95. Addition Rule (For Mutual And Non Mutual Exclusive Events)

Probability and Addition Rules

Summary

- **Probability** is defined as determining the likelihood of an event or experiment occurring.
- **Mutually Exclusive Events** are events that cannot occur at the same time, such as getting a head and a tail simultaneously in a single coin toss.
- The **Addition Rule for Mutually Exclusive Events** sums the individual probabilities of the events.
- **Non-Mutually Exclusive Events** are events that can occur simultaneously, such as drawing a card that is both a King and a Heart.
- The **Addition Rule for Non-Mutually Exclusive Events** sums the probabilities of the individual events and subtracts the probability of their intersection (the event where both occur).

Exam Notes

Defining Probability

Question: How should you define probability in an interview or exam setting?

Answer: Probability is the measure of **determining the likelihood of an event or an experiment** occurring.

Introduction to Probability

Probability is fundamentally about determining the likelihood of an event or experiment. It is essential in machine learning, particularly for classification algorithms where outputs are based on probabilities and threshold values to decide class membership.

Basic Examples

- **Coin Toss**: When tossing a coin, the sample space consists of two outcomes: **Head (H)** or **Tail (T)**.
  - The probability of getting a head is calculated as:

$$P(H) = \frac{1}{2} = 50\%$$

  - The probability of getting a tail is also 50%.
- **Rolling a Dice**: When rolling a standard die, there are six possible outcomes $\{1, 2, 3, 4, 5, 6\}$.
  - The probability of rolling a specific number, such as 1 ($x = 1$), is:

$$P(x = 1) = \frac{1}{6}$$

  - This logic applies to any single number on the die.

Mutually Exclusive Events

Mutually exclusive events are defined as two or more events that **cannot occur at the same time**. In a Venn diagram, these events appear as separate circles with no overlapping region.

Examples

- **Coin Toss**: You cannot get both a head and a tail in a single toss; the events are distinct and separate.
- **Rolling a Dice**: You cannot roll a 1 and a 6 simultaneously on a single die.

Addition Rule for Mutually Exclusive Events

To find the probability of either event A or event B occurring when they are mutually exclusive, you add their individual probabilities. This is known as the **Additive Rule**.

Formula:

$$P(A \text{ or } B) = P(A) + P(B)$$

Example Calculation (Coin Toss): To find the probability of getting a Head (H) or a Tail (T):

$$P(H \text{ or } T) = P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1$$

Example Calculation (Dice Roll): To find the probability of rolling a 1 or a 5:

$$P(1 \text{ or } 5) = P(1) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Non-Mutually Exclusive Events

Non-mutually exclusive events are events that **can occur at the same time**. There is an overlap between the events where both conditions are met simultaneously.

Example: Deck of Cards

When drawing a single card from a deck of 52 cards, consider the events of drawing a **King (K)** or a **Heart ($\heartsuit$)**.

- It is possible to draw a card that is both a **King** and a **Heart** (the King of Hearts).
- Because this combination exists, the events are non-mutually exclusive.

Addition Rule for Non-Mutually Exclusive Events

When events overlap, simply adding probabilities counts the intersection twice. To correct this, you must subtract the probability of the intersection (where both events happen).

Formula:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example Calculation (Cards): Calculate the probability of drawing a King (K) or a Heart ($\heartsuit$):

1. **Probability of King ($P(K)$)**: There are 4 Kings in a deck of 52.

$$P(K) = \frac{4}{52}$$

2. **Probability of Heart ($P(\heartsuit)$)**: There are 13 Hearts in a deck of 52.

$$P(\heartsuit) = \frac{13}{52}$$

3. **Probability of King AND Heart ($P(K \text{ and } \heartsuit)$)**: There is only 1 King of Hearts.

$$P(K \text{ and } \heartsuit) = \frac{1}{52}$$

Final Calculation:

$$P(K \text{ or } \heartsuit) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

96. Probability-Multiplication Rule(Independent And Dependent Events)

Probability Multiplication Rule: Independent & Dependent Events

Commands

- No software commands were used in this lecture.

Summary

- The **Multiplication Rule** is used specifically for determining the probability of **Independent** and **Dependent** events occurring together.
- **Independent Events** are events where the outcome of one does not affect the outcome of the other (e.g., tossing a coin multiple times).
- **Dependent Events** are events where the outcome of the first event affects the probability of the second event (e.g., drawing cards from a deck without replacement).
- The formula for **Independent Events** multiplies individual probabilities: $P(A \text{ and } B) = P(A) \times P(B)$.
- The formula for **Dependent Events** involves **Conditional Probability**: $P(A \text{ and } B) = P(A) \times P(B|A)$.
- **Conditional Probability** is a foundational concept for **Bayes' Theorem** and the **Naive Bayes** machine learning algorithm.

Exam Notes

Conditional Probability

Question: What is Conditional Probability and where is it applied?

Answer: **Conditional Probability** is the likelihood of an event occurring given that another event has already occurred. It is a critical concept in **statistics** and is frequently asked about in **data science interviews**. It serves as the mathematical foundation for **Bayes' Theorem** and the **Naive Bayes** classification algorithm in machine learning.

Independent Events

Two events are considered **Independent** if the occurrence of one does not affect the probability of the other occurring.

Example: Tossing a Coin

When tossing a coin multiple times, the result of the first toss does not influence the result of the second toss.

- **First Toss**: Probability of Heads (H) is $1/2$.
- **Second Toss**: Even if the first was Heads, the probability of Tails (T) on the second toss remains $1/2$.

Example: Rolling a Dice

Similarly, rolling a dice is an independent event. The probability of rolling a 1 does not affect the probability of rolling a 2 on the next roll; both remain $1/6$.

Multiplication Rule Formula (Independent)

For independent events, we replace the "and" operation with multiplication.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Calculation Example: What is the probability of getting a Head (H) on the first toss **and** a Tail (T) on the second toss?

$$P(H \text{ and } T) = P(H) \times P(T) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Dependent Events

Two events are considered **Dependent** if the occurrence of the first event affects the probability of the second event. This often occurs in sampling **without replacement**.

Example: Drawing Cards from a Deck

Consider the task of drawing a **King (K)** first, and then drawing a **Queen (Q)** from a standard 52-card deck without putting the King back.

1. **First Draw (King)**:
    - Total cards: 52.
    - Number of Kings: 4.
    - Probability $P(K) = \frac{4}{52}$.
2. **Second Draw (Queen)**:
    - Because the King was not returned, the total number of cards is now **51**.

- Number of Queens: 4.
- Probability $P(Q) = \frac{4}{51}$.

Multiplication Rule Formula (Dependent)

For dependent events, the formula must account for the change in the sample space. This uses **Conditional Probability**, denoted as $P(B|A)$ (Probability of B given A).

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Calculation Example: What is the probability of drawing a King (K) and then a Queen (Q)?

$$P(K \text{ and } Q) = P(K) \times P(Q|K)$$

$$P(K \text{ and } Q) = \frac{4}{52} \times \frac{4}{51}$$

Connection to Machine Learning

This concept of dependent events and conditional probability is directly linked to **Bayes' Theorem**. It is heavily utilized in the **Naive Bayes** machine learning algorithm for classification tasks.