139. Distance of a point from a plane

Distance of a Point from a Plane

Summary

- Finding the **distance of a point from a plane** is a core linear algebra concept used in machine learning algorithms like **Logistic Regression** and **Support Vector Machines (SVM)**.
- The distance d from a point s to a plane π (passing through the origin) is calculated using the formula: $d = \frac{w^T s}{||w||}$.
- Points located on the **same side** as the weight vector w result in a **positive distance**.
- Points located on the **opposite side** of the weight vector w result in a **negative distance**.
- In classification, the sign of the distance (+ or -) helps categorize data points into different classes .

The Concept of Distance in Classification

In machine learning, specifically for **classification problems**, the goal is often to find a **best-fit line or hyperplane** that effectively splits data into different groups . Calculating the distance of new data points from this boundary is essential for accurate categorization.

Geometry of the Plane

Consider an **n-dimensional plane** π that passes through the origin $(0, 0)$.

- **Equation**: The plane is defined by $w^T x = 0$.
- **Weight Vector (w)**: This vector is **perpendicular (orthogonal)** to the plane.

The Distance Formula

To find the distance between a point s (with coordinates $x_1, x_2, ..., x_n$) and the plane π, we use the following algebraic representation:

$$d = \frac{w^T s}{||w||}$$

- $w^T s$: The dot product of the weight vector and the point vector.
- $||w||$: The magnitude (norm) of the weight vector.

Mathematical Derivation via Dot Product

The numerator $w^T s$ can be expanded using linear algebra as:

$$w^T s = ||w|| \cdot ||s|| \cdot \cos(\theta)$$

Where θ is the angle between the weight vector w and the vector representing point s.

Positive vs. Negative Distance

The algebraic distance indicates which side of the plane a point resides on relative to the weight vector w.

Case 1: Positive Distance (Above the Plane)

If a point s is located "above" the plane (in the same direction as w):

- The angle θ is between 0° **and** 90°.
- $\cos(\theta)$ is positive, resulting in a **positive value** for the distance calculation.

Case 2: Negative Distance (Below the Plane)

If a point s' is located "below" the plane (opposite to the direction of w):

- The angle θ is **greater than** 90° .
- $\cos(\theta)$ becomes negative, resulting in a **negative distance**.

> **Note**: A "negative distance" does not mean a physical distance is less than zero; it is a mathematical convention indicating that the point is on the **opposite side** of the decision boundary .