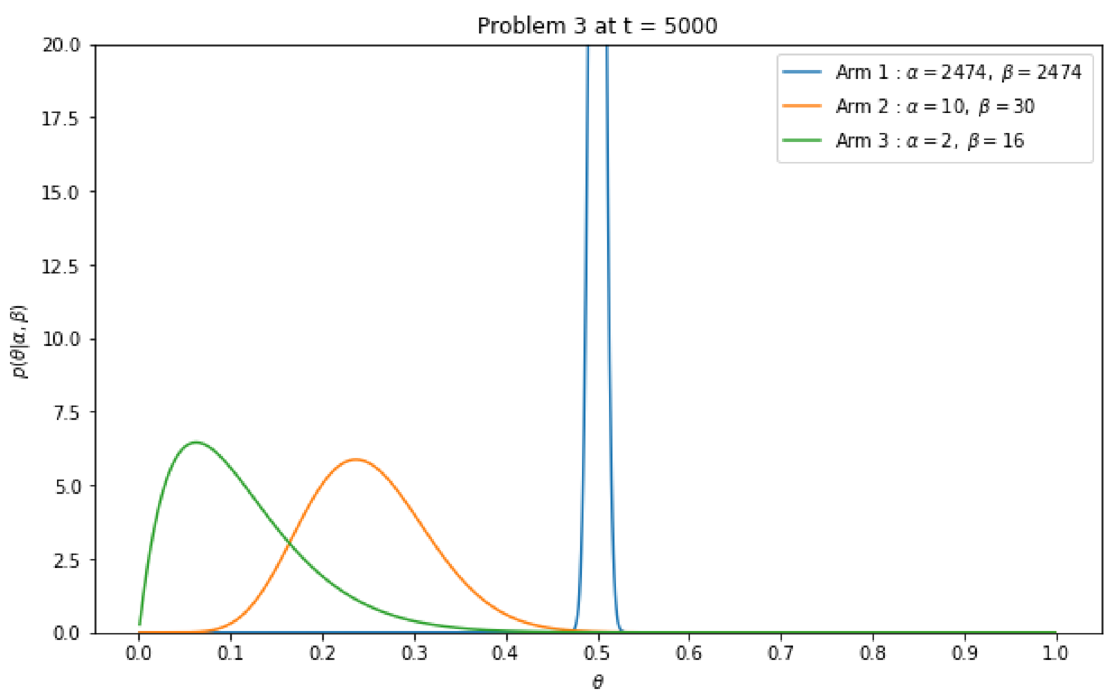
```
optimal_arm_stderr
[[ 0.01705872  0.01959592  0.          0.00994987  0.014     ]
 [ 0.00994987  0.          0.          0.          0.        ]
 [ 0.00994987  0.00994987  0.          0.          0.        ]]
Mean at time t = 1 is:  [ 0.5  0.5  0.5]
```



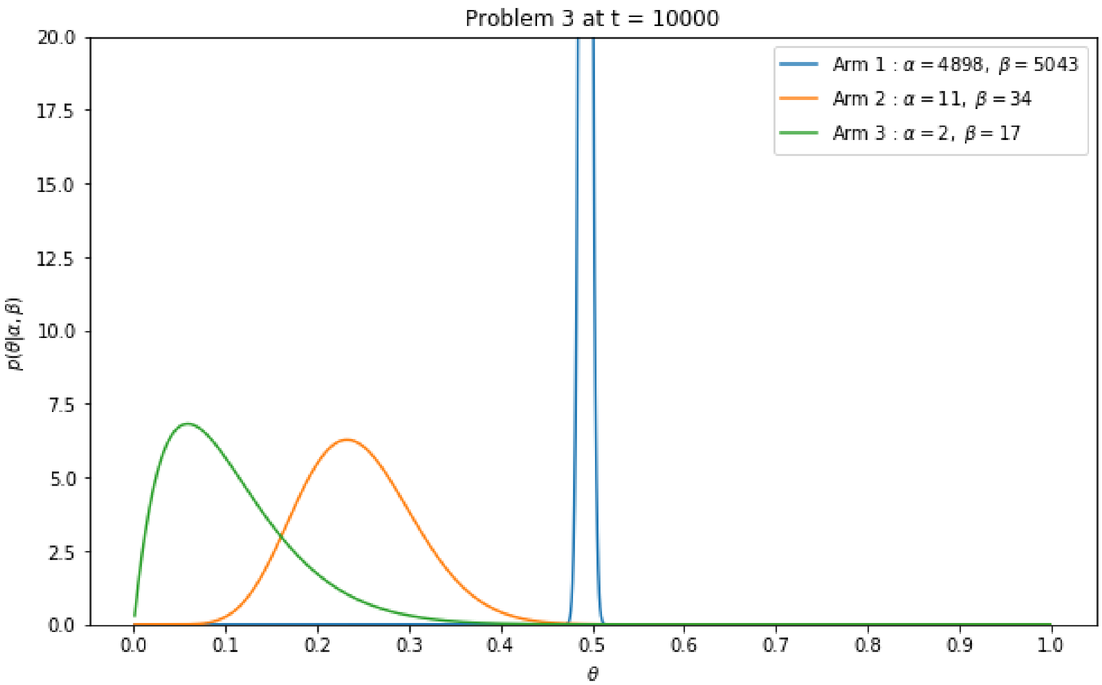

```
Mean at time t = 1000 is:  [ 0.47983454  0.24        0.14285714]
```



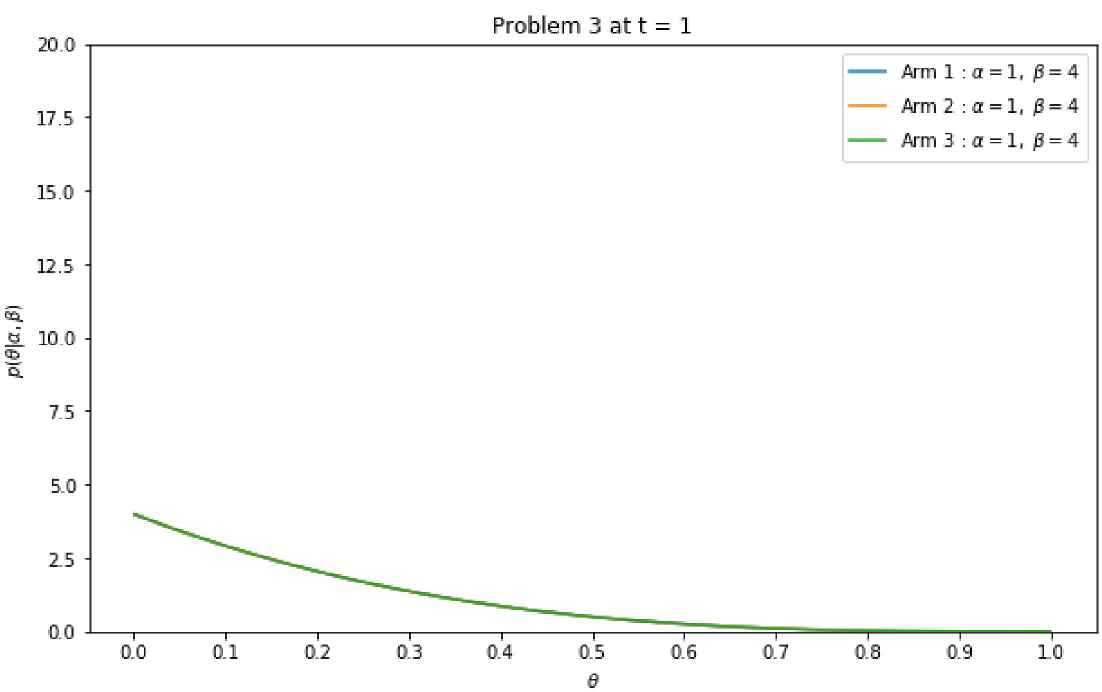

```
Mean at time t = 5000 is:  [ 0.5         0.25        0.11111111]
```



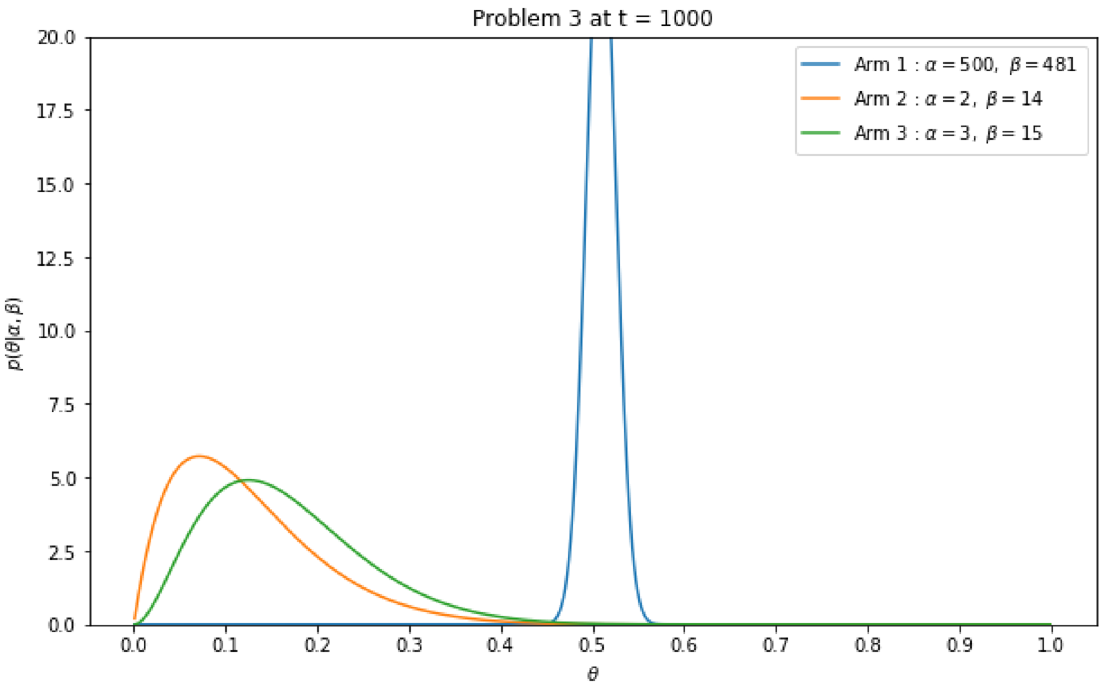
```
Mean at time t = 10000 is:  [ 0.49270697  0.24444444  0.10526316]
```



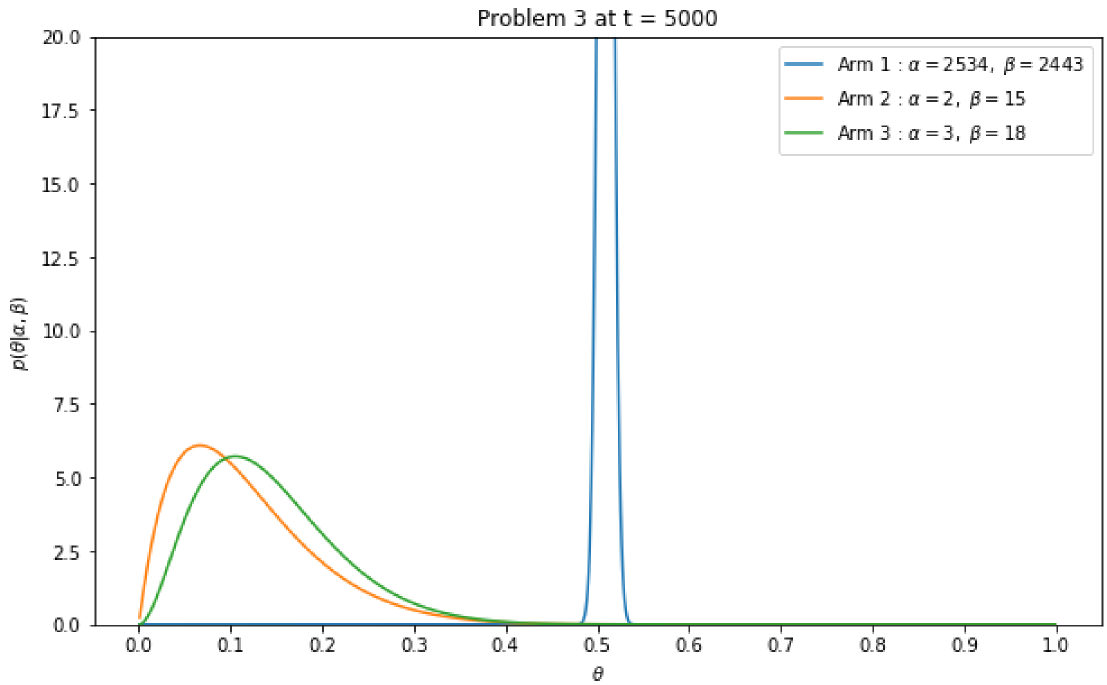

```
Mean at time t = 1 is:  [ 0.2  0.2  0.2]
```



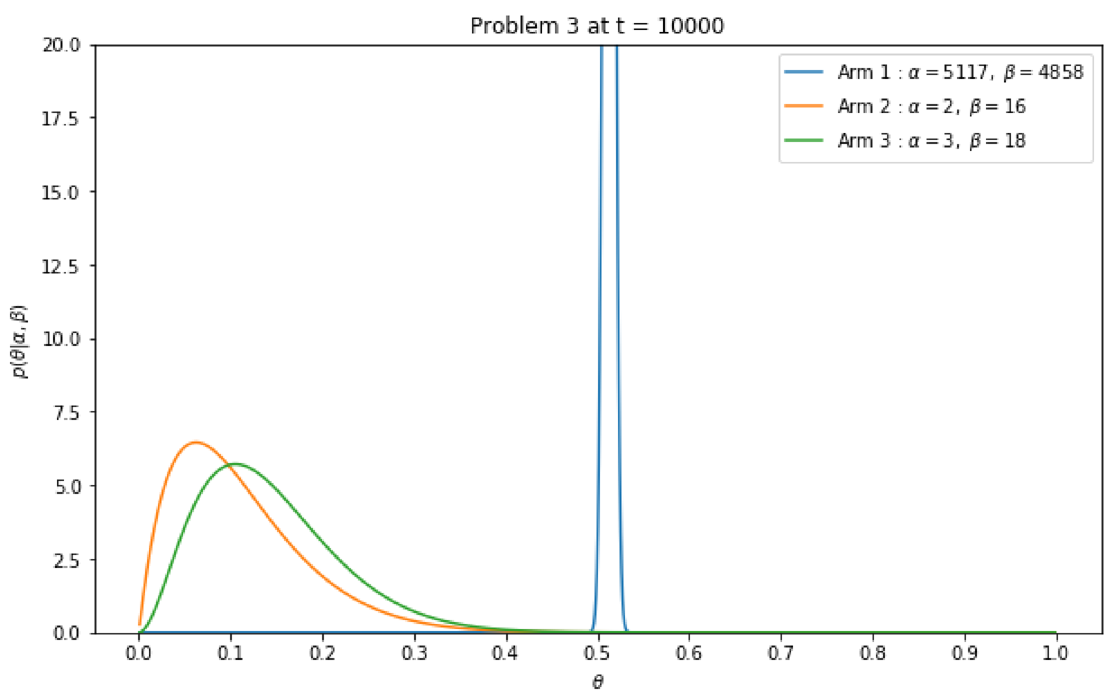
```
Mean at time t = 1000 is:  [ 0.509684    0.125       0.16666667]
```




```
Mean at time t = 5000 is:  [ 0.50914205  0.11764706  0.14285714]
```



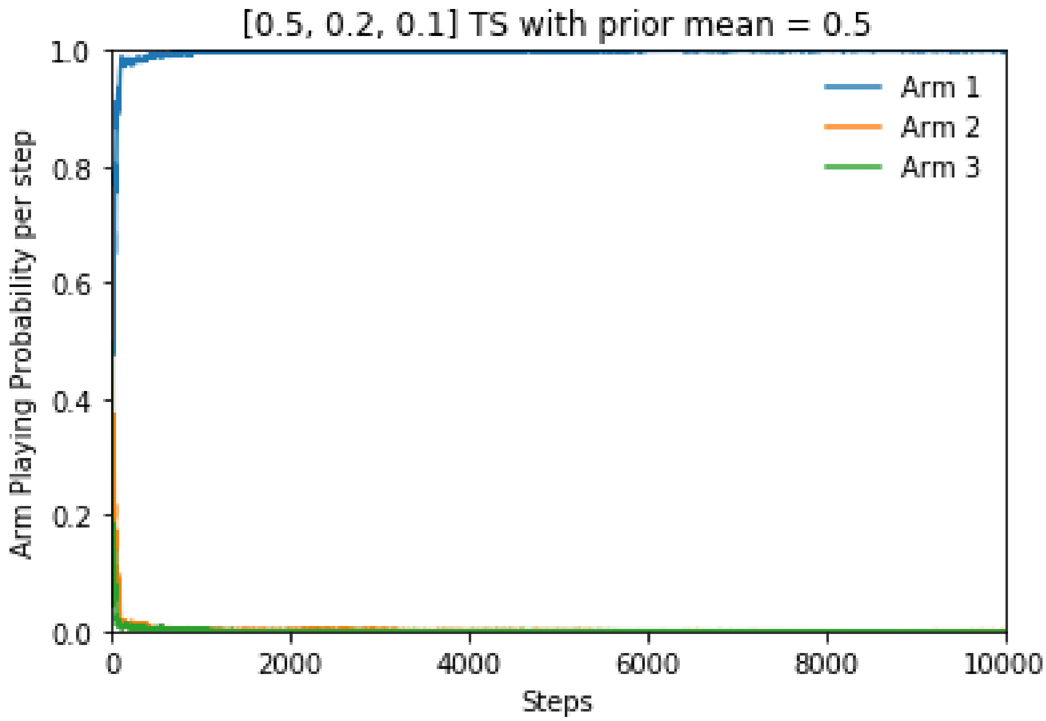

```
Mean at time t = 10000 is:  [ 0.51298246  0.11111111  0.14285714]
```

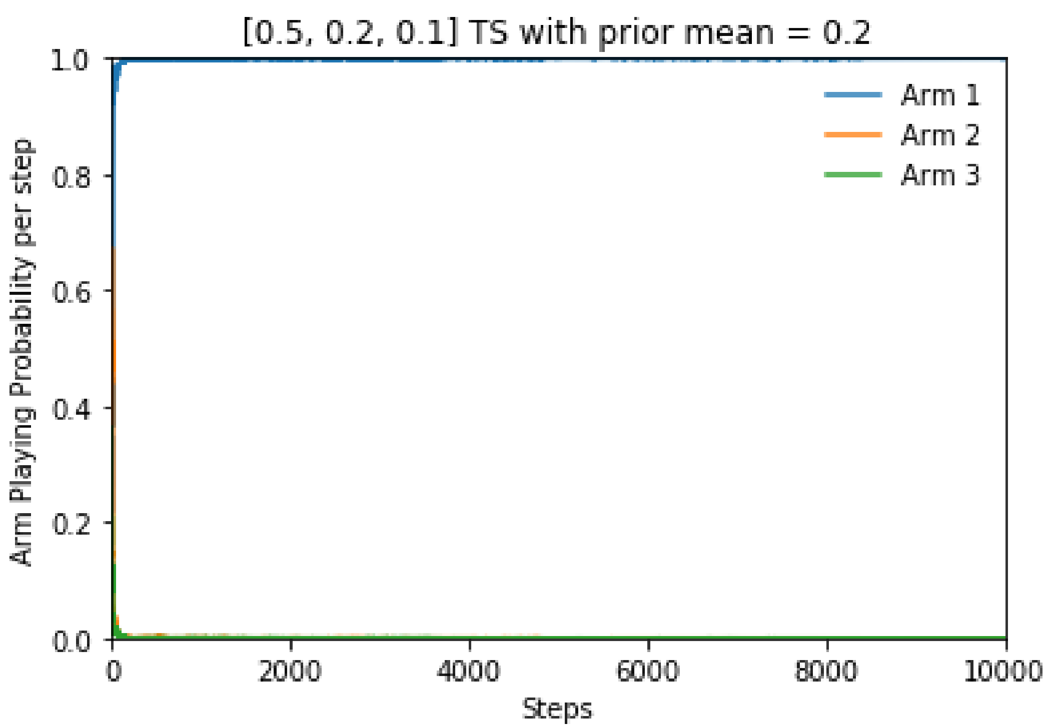


```
In [14]: # Saving variables
         dill.dump_session(filename)

In [15]: def get_arm_playing_prob(success,failure,trials=10000):

             sample_means = np.zeros([trials,len(success)])
             arm_played_times = np.zeros(trials)
             arm_paying_prob = np.zeros(len(success))
             for i in range(len(success)):
                 sample_means[:,i] = np.random.beta(success[i],failure[i],trials)

             for j in range(trials):
                 arm_played_times[j] = np.argmax(sample_means[j,:])

             for i in range(len(success)):
                 arm_paying_prob[i] = (arm_played_times == i).sum()/trials

             return arm_paying_prob

In [31]: # Plotting Arm Playing Probability Vs Time steps for TS
         def plot_arms_playing_prob(horizon,arm_play_Prob,problem):
             x = np.arange(horizon)
             for i in range(len(arm_play_Prob[1,:])):
                 plt.plot(x,arm_play_Prob[:,i])
             plt.xlabel('Steps')
             plt.ylabel('Arm Playing Probability per step')
             if len(arm_play_Prob[1,:]) == 3:
                 plt.legend(["Arm 1","Arm 2","Arm 3"],loc="best",frameon=False)
             else:
                 plt.legend(["Arm 1","Arm 2","Arm 3","Arm 4","Arm 5","Arm 6","Arm 7","Arm 8","
             if problem % 2 == 0:
                 plt.title("Problem 3: TS with prior mean = 0.2")
             else:
                 plt.title("Problem 3: TS with prior mean = 0.5")

             plt.xlim((0,10000))
             plt.ylim((0,1))
             plt.savefig('Arm_play_prob_'+str(problem)+'.png',dpi=300)
             plt.show()



In [17]: # Implemention Thompson Sampling for plotting arm playing probability at each time st
         def TSForPlottingArmProb(horizon,replications,arms_prob,alpha,beta,optimalpulls,proble
             optimal_arm = 0
         #     optimal_arm_pulls_per_round = np.zeros([horizon,replications])
         #     regret_per_round = np.zeros([horizon,replications])
             arm_playing_prob = np.zeros([horizon,len(arms_prob)])
```

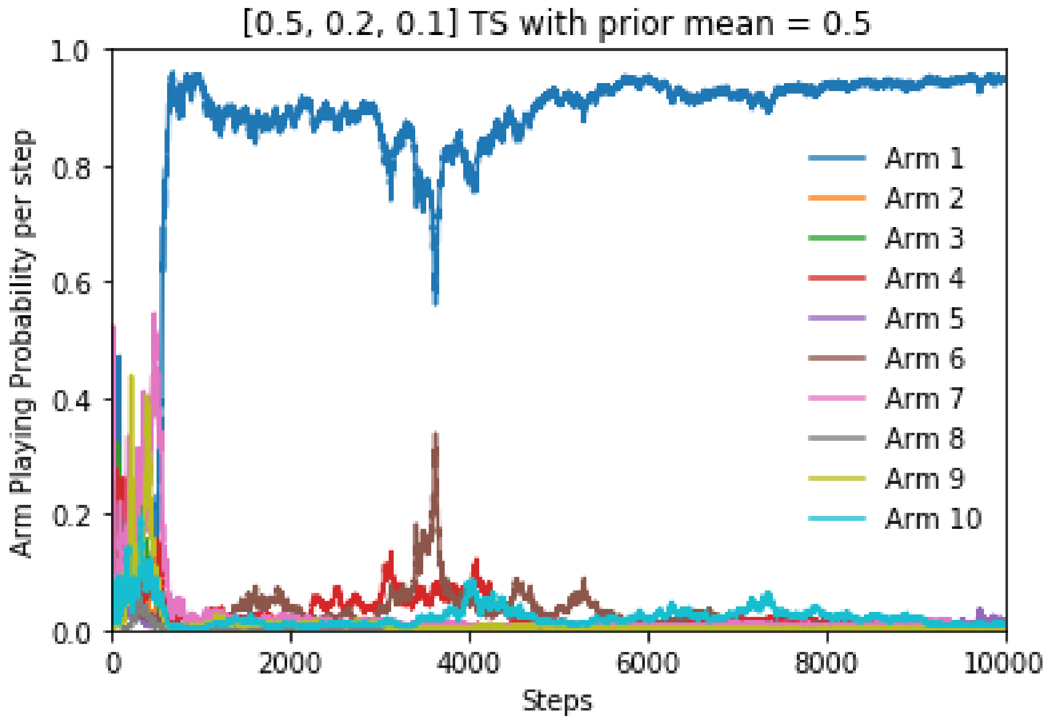
```
    savepoints = (0,1000,5000,9999)
    success_ret = np.zeros([len(savepoints),len(arms_prob)])
    faliure_ret = np.zeros([len(savepoints),len(arms_prob)])

    for r in range(replications):
        arm_pulls = [0]*len(arms_prob)
        success = np.array(alpha)
        failure = np.array(beta)
        t = 0
        s = 0
        while t < horizon:
            if t in savepoints and r == replications-1:
                success_ret[s] = success
                faliure_ret[s] = failure
                s+=1

            #Picking arm according to Posterior distribution
            sample_means = [0]*len(arms_prob)
            for i in range(len(arms_prob)):
                sample_means[i] = np.random.beta(success[i],failure[i])
            arm_selected = np.argmax(sample_means)

            arm_playing_prob[t] = get_arm_playing_prob(success,failure)

            arm_pulls[arm_selected] += 1
            temp = np.random.binomial(1, arms_prob[arm_selected])
            success[arm_selected] += temp
            failure[arm_selected] += 1 - temp

            t+=1

#     plot_arms_playing_prob(horizon,arm_playing_prob,problem)
    return arm_playing_prob
```

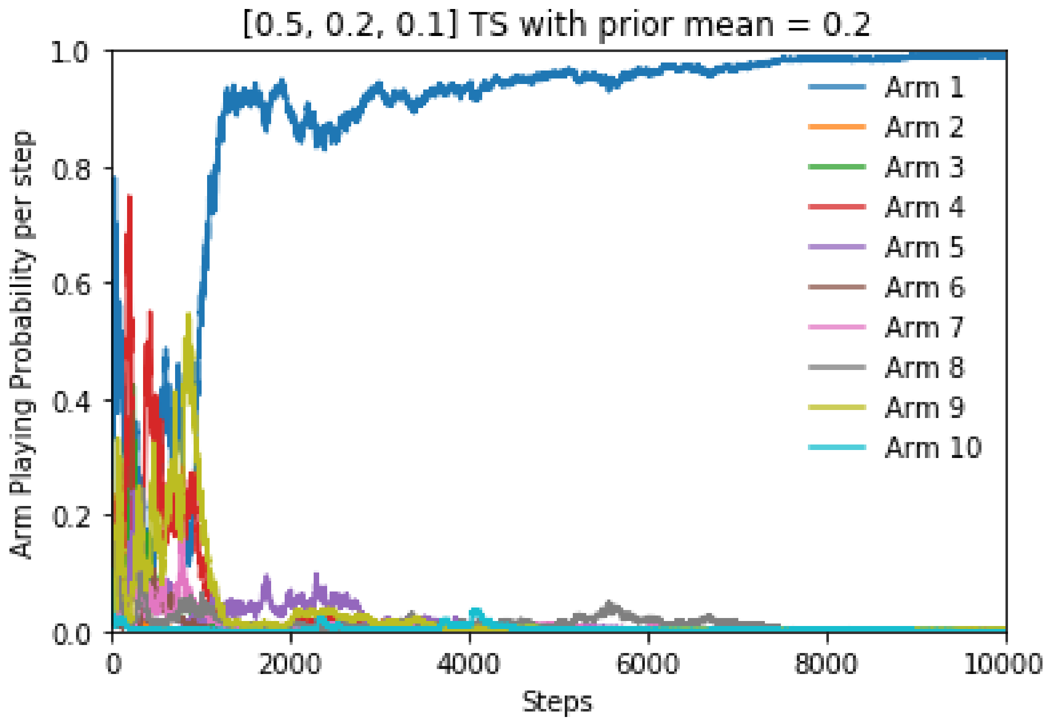
In [18]:
```
arm_playing_prob1 = TSForPlottingArmProb(10000,1,[0.5, 0.2, 0.1],[1,1,1],[1,1,1],'Avg
plot_arms_playing_prob(10000,arm_playing_prob1,1)

arm_playing_prob2 = TSForPlottingArmProb(10000,1,[0.5, 0.2, 0.1],[1,1,1],[4,4,4],'Avg
plot_arms_playing_prob(10000,arm_playing_prob2,2)
```


[0.5, 0.2, 0.1] TS with prior mean = 0.5
Arm Playing Probability per step
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10000
Steps
Arm 1
Arm 2
Arm 3

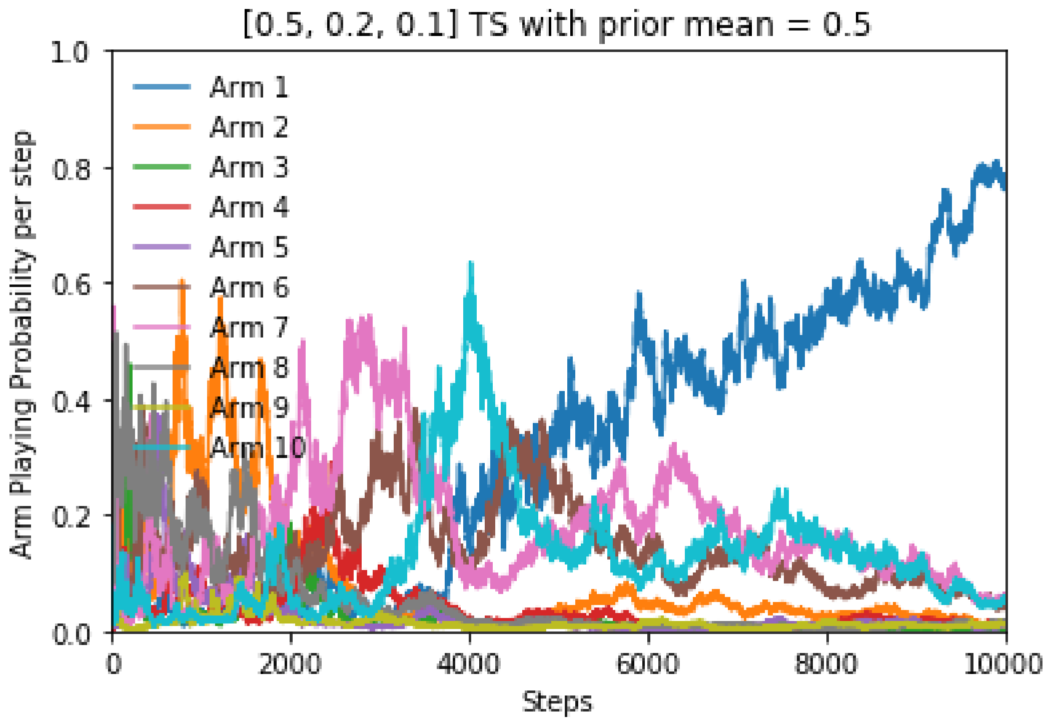

[0.5, 0.2, 0.1] TS with prior mean = 0.2
Arm Playing Probability per step
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10000
Steps
Arm 1
Arm 2
Arm 3

```
In [19]: arm_playing_prob3 = TSForPlottingArmProb(10000,1,[0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0
         plot_arms_playing_prob(10000,arm_playing_prob3,3)

         arm_playing_prob4 = TSForPlottingArmProb(10000,1,[0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0
         plot_arms_playing_prob(10000,arm_playing_prob4,4)
```

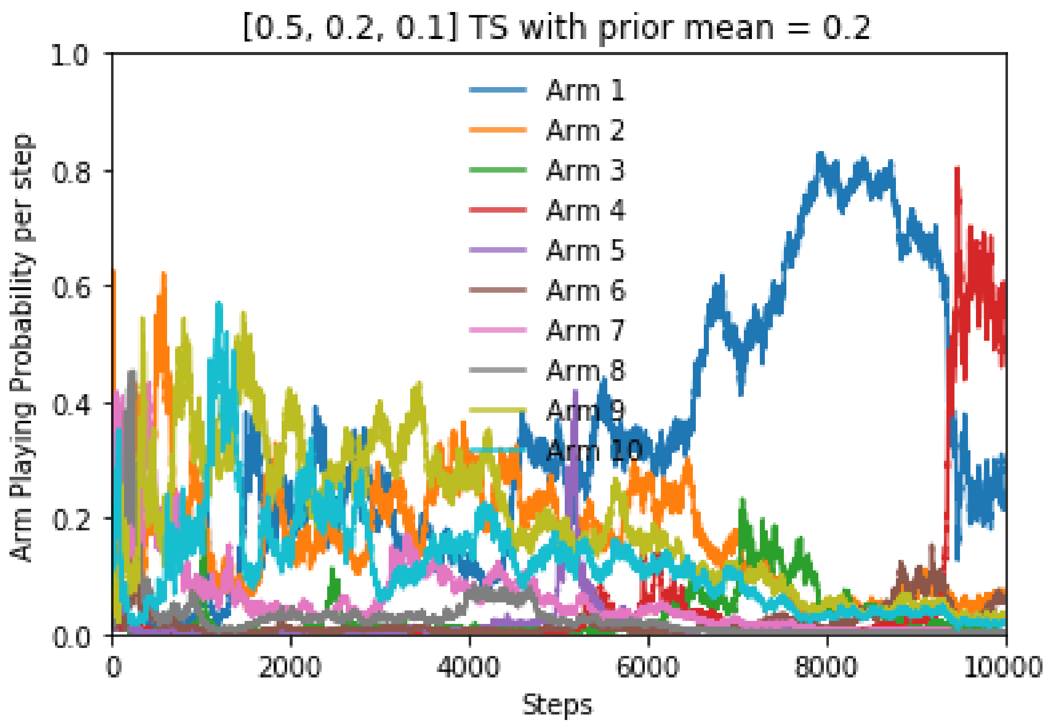





```
In [20]: arm_playing_prob5 = TSForPlottingArmProb(10000,1,[0.5, 0.48, 0.48, 0.48, 0.48, 0.48, 0
         plot_arms_playing_prob(10000,arm_playing_prob5,5)

         arm_playing_prob6 = TSForPlottingArmProb(10000,1,[0.5, 0.48, 0.48, 0.48, 0.48, 0.48, 0
         plot_arms_playing_prob(10000,arm_playing_prob6,6)
```



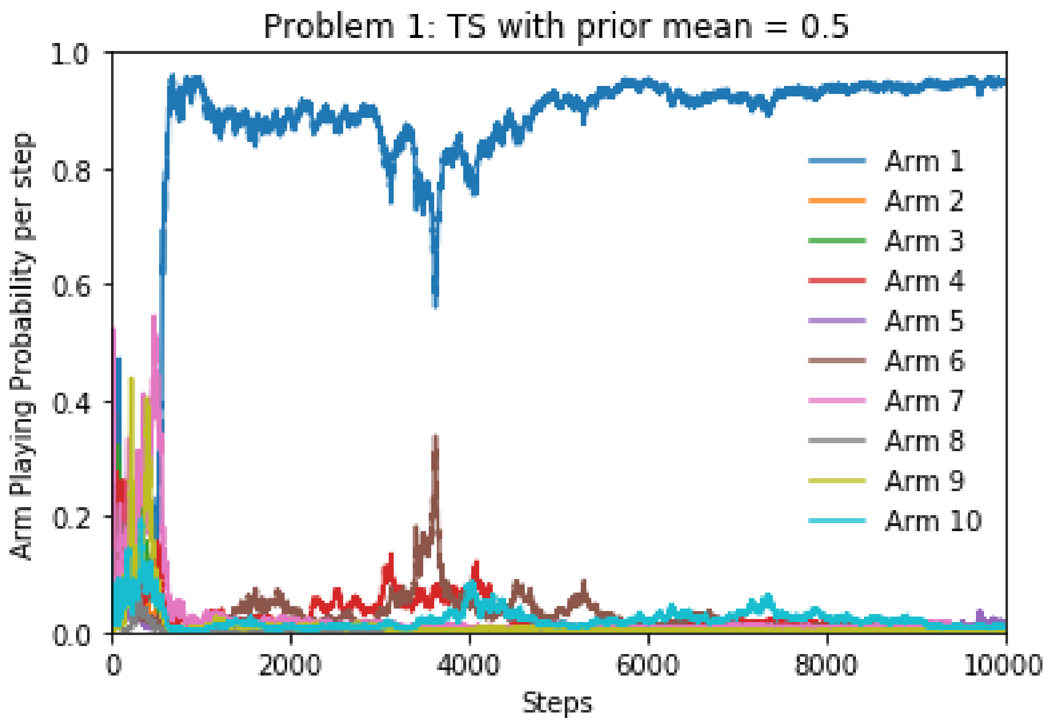
[0.5, 0.2, 0.1] TS with prior mean = 0.5
Arm Playing Probability per step
Steps
Arm 1
Arm 2
Arm 3
Arm 4
Arm 5
Arm 6
Arm 7
Arm 8
Arm 9
Arm 10
0.0
0.2
0.4
0.6
0.8
1.0
0
2000
4000
6000
8000
10000

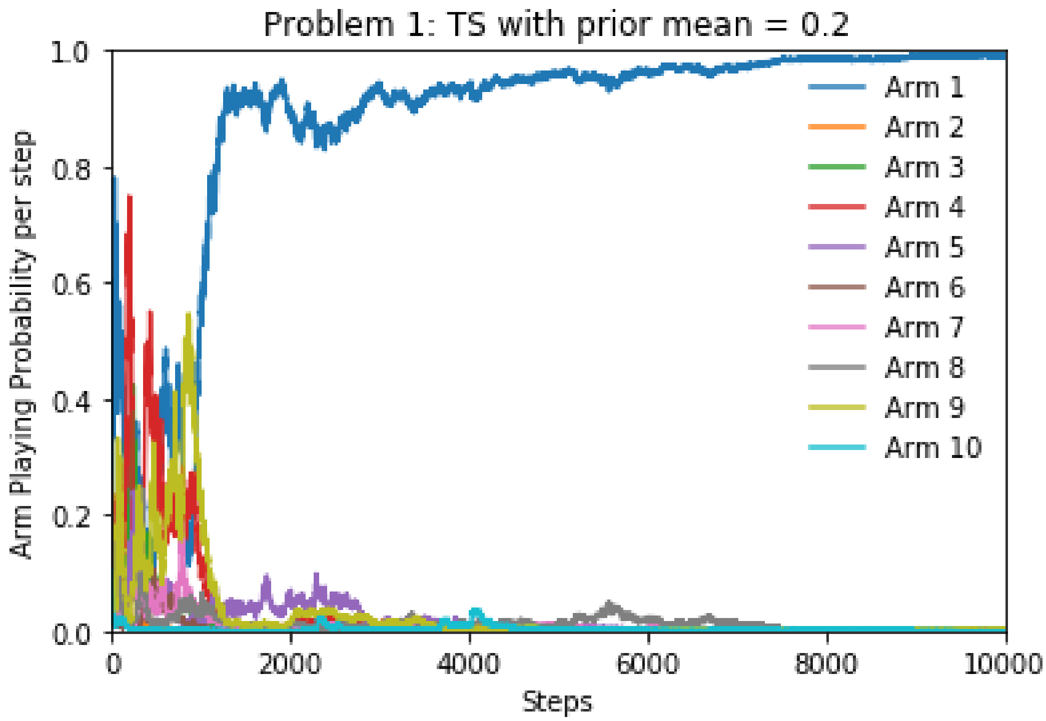



[0.5, 0.2, 0.1] TS with prior mean = 0.2
Arm Playing Probability per step
Steps
Arm 1
Arm 2
Arm 3
Arm 4
Arm 5
Arm 6
Arm 7
Arm 8
Arm 9
Arm 10
0.0
0.2
0.4
0.6
0.8
1.0
0
2000
4000
6000
8000
10000

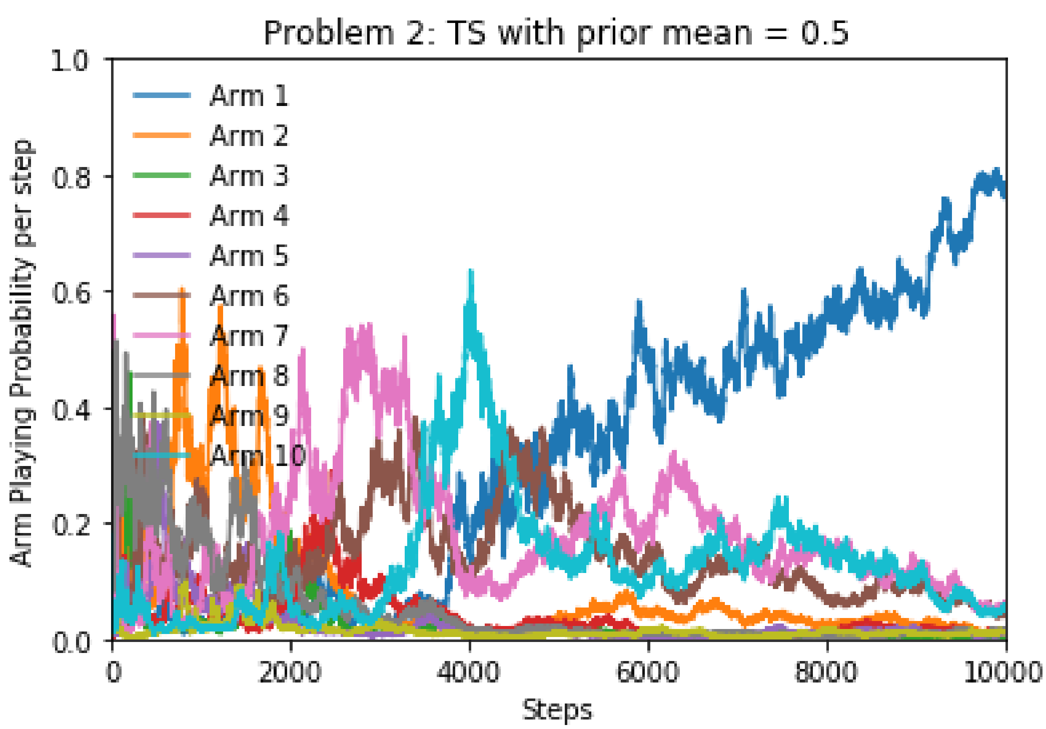
In [21]: 
```
dill.dump_session(filename)
```

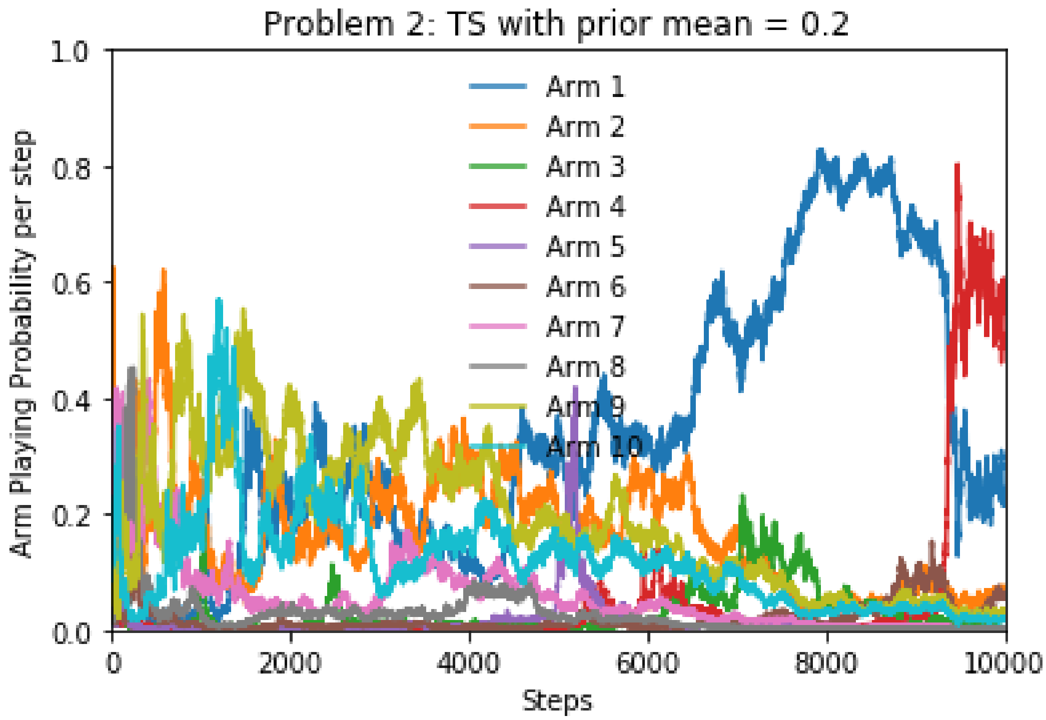
In [23]: 
```
plot_arms_playing_prob(10000,arm_playing_prob3,3)
plot_arms_playing_prob(10000,arm_playing_prob4,4)
```



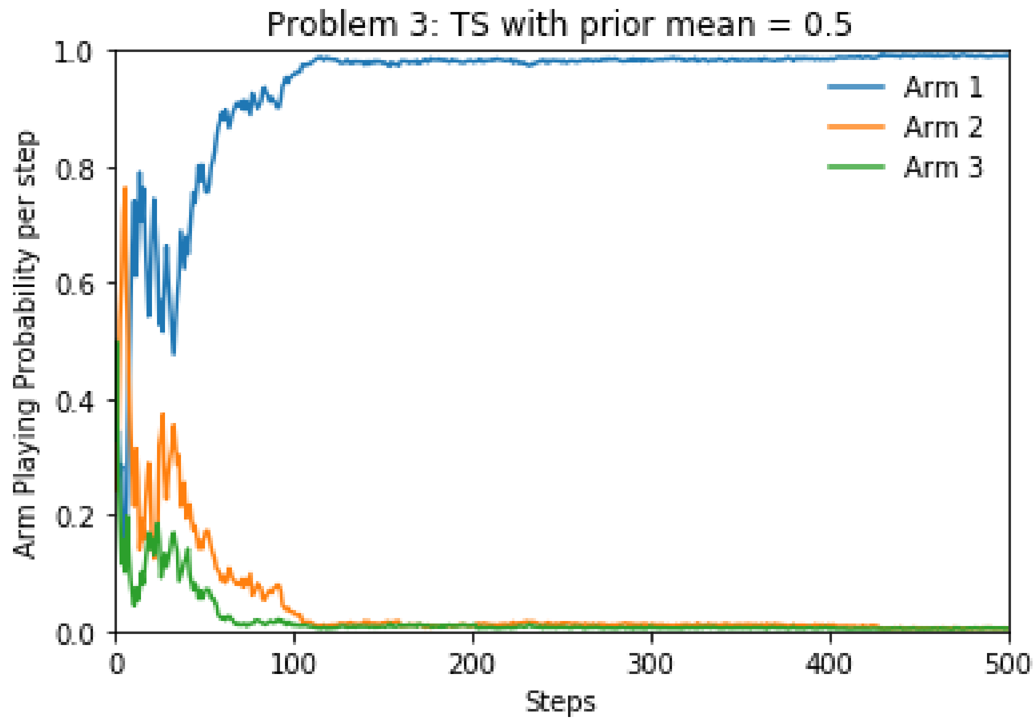


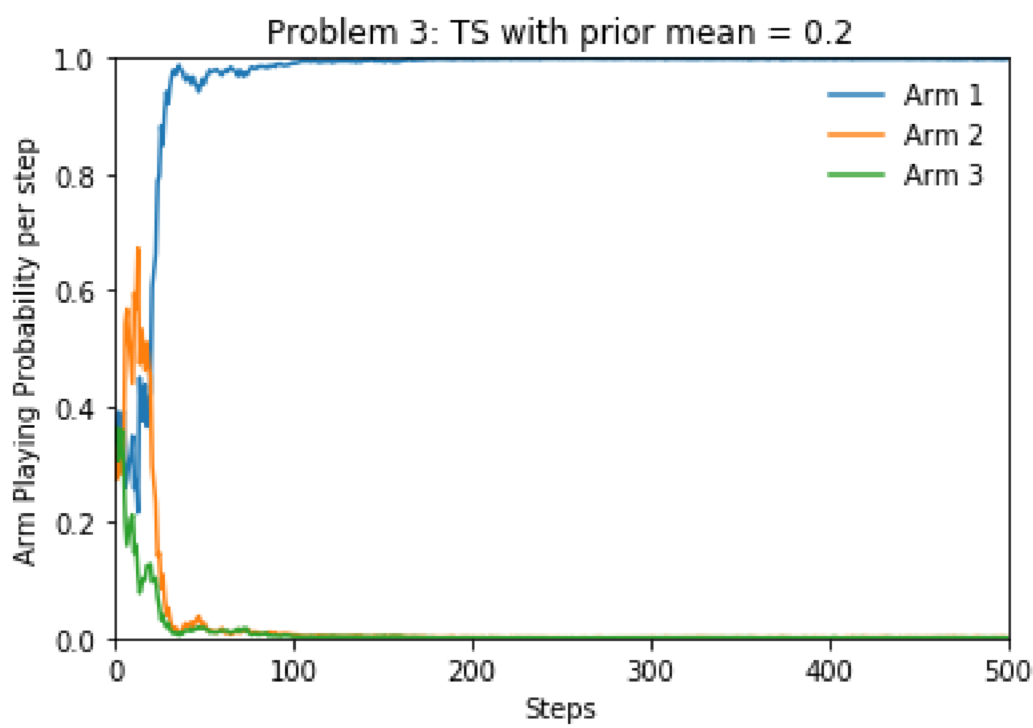

```
In [25]: plot_arms_playing_prob(10000,arm_playing_prob5,5)
         plot_arms_playing_prob(10000,arm_playing_prob6,6)
```






In [30]:
```
plot_arms_playing_prob(10000,arm_playing_prob1,1)
plot_arms_playing_prob(10000,arm_playing_prob2,2)
```


Problem 3: TS with prior mean = 0.5
Arm Playing Probability per step
Steps
Arm 1
Arm 2
Arm 3
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500


Problem 3: TS with prior mean = 0.2
Arm Playing Probability per step
Steps
Arm 1
Arm 2
Arm 3
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500

```
In [32]: plot_arms_playing_prob(10000,arm_playing_prob1,1)
         plot_arms_playing_prob(10000,arm_playing_prob2,2)
```